

Blending World Map Projections

Bernhard Jenny, Oregon State University, USA

Tom Patterson, US National Park Service, USA

ABSTRACT: The idea of designing a new map projection via combination of two projections is well established. Some of the most popular world map projections in use today were devised in this manner. One construction method is to combine two source projections along a common parallel (for example, Goode's homolosine combines the sinusoidal and the Mollweide projections). A second method calculates the arithmetic means of two projections (for example, the Winkel Tripel projection).

These two methods for creating new world map projections are included in the latest version of *Flex Projector*. *Flex Projector*, a freeware mapping application, offers a graphical approach for customizing existing projections and creating new projections. The *Mixer* is a new feature in the latest version that allows the user to blend two existing projections to create a new hybrid projection. Additional to the two established combination methods, the software includes a new method for blending projections specific to its visual design approach. With this new method, a unique trait of one projection is transferable to a second projection. For example, the straight parallels of the Eckert IV projection are replaceable with the arced parallels of the Winkel Tripel projection. *Flex Projector* allows for the blending of four different projection traits separately or in combination: (1) the horizontal length of parallels, (2) the vertical distance of parallels from the equator, (3) the distribution of meridians, and (4) the bending of parallels.

With this new approach to projection blending, creating new projections becomes simple and easy to control. The integration of the three methods into *Flex Projector* allows the user to transform raster and vector data to new projections, and to evaluate distortion characteristics of new projections. As an applied example, the paper introduces the new Pacific projection that is a blend of the Ginzburg VIII and Mollweide projections.

KEYWORDS: projection design, projection blending, *Flex Projector*, Pacific projection

Introduction: Visual design of world map projections

Flex Projector (www.flexprojector.com) is a, free, open-source, cross-platform application with a graphical user interface for designing world map projections. This article discusses the *Mixer*, a feature in *Flex Projector* for selectively combining two projections. The *Mixer* complements the other design tools found in *Flex Projector* and further simplifies the creation of new world map projections.

When developing *Flex Projector*, the goal was to give users without expert knowledge in mathematics an accessible tool for designing world map projections. The application creates pseudocylindrical and cylindrical projections, as well as polyconical projections with curved parallels. It allows users to shape the graticule, and provides visual and numerical feedback for assessing distortion properties. The design of the graphical user interface was done from the end user's perspective—ease-of-use and encouraging experimentation were priorities (Jenny and Patterson, 2007; Jenny et al., 2008). The intended users of *Flex Projector* are practicing mapmakers and cartography students. Details for the mathematics and algorithms that convert user settings to formulae for projecting digital data are covered in Jenny et al. (2010).

The inspiration for developing *Flex Projector* was Arthur Robinson's graphical approach to projection design. In 1961, working on a commission for the Rand McNally publishing house, Robinson created his eponymous world map projection, originally dubbed the Orthophanic, meaning correct-looking (Robinson, 1974). Robinson proceeded through an iterative process to create his pseudocylindrical projection, graphically evaluating the appearance and relative relationships of landmasses. He first estimated the values for parallel lengths and spacing, then the projection was drawn and the continents plotted. When he found early drafts less than satisfactory, compensating adjustments to the graticule were made and the continents re-plotted. This iterative process, a sort of graphic successive approximation, was repeated until it became obvious that further adjustments would produce no improvement, at least to the eyes of the author (Robinson, 1974, p. 151-152). Others agreed with Robinson. His projection has since become widely popular for making world maps, used by National Geographic Society (Garver, 1988) and other respected cartographic establishments.

The appeal of the Robinson projection is due in large part to the pleasant appearance of the graticule and the major landmasses. It presents the world in a handsome, partially oval container; the continents within it look correct in size and shape to most readers. The success of the Robinson projection is largely due to the fact that it is a compromise projection, that is, it neither preserves angles nor areas. Because designing projections always involves compromises, a projection adhering strictly to the conformal or equal-area property by necessity must distort geographic shapes, often grossly. Conformality (the preservation of angles) is a property ill suited to general world maps (Canters, 2002). In contrast, most cartographers value the equal-area property, as the comparison of area sizes is made easier. Additionally, some cartographic methods require an equal-area base, for example, choropleth maps (showing values usually normalized by area by differently shaded areas) or dot maps (where the relative density of dots changes with areal distortion). However, when strict adherence to the equal-area property is not required, a compromise projection often shows the shapes of continents with a more pleasant appearance than equal-area projections (Canters, 2002). *Flex Projector* and its *Mixer* feature allow cartographers to design compromise projections that balance the competing priorities of equal-area fidelity and pleasing appearance.

Upon opening *Flex Projector*, the user sees a graphic user interface comprised of three components (Figure 1). The panel in the upper left is a world map in the Robinson projection, the default. To the right of the map is a panel with sliders that control the

shape of the projection, and tempting the user to experiment. Moving any of the sliders results in an immediate change to the Robinson projection, which then ceases to be a Robinson projection and starts on its way to becoming an entirely new projection. Four groups of sliders exist, for adjusting the length of parallels, their vertical distribution, their bending, and the distribution of meridians. Below the map is a table with distortion indices, which reports in real-time the amount of distortion contained in the modified projection, including comparisons to common world map projections. The tools in *Flex Projector* provide a means to design projections in the same manner as Robinson did nearly 50 years ago—more accurately, quickly, and with much less tedium.

This paper extends this graphical approach and introduces graphical tools for blending existing projections to create new world projections. Three different methods for blending projections are included, which offer complementary approaches to the design of world map projections, and are often faster and easier to control than the method described above.

The discussion is structured as follows: First, we examine existing methods for combining projections applied in the past to create a variety of projections. We identify three groups: combining along lines of latitude, arithmetic means, and interpolating with varying weights. The second section discusses three methods implemented in *Flex Projector*, including a new approach allowing the user to combine selected traits of two projections to create a new projection. The last section then discusses the design and characteristics of the Pacific projection, before we conclude with a few final remarks.

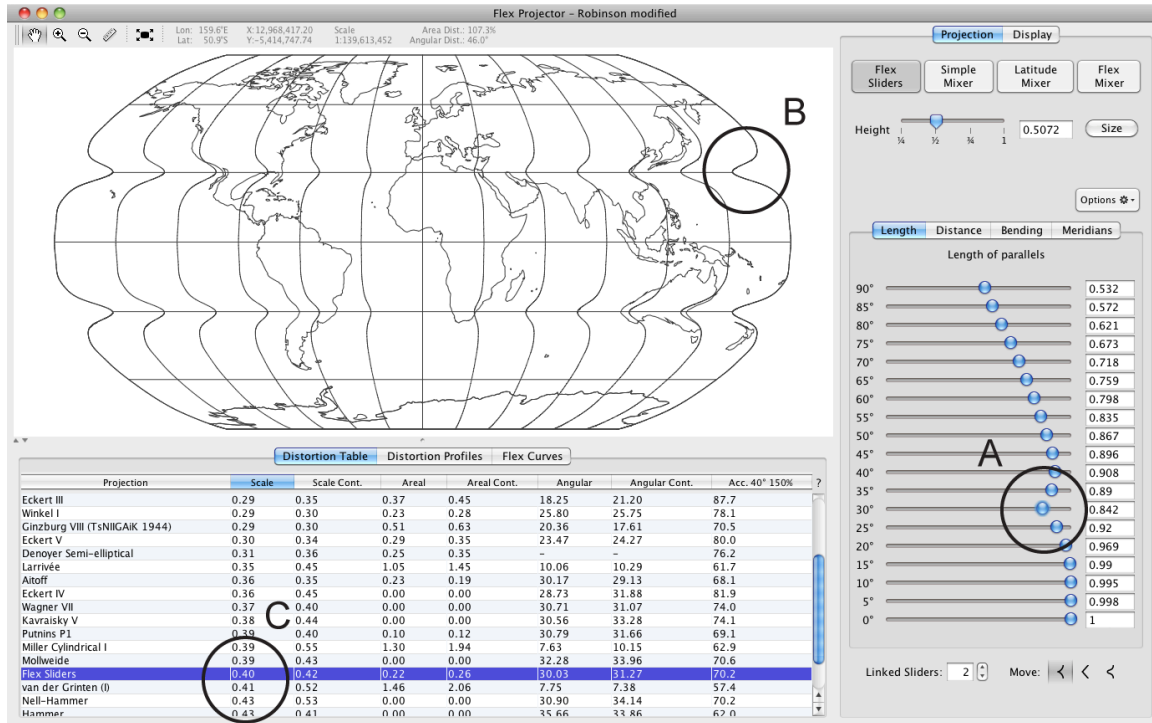


Figure 1: Screenshot of *Flex Projector*: Moving sliders changes the length of parallels based on increments of five degrees of latitude (A) which in turn changes the projection shape (B) and the distortion ranking (C).

Combining projections

The idea of designing a new map projection by combining two existing projections is well established. Some of the most popular world map projections in use today were devised in this manner. In general, the goal is to merge the desired characteristics of two projections, while eliminating disadvantages. For example, the pointed poles of the sinusoidal projection add considerable angular distortion to polar areas, while projections with a polar line, such as the Robinson projection, introduce less shape distortion at poles. To date, cartographers have developed various techniques for combining world map projections, which can be grouped in three categories.

1. Combined projections

The first group are hybrid projections made by fusing together parts of other projections. For world map projections, this typically involves joining two pseudocylindrical projections along a common parallel. For example, Goode (1925) combined the Sanson sinusoidal and the Mollweide projection at $40^{\circ} 44' 12''$ north and south latitude, which is the latitude of equal scale. The resulting Goode homolosine projection is most common in its interrupted form. Others proposing non-continuous combined projections include, for example, Érdi-Krausz (1968), Hatano (1972), and McBryde (1978) (also see Canters, 2002, p. 154, and Snyder 1993, p. 217–220 for overviews). A trait of most non-continuous combined projections is a discontinuity in the first derivatives at the latitude where the two projections join. This typically appears as a sharp crease where the meridians meet. This discontinuity can be visually disturbing, especially when it is concave, as is the case for Goode's homolosine projection. For some source projections, mathematical methods exist for eliminating the discontinuities in the destination projection. For example, Gede (2011) eliminates the visual join of the Érdi-Krausz projection.

2. Arithmetic means of two projections

Calculating the arithmetic means of two different projections is the technique for devising a large number of projections. The two “starter” projections are often a cylindrical projection, such as the plate carrée, and a pseudocylindrical projection with meridians converging at pole points. Examples include projections devised by Eckert (1906), Putniņš (1934), and Winkel (1921). For example, the Winkel Tripel projection is the arithmetic mean of an equirectangular projection and the Aitoff projection (Winkel, 1921; Snyder, 1993, pp. 231–232); and Eckert V is an average of the plate carrée and the sinusoidal (Eckert, 1906). Foucaut (1862), Hammer (1900), Nell (1929) and Tobler (1973) have averaged the cylindrical equal area and the sinusoidal (after Snyder, 1977). Some of these projections are equal area, which is only possible if either the x or y coordinate is not obtained by averaging the two source projections, but by calculation from an equation. This is also the technique used by Gall (1929) for his combination of the sinusoidal and the Mollweide.

3. Interpolating projections with varying weight

Interpolating projections using variable weighting is an extension of the previous technique. In examples discussed by Anderson and Tobler (s.d.), the imposed weighting varies with the latitude, decreasing from one at the equator to zero at the poles. Tobler (1973) has applied this technique to create various equal-area projections.

It remains to be mentioned that a variety of alternative methods for modifying a single projection exist. Canters (2002, p.115ff) distinguishes between polynomial transformations for projections, and the modification of projection parameters, including Wagner's powerful *Unbeziffern* (or re-numbering) method (Wagner, 1949). These methods modify a single source projection to create a new projection, and do not combine two source projections.

The *Flex Projector Mixer*

Flex Projector offers three methods to combine projections. The first method joins two projections along a selected latitude; the second method computes an arithmetic means of two projections; and the third method is a new approach to combine selected characteristics of two projections. As described in the introduction of this paper, *Flex Projector* aims at providing a graphical approach to the design of map projections. For all three methods the user loads two map projections from pop-up lists on the right side of the main window (Figure 2). Moving the sliders at the top right interactively combines the loaded projections. Moving a slider to the left or right proportionally controls the influence of each projection. As the user experiments, changes appear instantaneously on the large composite world map (Figures 2 and 4).

1. Combined projections

With the *Latitude Mixer*, the user can choose the latitude along which the two projections are combined. The shared parallel generally does not have the same length with both projections, which requires one of the projections to be scaled. The scale factor is normally computed automatically, or can be adjusted manually (although this option is probably only useful for didactical purposes). A second slider defines a latitude band for linearly interpolating around the shared parallel, which can smooth the crease along the parallel where the two projections join. A third slider adjusts the height-to-width ratio of the combined projection.

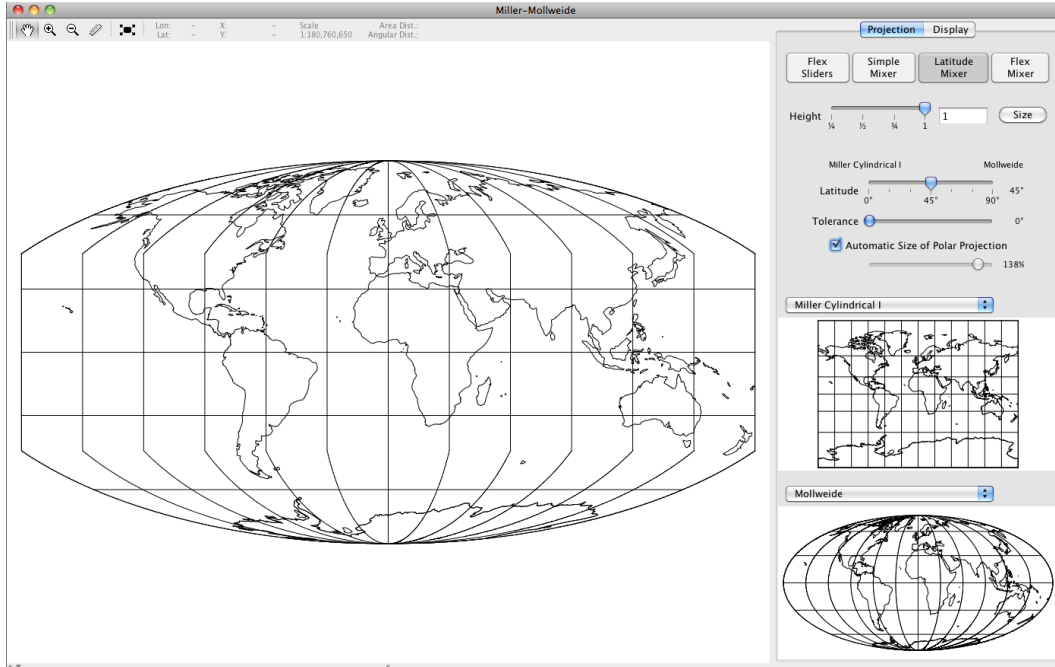


Figure 2: *Latitude Mixer* panel in *Flex Projector* combining the Miller Cylindrical I (center right) and Mollweide (bottom right) projections at 45 degrees latitude.

2. Arithmetic means of two projections

The *Simple Mixer* computes the means of two source projections. For example, the “DNA” of the new projection depicted in Figure 3 is 35 percent Ginzburg VIII and 65 percent Eckert IV. The Ginzburg VIII projection was chosen because of its appealing depiction of landmasses at moderate latitudes. The equal-area Eckert IV was chosen to compensate the overly large polar areas of the Ginzburg VIII. Additionally, scaling the height-to-width ratio to 0.9 depicts major landmasses with more graphically pleasing proportions.

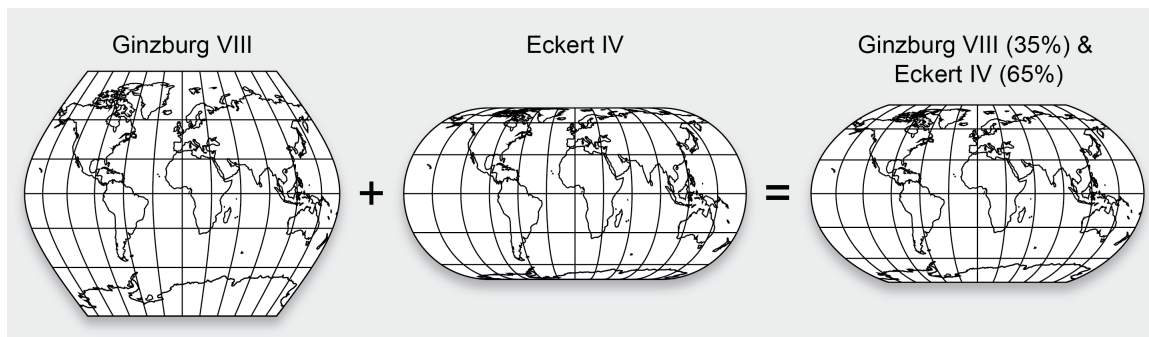


Figure 3: A projection created by computing the weighted means of the Ginzburg VIII (35%) and the Eckert IV (65%) projections and scaling vertical coordinates by the factor 0.9.

The authors also experimented with options for graphically adjusting blending weights with latitude, the third idea for combining projections discussed in the previous section. Interactive spline curves—similar to the Curve Adjustments panel in Adobe Photoshop—

were added to a prototype version of *Flex Projector*. The curves allowed users to adjust the weight with latitude. Because the functionality of this tool is relatively difficult to grasp for novice users, and because the effect is difficult to control, the current version of *Flex Projector* does not include this option.

3. *Selective combinations*

This third technique for creating projections selectively transfers a unique trait from one projection to another, such as replacing the straight parallels of the Eckert IV projection with the arced parallels of the Winkel Tripel projection. No other characteristics of the Eckert IV would change.

Four different projection properties can be combined: (1) the horizontal length of parallels, (2) the vertical distance of parallels from the equator, (3) the distribution of meridians, and (4) the bending of parallels. The user can adjust weights using four sliders (Figure 4 top right). For example, when setting the weight for the horizontal length of parallels to 30%, the parallel lengths of the mixed projection are a combination of 30% of the parallel length of the first source projection and 70% of the second projection. The same principle applies to the other three properties. If neither of the two source projections has bent parallels or irregularly distributed meridians, mixing these attributes would not change the final projection and the corresponding sliders are accordingly deactivated.

The simple graphical interface hides an algorithm from the user that proceeds in three steps. In the first step, *Flex Projector* “deconstructs” the two selected source projections by converting them into tabular form. A projection is commonly defined by a pair of transformation formulae with the form $X = f(\varphi, \lambda)$ and $Y = g(\varphi, \lambda)$ that convert longitude λ and latitude φ into projected Cartesian X/Y coordinates. The first step creates four tables of numerical values for each projection using the corresponding pair of transformation formulae. Three of the resulting tables contain values for every 5 degrees of increasing latitude (the length, vertical distribution and bending of parallels) and one table contains values for every 15 degrees of increasing latitude (the horizontal distribution of meridians). Both source projections are converted to these tabular forms, resulting in 2×4 tables.

In a second step, the four pairs of tables are blended, using the four user-defined weights. The four pairs of tables are merged to four blended tables by computing a weighted average of each pair of corresponding tabular values.

The final third step converts geographical longitude / latitude coordinates to Cartesian X/Y coordinates. The conversion interpolates spline curves through the values stored in the four tables, and then applies an extended version of the method presented by Robinson (1974) for projecting geographic coordinates to Cartesian coordinates (see Jenny et al. (2010) for details).

After mixing the four different projection traits with this method, the user may then fine-tune the new projection by adjusting individual values of one of the four tables with *Flex Projector's* graphical user interface, as shown in Figure 1.

It must be mentioned that the described technique does not work perfectly for all projections. The reason is that the first step in the algorithm (the transformation from formulae to tabular values) sometimes does not accurately replicate the original projection. While cylindrical and pseudocylindrical projections with regularly distributed meridians are matched perfectly, only approximate transformations are possible for polyconic projections (with arcing parallels), or projections with irregularly spaced meridians.

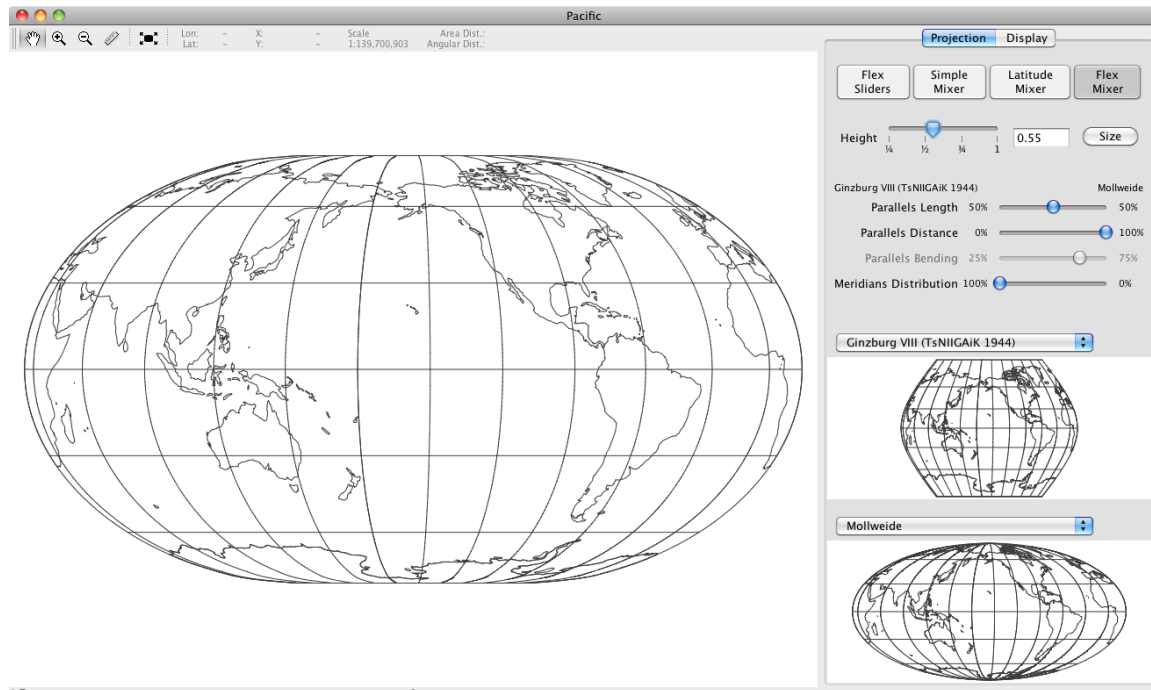


Figure 4: *Flex Mixer* panel in *Flex Projector*: The Pacific projection (left) is a blend of the Ginzburg VIII (center right) and the Mollweide (bottom right) projections. Sliders at the top right control the blending of the three active parameters (with blue buttons) that define the hybrid Pacific projection.

The Pacific projection

As a practical example on how to use the *Mixer*, we created the hybrid *Pacific* projection by combining the Ginzburg VIII and Mollweide projections (Figure 4). The design intent was a world map with a rather conventional appearance centered on 160 degrees west longitude to focus on the Pacific Ocean, and with relatively little areal distortion. We chose the Mollweide projection because it is oval in shape, which complements the roundness of the Pacific basin, and equal-area. The Ginzburg VIII contributes unevenly distributed meridians that are widely spaced at the projection center and compressed at the map margins, a useful feature for emphasizing the Pacific in relation to other parts of the world. Creating the Pacific projection involved adjustments to three parameters controlled by the sliders at the top right of the *Mixer* panel (Figure 4). Because neither the Ginzburg VIII nor Mollweide projection has parallels that bend, this parameter is disabled in the graphical user interface. Adjustments to the sliders included:

Length of Parallels – Setting the slider in the middle at 50% gives equal weight to the Ginzburg VIII and Mollweide projections for this parameter. This combination gives the Pacific projection highly rounded pole lines that merge smoothly into the lateral meridians.

Distance of Parallels – Setting the slider at 100% by dragging it all the way to the right toward the Mollweide projection weighted this parameter entirely from that projection. The Pacific projection as a result has a Mollweide-like vertical distribution of parallels.

Distribution of Meridians – Setting the slider at 0% by dragging it all the way left weights this parameter entirely toward the Ginzburg VIII projection, thus increasing the area of the Pacific Ocean because the meridians near the center point are more widely spaced than those at the map margins.

Once work in the *Mixer* is finished, the user can further enhance the combined projection using the other tools in *Flex Projector*. In the case of the Pacific projection, we increased the height-to-width proportions from 0.507 to 0.55 to make the map taller and accentuate the round shape of the Pacific Ocean (Figure 5). By saving the final Pacific projection as a small text file we could use it again in *Flex Projector* for producing publishable-quality maps with imported shapefiles and raster geodata.

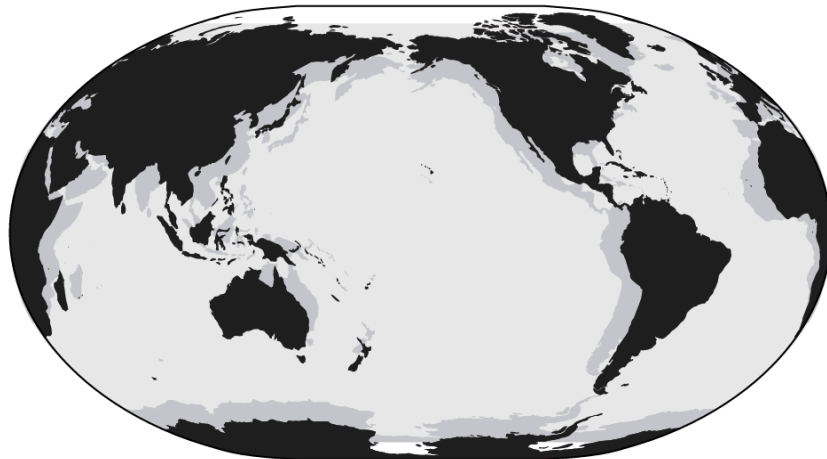


Figure 5: The Pacific projection (black) overlaid on the Robinson projection (gray) with each projection having the same width. The taller Pacific projection devotes relatively more area to the Pacific Ocean than does the Robinson.

Projections created with the *Mixer* exhibit distortion values similar to the projections from which they derive. For example, because the Pacific projection is part Mollweide projection, which is equal-area, it ranks favorably for areal distortion. It is not free of areal distortion, however, because of the influence of the non-equal-area Ginzburg VIII. The *Flex Projector* distortion tables (Figure 6) show how the Pacific projection ranks against common world map projections for the other distortion categories—it is unexceptional. When comparing the distortion values in Figure 6 lower values are better for all categories except for the *Acceptance Index* (far right column), where higher is better. The tint in Figure 7 shows the extent of acceptable area on the Pacific projection. The acceptable area is the area with an angular distortion of less than 40° and areal

distortion between 66% and 150%. For a full explanation of the distortion tables and acceptance index refer to Jenny et al. (2010).

Projection	Scale	Scale Cont.	Areal	Areal Cont.	Angular	Angular Cont.	Acc. 40° 150%
Cylindrical Equal-Area	0.57	0.82	0.00	0.00	30.90	38.53	71.9
Nell-Hammer	0.43	0.53	0.00	0.00	30.90	34.14	70.2
Eckert IV	0.36	0.45	0.00	0.00	28.73	31.88	81.9
Sinusoidal (Sanson-Flamsteed)	0.51	0.49	0.00	0.00	39.01	38.24	57.1
Kavraysky V	0.38	0.44	0.00	0.00	30.56	33.28	74.1
Mollweide	0.39	0.43	0.00	0.00	32.28	33.96	70.6
Goode Homolosine	0.46	0.46	0.00	0.00	36.39	36.21	58.9
Wagner VII	0.37	0.40	0.00	0.00	30.71	31.07	74.0
Putnins P1	0.39	0.40	0.10	0.12	30.79	31.66	69.1
Winkel II	0.27	0.31	0.17	0.23	24.20	25.66	87.4
Flex	0.37	0.37	0.17	0.10	30.68	29.78	70.5
Winkel Tripel	0.26	0.26	0.18	0.26	23.28	22.77	85.0
Robinson	0.27	0.32	0.19	0.23	21.26	23.74	84.3
Winkel I	0.29	0.30	0.23	0.28	25.80	25.75	78.1
Aitoff	0.36	0.35	0.23	0.19	30.17	29.13	68.1
Denoyer Semi-elliptical	0.31	0.36	0.25	0.35	-	-	76.2
Kavraysky VII	0.23	0.27	0.28	0.38	19.15	20.88	86.5
Eckert V	0.30	0.34	0.29	0.35	23.47	24.27	80.0
Wierenskiold I	0.40	0.45	0.31	0.31	31.54	32.93	72.4
Wagner VI	0.26	0.32	0.34	0.48	20.41	22.52	80.4
Eckert III	0.29	0.35	0.37	0.45	18.25	21.20	87.7
Fahey	0.25	0.29	0.43	0.56	16.14	17.32	83.2
Ginzburg VIII (TsNIIGAIK 1944)	0.29	0.30	0.51	0.63	20.36	17.61	70.5
Equidistant Cylindrical (Plate Carrée)	0.29	0.41	0.57	0.82	16.84	21.48	74.3
Larrivée	0.35	0.45	1.05	1.45	10.06	10.29	61.7
Miller Cylindrical I	0.39	0.55	1.30	1.94	7.63	10.15	62.9
van der Grinten (I)	0.41	0.52	1.46	2.06	7.75	7.38	57.4
Mercator	0.57	0.82	4.79	7.83	0.00	0.00	57.4

Figure 6. *Flex Projector* distortion tables. The Pacific projection is the highlighted row identified as “Flex.”

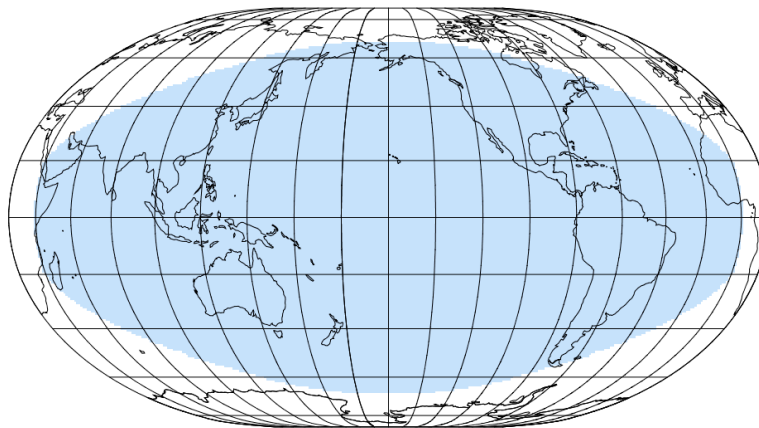


Figure 7. The tint indicates the area of acceptable angular and areal distortion for the Pacific projection. Most of the Pacific with the exception of waters adjacent to Antarctica show angular distortion of less than 40° and areal distortion between 66% and 150%.

Conclusion

Flex Projector simplifies the design of new hybrid projections. It can selectively blend individual characteristics of existing projections. If necessary, the blended projection can serve as a good starting point for additional fine-tuning using the graphical user interface for adjusting the curves and other options (as shown in Figure 1). It is also possible to load a projection created in the *Mixer* back into the *Mixer* to combine it again with other projections. Repeating this process can yield an almost infinite variety of new projections.

In the *Mixer*, it is easy to design new projections that not only are visually pleasing, but also have excellent distortion characteristics. For example, the two equal-area projections blended in Figure 8 yield a new pseudocylindrical projection with an overall shape similar to the Robinson projection and with less areal distortion (0.05 vs. 0.19, 0.0 is equal-area), albeit at the expense of additional deformation for continental shapes. According to Anderson and Tobler (s.d.) ‘blended map projections are splendid projections’. We think that users of the *Flex Projector Mixer* will come to the same conclusion.

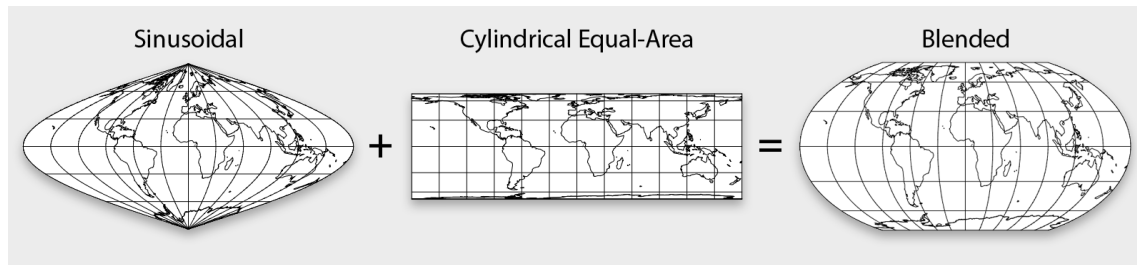


Figure 8. Combining two equal area projections (left and middle) produces a blended hybrid (right) that is nearly equal area.

References

- Anderson, P.B. and Tobler, W.R. (s. d.). Blended map projections are splendid projections, http://www.geog.ucsb.edu/~tobler/publications/pdf_docs/inprog/BlendProj.pdf (accessed 3 August 2011).
- Boggs, S. (1929). A new equal-area projection for world maps, *Geographical Journal*, 73-3, pp. 241–245.
- Canter, F. (2002). *Small-scale Map Projection Design*, Taylor & Francis, London.
- Eckert, M. (1906). Neue Entwürfe für Weltkarten. *Petermanns Mitteilungen*, 52-5, pp. 97–109.
- Érdi-Krausz, G. (1968). Combined equal-area projections for world maps, *Hungarian Cartographical Studies*, pp. 44–49.
- Foucaut, H.C. de Prépetit (1862). *Notice sur la construction de nouvelles mappemondes et de nouveaux atlas de géographie*, Arras, France.
- Garver, J.B. (1988). New perspective on the world, *National Geographic*, 174, pp. 910–913.
- Gede, M. (2011) Optimising the distortions of sinusoidal-elliptical composite projections. A. Ruas (ed.), *Advances in Cartography and GIScience*. Volume 2: Selection from ICC 2011, Paris, Lecture Notes in Geoinformation and Cartography 6, DOI 10.1007/978-3-642-19214-2_14, Springer-Verlag Berlin Heidelberg 2011, pp. 209–225.

- Goode, J.P. (1925). The homolosine projection: a new device for portraying the earth's surface entire, *Annals of the Association of American Geographers*, 15-3, pp. 119–125.
- Hammer, E. (1900). Unechtzylindrische and unechtkonische flächentreue Abbildungen, *Petermanns Geographische Mitteilungen*, 46, pp. 42–46.
- Hatano, M. (1972). Consideration on the projection suitable for Asia-Pacific type world map and the construction of elliptical projection diagram, *Geographical Review of Japan*, 45-9, pp. 637–647.
- Jenny, B. and Patterson, T. (2007). *Flex Projector*, <http://www.flexprojector.com> (accessed 3 August 2011).
- Jenny, B., Patterson, T. and Hurni, L. (2008). Flex Projector—interactive software for designing world map projections, *Cartographic Perspectives*, 59, pp. 12–27.
- Jenny, B., Patterson, T. and Hurni L. (2010). Graphical design of world map projections. *International Journal of Geographic Information Science*, 24-11, p. 1687–1702.
- McBryde, F.W. (1978). A new series of composite equal-area world maps projections, International Cartographic Association, 9th International Conference on Cartography, College Park, Maryland, Abstracts, pp. 76–77.
- Nell, A.M. (1929). Äquivalente Kartenprojektionen, *Petermanns Geographische Mitteilungen*, 36, pp. 93–98.
- Putniņš, R.V. (1934). Jaunas projekci jas pasaules kartēm, *Geografiski Raksti, Folia Geographica 3 and 4*, p. 180–209. [Latvian with extensive French résumé].
- Robinson, A. (1974). A new map projection: Its development and characteristics, in *International Yearbook of Cartography*, ed. by Kirschbaum G.M. and Meine K.-H., pp. 145–155, Kirschbaum, Bonn-Bad Godesberg, Germany.
- Snyder, J.P. (1977). A comparison of pseudocylindrical map projections, *The American Cartographer*, 4-1, pp. 59–81
- Snyder, J.P. (1993). *Flattening the Earth: Two Thousand Years of Map Projections*, Chicago, University of Chicago Press.
- Tobler, W.R. (1973). The hyperelliptical and other new pseudo cylindrical equal area map projections, *Journal of Geophysical Research*, 78-11, pp. 1753–1759.
- Wagner, K. (1949). *Kartographische Netzentwürfe*, Leipzig: Bibliographisches Institut.
- Winkel, O. (1921). Neue Gradnetzkombinationen, *Petermanns Mitteilungen*, 67, pp. 248–252.