RESOLUTION REVISITED

Ferenc Csillag
Department of Geography, Syracuse University
Syracuse, NY 13244
(FCSILLAG@SUNRISE.bitnet)

ABSTRACT

This paper discusses the nature of models applied primarily for environmental data, where, theoretically, data collection is not restricted in terms of resolution. Once these data are entered into a geographical information system, its data structure should also be adjusted to the underlying model. This adjustment can determine a range of scales for spatial primitives to be efficiently handled by the system. The paradox of data models, in terms of what is an object rather than a group of points, is shown with an example. It is concluded that there may not be a generally best resolution for a given environmental variable to be mapped.

INTRODUCTION

Resolution, as such, would be most frequently defined in dictionaries as technical limitation or characteristics of some kind of a system. Obviously it is associated with "the minimum difference between two independently measured or computed values which can be distinguished by measurement or analytical methods" (NCDCDS, 1988). Concerning a geographical information system (GIS), this definition would determine our task: Target objects to be mapped should be defined so that they can be distinguished from each other. This formal requirement would subsequently determine the amount of necessary detail to represent these objects. With abstract spatial entities defined, attribute properties can easily be assigned to them: census tracts have a population, square meters do not. In most instances of environmental mapping, however, the problem is faced from a different angle. First, the spatial entities should be defined according to which attributes can be assigned, since, for example, a census tract may not have high suitability for wheat. Secondly, the definition above treats distinguishability as a dichotomous variable and does not specify levels of accuracy. This is primarily due to the still existing gap in understanding the relationship between spatial and non-spatial resolution (see Dueker, 1979 for early reference) that can be referred to respectively as a recognition/identification problem in the mapping space and in the feature (or measurement) space (see Fig.1).

Let us treat the above outlined apparent contradiction in a "historical" context, i.e. with the analysis of the considerably long history of philosophical and sophisticated discussions in the GIS-era about the relationship of geometry and attributes, as well as their respective accuracies. There are numerous approaches to such issues from geosciences, cartography, statistics, etc., but unfortunately the more authoritative definitions read, the more confusing they are.

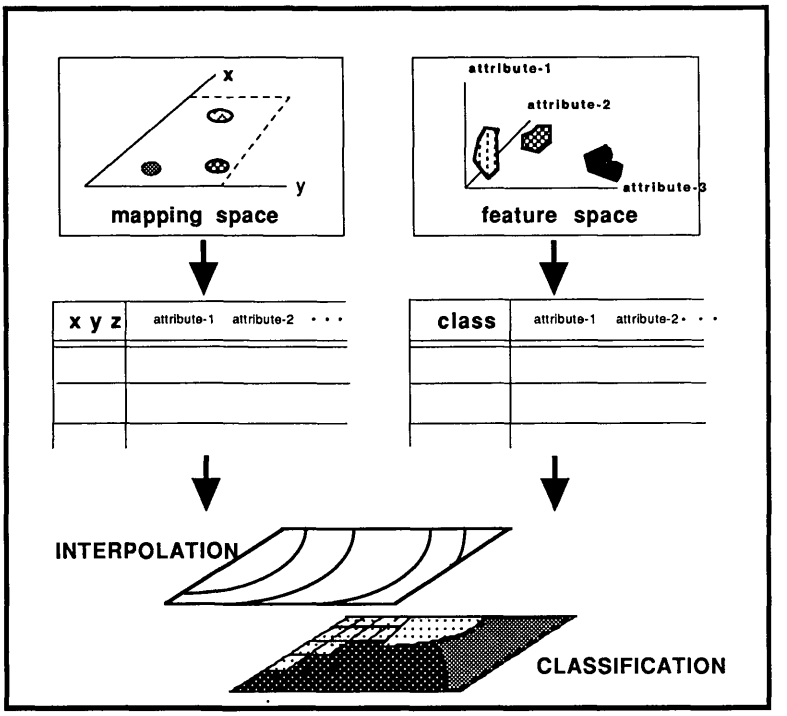

Figure 1.
Schematic representation of the relationship
between spatial and non-spatial data characteristics

The examination of the problem is organized as follows. Section 1 describes the major distinct approaches, which deserve much attention. I would argue that, although some of the technical ideas have been around for two decades or more, their authors might have wanted to use and interpret them in an inadequate way. Therefore a significant section is devoted to the mathematical models and some examples are elaborated on to prove their use. Section 2 then introduces an uncertainty relationship between spatial resolution and attribute accuracy. It is an extension of the "control one, measure another" scheme (Sinton, 1978), because it shows ***how*** resolution will vary in the mapping space once attribute accuracy is fixed, and vice versa. Section 3 presents an illustrative ecological site characterization example.

APPROACHES TO RESOLUTION

The nature of the approaches to resolution issues in mapping varies because the primary task is considered to be different: (1) in the geosciences it is assumed to be based on stochastic signal reconstruction which is also the most popular view of those in remote sensing and

image processing, (2) in "conventional" cartography it is more or less loosely linked to scale and observable detail, while (3) in the jargon of digital cartography ("data modelers") representation and model-fitting are the preferred key terms. For this discussion let us use an "ultimate" working definition of our task: derive information, or in other words, make a prediction at a "non-visited" site, where site refers to both mapping and feature spaces. Additionally, when discussing these approaches, one should not forget that all our mathematical tools operate on the foundations of mathematical models, i.e. much of our effort is focused on constructing meaningful models and the sometimes lengthy demonstration of mathematical apparatus must not hide this significant first step.

Geosciences - sampling, interpolation and variability

There is an obvious assumption about objects, or processes in space, namely, the larger the sample we have, the better. Since usually a number of constraints (e.g. time, storage, money) limit our ability to sample "infinitely", models, predicting our information loss with sampling, are of extreme interest. Therefore, not surprisingly, following paths of the "digital revolution of the 50's" in geosciences (see Clearbout, 1976, Webster, 1977 for reviews), references to the ***sampling theorem*** have emerged in the general cartographic literature (e.g. Tobler, 1969, Csillag, 1987, Tobler, 1988). There are three very attractive aspects to this approach: (1) it can be utilized in sampling design, (2) it provides handy tools for interpolation as well as filtering, and (3) it is computationally very efficient.

Once one adopts this approach, the underlying mathematical-statistical assumptions of the model should be clearly understood. A significant part of the discussion below is written in order to outline the background of the choices one can have when applying mathematical models. It turns out, that in some cases certain assumptions are made not because they provide more reasonable basis, but because of the practical reason that otherwise certain problems could not have been handled. First of all, in this particular case, having a sample of size **n**, the model is concerned with $\xi(x_1),\ldots,\xi(x_n)$ stochastic variables having joint normal distribution. It is crucial to everyday practice that we hardly have any tools to check this assumption. It is especially difficult, because the sample taken at **n** locations is a single realization of the variables. Furthermore, it is assumed that the expected value of this distribution is zero, and the variance is finite. So with this model we are confined in our prediction to the case, when, somehow, our original problem has been reduced to a zero-mean variable. With these assumptions we can prove that the covariance exists (i.e. $COV[\xi(x_i),\xi(x_j)]<\infty$) and it is positive semi-definite. It is our task now to construct an estimate of our distribution so the variance of the difference between the model and the estimate should be minimum. It is only due to the joint-normality assumption that our search for the estimate can be restricted for linear functions, i.e. in the form of weighted sum:

$$(1) \qquad \hat{\xi}(x) = \sum_i \lambda_i \xi x_{(i)}$$

The major problem in constructing our estimate is that we may not have sufficient information about the covariance, therefore further assumptions will be necessary. For instance, stationarity is a quite frequent assumption in order to reduce dramatically the number of elements to be estimated in the covariance matrix.

It is probably the advantage of modeling with linear functions that makes interpolation and filtering so popular in applying these tools (for math-intensive review of spectral analysis see Bracewell, 1965, or Bendat and Piersol, 1986). However, even if our assumptions are valid, there are lmany manners of abuse. When I say abuse, I mean that you can rarely find anyone who would apply these techniques, usually available by pressing a button, having tested accuracy constraints.

Let us just consider two simple cases for demonstration, linear interpolation and moving averaging. For the former case, suppose that we have taken sparse samples. Disregarding the distortion that may be due to undersampling, (i.e. less frequent sampling than half of the shortest wavelength represented), let us linearly interpolate among our data points! The total RMS-error (Bendat and Piersol, 1986), the square root of the mean difference between the original and the interpolated signal over the entire Nyquist-interval, will be

$$(2) \qquad E_{RMS}(f') = 2 - \mathrm{sinc}^2(f')(2+(2(\P f')^2/3))$$

where sinc denotes the sine-cardinal function [sinc(a)=sin(¶a)/¶a], while f' denotes dimensionless frequency (equals frequency times sampling distance). As Figure 2 clearly illustrates, linear interpolation can severely distort higher frequency signals. If, for example, one would interpolate 1,2,4,... points between existing data points, the maximum error term (from Eq.2. at f'=0.5) would be -5.63, -26.83, -50.21 in decibels, and 52, 4, 0.3 in relative percentage, respectively, providing upper limits for accuracy.

Considering moving averaging, it is again the frequency-dependent distortion that should be pointed out. In general, filtering can be written in the form

$$(3) \qquad y_k = \sum_i c_i x_{k+i} \qquad (i = -N,N)$$

for which moving averaging is a special case with $c_i = 1/(2N+1)$ for all i's. The amplitude response (or frequency modulation function, S(f')) can be obtained with the Fourier-transform of the (filter) coefficients. In this particular case it is in the form of a geometric series:

$$(4) \qquad S(f') = \sum_i c_i \exp\{-j2\P if'\} = 1/(2N+1) \sum_i \exp\{-j2\P if'\} =$$

$$= 1/(2N+1) \exp\{-j2\P Nf'\} [1 + \exp\{-j2\P f'\} + ... + \exp\{-j2\P 2Nf'\} =$$

$$= \mathrm{sinc}(2N+1)f'/\mathrm{sinc}(f')$$

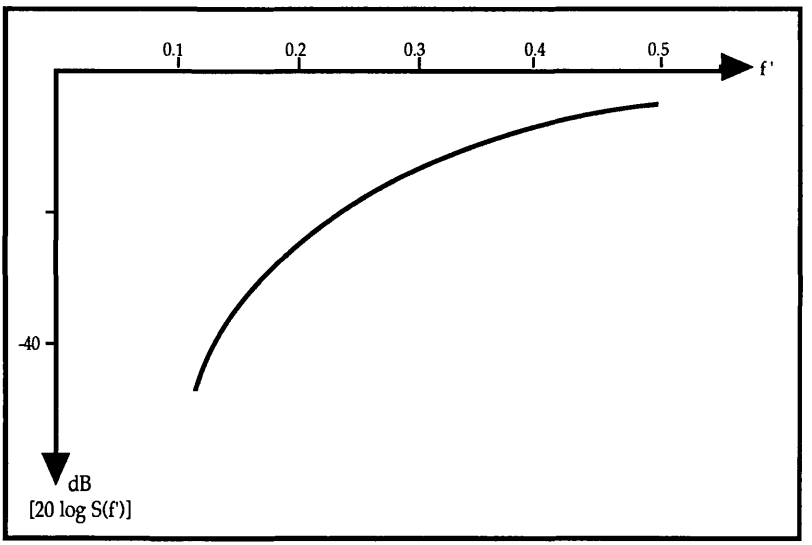

Figure 2.
Total RMS-error of linear interpolation for the Nyquist-interval
[The vertical axis is given in dB:=20logS(f')]

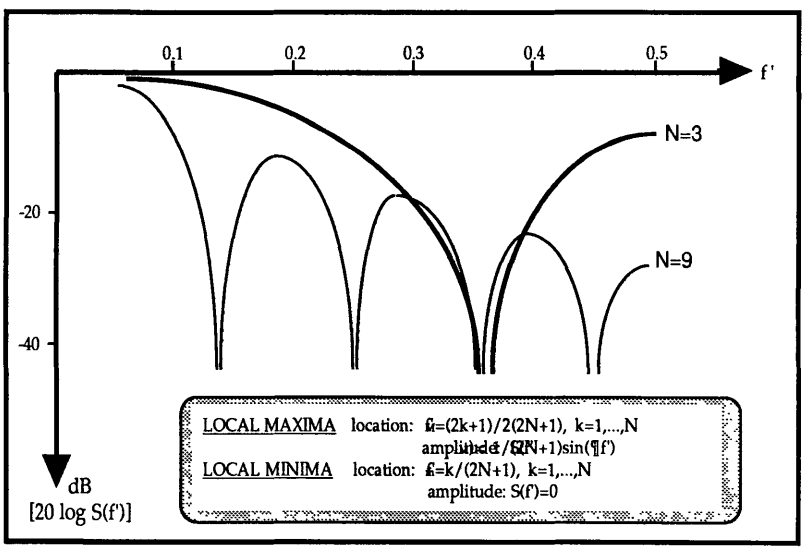

Figure 3.
The amplitude modulation transfer function of moving averaging for filter-size 3 and 9. Local extremes can be calculated with the given formulae, based on S(f'), where N denotes the length of the filter.

Some characteristics of S(f') for "everyday-size" moving average filters are displayed on Figure 3. It should be noted again, that these filters, generally applied heuristically, are close to our expectations at low frequencies, but at higher ones they seem to misbehave.

Such methods of spectral analysis aim to construct our estimate of the covariance matrix based on the strict stationarity assumption. A close relative, called kriging, became popular and uses the assumption of second order stationarity (Journel and Huijbregts 1978). The estimation procedure, in this case, is even further reduced, since it aims at the most commonly independent, parametric estimation of the substitute of the covariance function, called a semi-variogram (McBratney and Webster 1986). We should point out that it is the equivalence of the squared deviation from the mean and the normalized square difference between all pairs, known since the early days of mechanics, that is behind this methodology.

There are some further necessary remarks to be made about kriging. The estimation procedure with the stationarity assumption already eliminated a number of unknown parameters, and the covariance became a function of distance. Thus the covariance matrix is only dependent on the spatial arrangement of the sample that is, again, computationally efficient. However, the estimation procedure becomes highly dependent on the values of the semi-variogram at small distances, i.e. the nugget value, (Ripley 1981), and becomes statistically unstable when this value is not zero (Mardia, 1980, Philip and Watson, 1986). Still, the popularity of kriging is due to its close links to spatial variation (variability, heterogeneity, etc.) and the seemingly straightforward manner in which it treats continuous functions characterizing such, otherwise hardly mappable, phenomena.

The spread of these methods in GIS-applications can probably be attributed to their ability to give direct estimates of deviation from an expected value for points, as well as for areas (Journel, 1986). The spatial mean derived this way for arbitrary spatial partitioning has been widely applied in environmental sciences as well as in remote sensing (Burgess and Webster, 1980, Woodcock and Strahler, 1984). This implies that our software eventually can map not only a certain variable, but its reliability.

Cartography - scale, precision and detail

My impression is that cartographers do not like the term resolution (Robinson et al., 1984, Campbell, 1991). Implicitly, however, a kind of a rule of thumb is used according to Tobler (1988): Since the smallest physical mark which the cartographer can make is about one half of a millimeter in size, one can get a fairly good estimate of resolution in meters by dividing the denominator of map scale by two thousand.

This rule is certainly far from being absolute. The real art in cartography is to represent objects even if they are smaller than this nominal resolution because of "relative importance". Discussions about generalization, in fact, clearly reflect this paradox. For example:

> "Cartographically speaking, it is essential to retain both the details required for geographical accuracy and required for recognizability within a digital data base.
> ...
> To preserve accuracy and recognizability automatically during map generalization, one must be able to describe digitally the details that must be preserved." (Buttenfield, 1989)

Inevitably, cartographers, in the "traditional" sense (Vasiliev et al., 1990), are concerned with visually conceivable objects, i.e. map elements whose geometric and attribute characteristics are merged forming a graphic attribute. Thus the distinct boundary between precision and accuracy seems to be intentionally loosened.

In the previous section, for example, precision could have been understood as the definite upper limit of accuracy in both mapping and feature spaces, while here it is related only to location, and the content has been switched to recognizability. Consequently, this approach forms a counterpart of the one discussed above with extreme "geometrization" of the resolution issue.

Data model(er)s and structures - raster vs. vector

In one of the most recent summaries on accuracy-related research in GIS (Goodchild and Gopal, 1989), resolution had a roughly equal number of references (18) in the index with filtering (9) and interpolation (10), and generalization, on its own (17), was very close. This may mislead us into thinking of a delicately balanced approach.

The conventional separation of spatial data into geometry and attributes has not left this community yet. Such a separation is consistent with an entity-relationship model of phenomena, with geometry defining the objects, which then have attributes and relationships (Mark and Csillag, 1989). And there seems to be a borderline: Those who go for the priority of geometry (mapping space), having their roots in e.g. cartography or surveying, take a model of space most commonly called "vector", while those who emphasize the significance of classification, most probably rooted in geosciences, would adopt a model usually called "raster". Geometry and attributes, however, have in many cases intrinsic links to each other, therefore any treatment of one in isolation from the other will have a high risk of misrepresenting the phenomenon.

There is also a substantial difference between accuracy concepts in the vector and raster models (Chrisman, 1989, Mark and Csillag, 1989). The former, modeling space occupied by objects, attaches accuracy measures to representation of geometry (mapping space), while the latter, partitioning space into units which then will have attributes, prefers to assign such measures to the classification of attributes (feature space).

Regarding previous comments on the philosophy of modeling, once we have adopted a model, there are no mathematical-statistical tools to exchange it for another model; one can either apply it successfully with

proper predictions, or can fail to get close to reality. In light of this, there is no valid conclusion available to decide which model is "better".

As far as choice or design of data structures is concerned, lots of efforts have been devoted to handling numerous kinds of objects (spatial primitives) simultaneously, and to implement their manipulation as transparently as possible (Goodchild, 1987). Thorough research has been carried out on the design of the functionality of GIS software focusing interest on user needs in terms of data volume and manipulation requirements.

Once a system is implemented on this basis, efforts to achieve a predefined classification accuracy may lead to either cumbersome recursion, or overdesigning the capabilities of the system. It seems to be more popular for "GISers" to provide performance tests only in terms of "geometrical representation", however, the community still lacks those tests on matching categorization requirements. Vector viewers specially claim that the raster approach overemphasizes geometric properties, while the vector model permits the attribute to be attached to the appropriate spatial object. Indeed, that is why there is emphasis on the links between geometry and attributes: the appropriate objects are not known *a priori*. It seems to me that the methodology of cartographers has been preferred to modeling uncertainty.

Unless data structures, efficiently handling a set of spatial primitives, are not adjusted to the inherent data characteristics, including accuracy, heterogeneity and the like, there will be no guarantee that a given representation can fulfill the requirements of classification accuracy. On the other hand, whenever the attribute domain was in focus, a very limited set of spatial characteristics, like a single fractal dimension, was taken into consideration (Goodchild and Dubuc, 1987). It would be properly modest to say that we have understood, and more or less successfully modelled, spatial data in the mapping space, while the exploration of feature space is still ahead.

THE CARTOGRAPHIC UNCERTAINTY RELATIONSHIP

The solution of the problem of making reliable maps (i.e. where both locational and classification accuracy is known and limited) has to be accompanied by the recognition that "accurate" and "erroneous" are not just two disjoint sets, but rather should be viewed as a continuum. When map users consider accuracy issues, they certainly want "the best". In simple words, if 10 m and 90% were printed on a tourist map as accuracy limits, they would like to assume that any dark green patch represents a forest with the same locational and thematic accuracy. And this is the point where real data may cause so much trouble to professional modelers. All of our tools dealing with spatial data, and *de facto* our geographical information systems, are context-dependent.

It requires manageable definitions of "objects to be mapped". We may want to ask, for instance in the previous example, whether our definition of a forest is useful at all: Can one, two, three... trees be a forest? Or, if one knows for sure that there is no forest covering more than 10,000 square miles in an area, is that a useful piece of information? Such

questions should not look absurd. When soil scientists are calculating the risk of missing a(n infinitely narrow) boundary in the field, landscape architects assign a value of ecological potential for a 5 km * 5 km area, or economists rank countries based on per capita income, etc., they are dealing with very similar problems: Complex human concepts (variables, categories and relationships) are "projected" into Euclidean space in a manner that their potential for further inference is maximized. In other words, spatial homogeneity criteria are defined so that uncertainty is tolerable.

There is a significant mathematical-statistical arsenal to study such criteria. Beyond classical works in autocorrelation studies (e.g. Griffith 1988) more recently attribute classification with spatial constraints has been introduced (Gordon, 1987) more or less independently from mainstream GIS-related research (Chrisman, 1986).

Most importantly to our topic it has been shown for environmental variables that homogeneity criteria based on a given categorization reveal spatial variation (Csillag and Kertesz 1990). Generally speaking, ***there is a contradiction between the requirements of constant attribute accuracy and constant spatial resolution.*** The general concept that fixed these parameters independently over an entire data set cannot be held. If given that recognition probabilities for a class-set are predefined, there is no guarantee that a certain spatial resolution will match any homogeneity criterion. Conclusively, there may not be a unique, generally best resolution for a data set; either accuracy or resolution will exhibit variation.

ECOLOGICAL SITE CHARACTERIZATION - AN EXAMPLE

Let us illustrate the above outlined ideas with a practical environmental mapping example. The task of information processing in this case is to quantitatively describe ecological site characteristics of a salt-affected low-grass prairie (Toth et al., 1990a).

This landscape covers more than 100 km^2 in the Hortobágy-region in E-Hungary, and it can be characterized by abrupt changes in soil conditions, surface grass cover, microrelief with very sharp boundaries (Rajkai et al., 1988). Additionally, the descriptive measures of the apparent surface pattern are highly ***scale dependent***, consequently there have been numerous efforts to determine the spatial behavior of underlying variables. The primary tool of these investigations was geostatistics, but several botanical and cartographic considerations were also taken into account.

The section below is focused on the following problem. Given a set of interrelated variables their spatial characteristics are determined in order to find the most suitable resolution to sample and map them. If these characteristics turn out to be different, a pointwise classification based on these variables will lead to heterogeneous patches. Having a control categorical variable, the spatial variability of the individual variables can be described by patches. How can those patches be found, for which all spatial variances will be lower than an acceptable threshold?

This way one can identify class-membership for any given location with predefined accuracy.

The variables included in this study cover a wide range related to salinity status, soil chemistry, soil texture, etc., as well as microrelief and a number of botanical variables. The typical alkali soils in the Hortobágy National Park, mainly heavy-clay solonetz soils, can be characterized by varying depth of A horizon (Rajkai et al., 1988), and that variation corresponds to the dramatically different surface conditions. It is an erosion process on an almost completely flat plain induced by local disturbance (Toth et al., 1990a). Eroded surfaces occur as micro-valleys, and there is a well-known toposequence from elevated spots through the slopes down to the valleys. As the A horizon is washed away pH, salt-content (S%), and exchangeable sodium percentage (ESP) increase, while ecological diversity decreases. This spatial pattern which is seemingly dominant in the meter range horizontally and in the centimeter range vertically produces a highly complex terrain over the whole extended area.

The mapping strategy must be based on understanding the interrelationships between soils and vegetation forming a complex ecological system. A 15 m by 15 m plot was selected as a test-site for detailed analysis, a number of 60 - 500 m long transects were sampled, while remotely sensed data were collected for regional extrapolation, inventory and monitoring (Toth et al., 1990b). From an environmental point of view the task is to assign description of spatial variation to patches, in terms of variables and resolution, which otherwise would appear as equally homogeneous in terms of salinity status.

Figure 4. summarizes some of the data collected for the test-site. Systematic sampling was applied along the 1.5 m by 1.5 m grid, while stratified random sampling was carried out for the distinct floors of the toposequence, i.e. for hills, slopes and valleys. Soil samples were collected for 100 cm^3 samples, i.e. with approximately 5 cm^2 ground resolution, while botanical data for individual species and total coverage were recorded corresponding to 50 cm by 50 cm quadrats.

The geostatistical evaluation of measurements revealed that there are sharp differences between the spatial characteristics of individual variables, even though they play more or less similar roles in describing salinity status. For example, while pH clearly showed well-defined spatial structure on the test-quadrangle with a characteristic range of about 11 m, that of clay percentage came out to be about 14 m with very high nugget, but salt-content had an unbounded semi-variogram. If one wanted to characterize a given surface within the region, there were always variables, which showed too high estimation variance, or others must have been oversampled. Therefore, an optimum sampling scheme for classification of salinity status based on these variables could not be computed.

Stratified sampling was controlled by botanical data Elevated spots are characterized by more complex associations and more surface cover, while valleys are dominated by one species. This is due to the dramatic difference between their salinity status: Where the A horizon is present, pH and salt-content is lower, while on eroded spots severe salinization

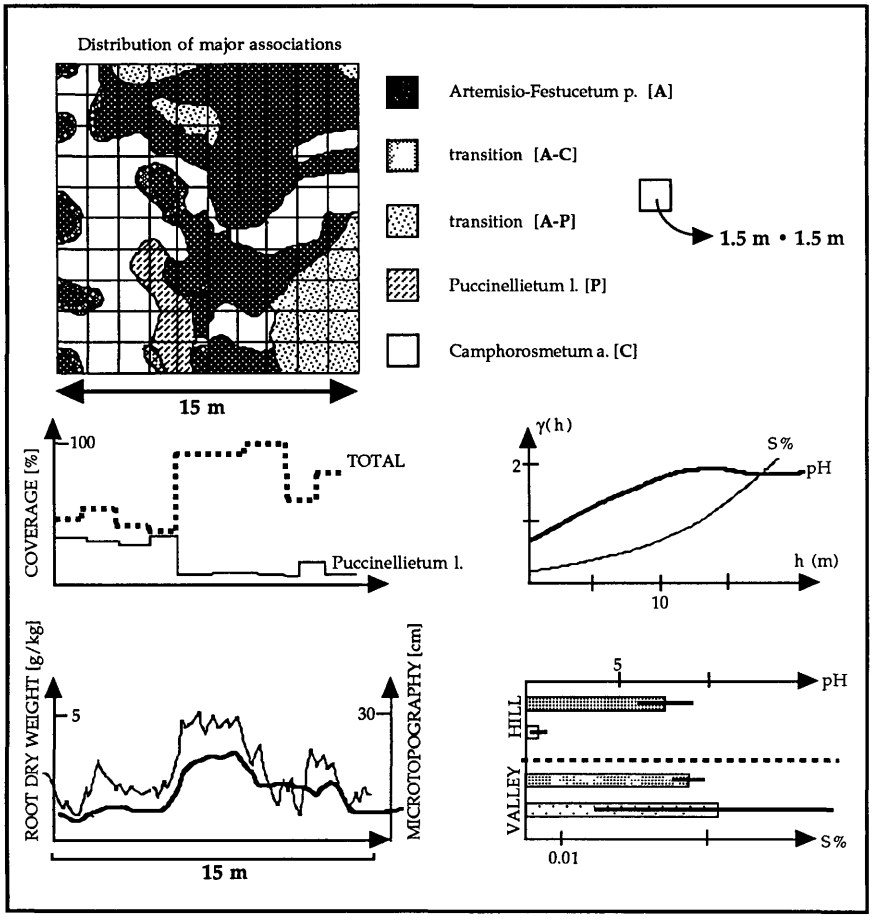

Figure 4.
Environmental data for resolution study (Hortobágy National Park, E-Hungary) - systematic and stratified random sampling
[Distribution and classification of major botanical associations on the test-site (sketch-map, top); Descriptive data along cross transect for vegetation quadrats (top left graph) and for related variables (low left graph); Semi-variograms of two soil properties with curves to guide the eye (top right graph); Descriptive statistics (mean and standard deviation) for pH and salt-content for two-classes of the toposequence (low right graph) - see text for details]

occurs. Although this relationship supported the initial classification, descriptive statistics showed an interesting side-effect. On the hills pH and root-dry weight had significantly higher variance than in the valleys, or on the slopes, while this relationship was reversed in case of, for instance, salt-content. This observation leads again to a conflict, if one wishes to determine the necessary number and distribution of samples to classify a given location.

As a summary of this example the hierarchical nature of the possible solution should be pointed out. On a general soil map this area would be shown as a "highly variable salt-affected" area. Neither does this description contain explicit information about the amount or nature of this variation, nor does it provide reasonable estimates of the key variables by means of descriptive statistics. Having a detailed survey data set, say in a GIS, overlaying pH on salt-content leads to different results depending on which salinity ***class gets preference*** in determining classification criteria. It is because the objects to be mapped, in this case salinity classes, have class-dependent links between the mapping space and the feature space. Therefore, for example, more saline surfaces can be better identified with finer resolution, taking into account more non-spatial variation in salt-content, than non-eroded surfaces, and so on. Furthermore, this information can eventually be incorporated in the data structure as well.

CONCLUDING REMARKS

The evolution of geographical information analysis has resulted in conflicts with the common sense of "resolution". It has been shown that there is inherent uncertainty involved in data models applied in geographical information systems. Several approaches have been applied to spatial data to deal with this uncertainty, but they handle the mapping and feature space separately. In environmental mapping, when resolution of spatial sampling is theoretically unrestricted and classification does not define the spatial objects themselves, the problem of determining ***an*** optimal resolution, which provides a given constant attribute accuracy leads to a contradiction. A soil mapping example outlines that the most promising path for further research is context-dependent merging of criteria defined in mapping and feature space, rather than separating them as independent properties of objects to be mapped. The various statistical tools one can apply through data models permit not only control of accuracy, but they can contribute to the evolution of data structures, which incorporate this information. These data structures should be object-oriented, since there are no objects unless they can be recognized with certain probability, and can be located with certain accuracy.

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