SIMULATION OF THE UNCERTAINTY OF A VIEWSHED

Peter F. Fisher,

Department of Geography,
Kent State University,
Kent, Ohio 44242-0001, U.S.A.
PFISHER1@KENTVM.BITNET

SUMMARY

One of the most widely available procedures packaged with GIS for the analysis of a Digital Elevation Model (DEM) is the identification of the viewable area, or the viewshed. The elevations recorded in the DEM do, however, contain error, and the USGS, for example, publishes a Root Mean Squared Error (RMSE) for each DEM. Research reported here assesses the uncertainty of locations being within a viewshed, given the published error for the DEM. In this research, repeated error fields are simulated with variable spatial autocorrelation, and added to the original DEM. The viewshed is then determined in the resulting noisy DEM. Results show that using the basic assumption of spatial independence in the error which is implicit in the RMSE remarkably few points are reliably within the viewshed. With spatially autocorrelated noise, the reliability is higher, but still should be cause for concern to many using viewshed procedures.

INTRODUCTION

Research on the propagation of error within GIS operations has focused upon the polygon overlay operation (MacDougall, 1975; Newcomer and Szajgin, 1984; Chrisman, 1989; Maffini et al., 1989; Veregin, 1989), at the expense of other GIS data types and functions. The experiments reported here examine one aspect of the propagation of error from a Digital Elevation Model (DEM) into the derivative product showing visible locations, sometimes known as a viewshed (see also Felleman and Griffin, 1990; Fisher, 1990).

This paper starts by briefly discussing the viewshed operation, and the nature of error in DEM data. The general methodology of simulating error is then discussed, followed by its application to a real location.

VIEWSHEDS AND DEMS

The basic algorithm in establishing the viewshed examines the line-of-sight between two points (the viewpoint and a target), and assesses whether any land or object rises above that line-of-sight. If it does then the target is not within the viewshed of the viewing location, but if no land rises above the elevation, then the target is within the viewshed. In establishing the viewshed either all possible targets in the area of a database (Clarke, 1990, 227-228), or only those within some constrained portion of the area (Aronoff, 1989, 234), may be considered. Several studies have explored differences in viewshed algorithms (Anderson, 1982; DeFloriani et al., 1986; Sutherland et al., 1974), and Felleman and Griffin (1990) have compared the output of four different GIS-based implementations of the viewshed operation. They show the viewsheds delimited to be very different. This difference is not particularly surprising given the multiple decisions to be made in designing the implementation of the viewshed operation. For example, decisions have to be made as to whether the viewpoint in a gridded DEM is anywhere within the viewpoint gridcell, or is just the mid-point; similarly, should the surface be treated as the stepped phenomena it is coded as, or an interpolated surface? The outcome of such algorithm-design decisions may produce dramatically different viewshed results in some DEMs.

The viewshed is invariably reported as a binary product, a target location is either within or without the viewshed of the viewpoint. No shades of uncertainty are admitted; neither the likelihood nor the probability of a point being within, or of being without the viewshed is reported. In the light of the considerable interest in database accuracy this seems remarkable, especially when each DEM is required to be accompanied by an error report (USGS, 1987).

The USGS has adopted the Root Mean Squared Error (RMSE) for reporting accuracy in their DEM products (USGS, 1987). The RMSE for any one DEM is based on the comparison between the elevations of at least twenty locations on the map, and their elevations recorded in the database. It should be noted that most USGS source maps are stated to conform to the National Map Accuracy Standards, which themselves state that "at no more than 10 percent of the elevations tested will contours be in error by more than one half the contour interval", as established by comparison with survey data (Thompson, 1988, p 104). In generating a DEM from a map, therefore, at least two stages are present when error may be introduced: map compilation and DEM generation from the map. The error reported for the DEM only refers to the second of these, and it is only that error that is examined here. Some DEMs are generated directly from aerial photographs by the Gestalt Photo Mapper II, and, in this case, the error may only be introduced in one stage.

Error in DEMs is then widely acknowledged, and has been the subject of some study. That study has, however, concentrated on the nature and description of the error, not its propagation into any derivative products. The only work known to the current author which provides any evaluation of error propagation is by Felleman and Griffin (1990). They have compared implementations of the viewshed operation, and simulated error in the DEM before calculating the viewshed, as is reported here. They examined 3 viewpoints in 2 test areas for each of which 10 error simulations were run. Results are, however, only reported for one test location.

METHOD

SIMULATING ERROR

A Monte Carlo simulation and testing approach is taken to studying the propagation of DEM error here. In this approach, randomizing models of how error occurs are established, and then coded as computer procedures. The resulting computer program may be used to generate multiple realizations of the random process. Many workers have used original data in combination with realizations of the defined random process to establish the statistical significance of the original data with respect to the random process (Besag and Diggle 1977). Thus Openshaw et al. (1987) executed 499 realizations of the random process to locate two significant clusters of incidents of childhood leukemia in northern England.

How the error is distributed across the area of any one DEM is currently unknown, and factors that may effect the distribution of error is largely unresearched. The inference of the error reporting used by the USGS is that the error at any point occurs independently of that at any other point (i.e. the error is not spatially autocorrelated). Therefore, the following algorithm may be implemented (Fisher, in press):

1. Define a standard deviation of a normal distribution (S = RMSE);

2. Read Original_Value for the current cell:
   - 2.a Using the Box-Muller (or some other) algorithm generate a random number drawn from a normal distribution with mean = 0 and standard deviation = S;
   - 2.b Add the random number to the Original_Value for the current cell, to give the New_Value;

3. Repeat 2 for all cells in the Map_File.

This assumes that the standard deviation of a normal distribution is equivalent to the RMSE. In the absence of any other information on error structure, this may not be unreasonable. Such independent error is, however, very likely to contribute only a small portion of the overall error. High spatial autocorrelation is probably present, and banding can often be seen in the DEM data. To accommodate the occurrence of spatial autocorrelation, a version of the algorithm given by Goodchild (1980) was implemented, using Moran's I to measure the autocorrelation (Goodchild 1986; Griffith 1987). It works thus:

1. Define a target autocorrelation (I_t), and a standard deviation of a normal distribution ($S = RMSE$);

2. For each cell in the DEM generate a random value, with a normal distribution with mean = 0 and standard deviation = S (see first algorithm);

3. Calculate the Spatial Autocorrelation of the field (I_1);

4. Randomly identify two cells in the DEM:
   - 4.a Swap the values in the two cells;
   - 4.b Calculate the new spatial autocorrelation (I_2);
   - 4.c IF $I_t > I_1$ AND $I_2 > I_1$ THEN retain the swap, and $I_1 = I_2$
     OR
     IF $I_t < I_1$ AND $I_2 < I_1$ THEN retain the swap, and $I_1 = I_2$
     ELSE swap the two cells back to their original values;

5. Repeat 4 until ($I_t - I_1$) is within some threshold.

6. For each cell in the original DEM, add the value in the corresponding autocorrelated field.

This algorithm is simple and can be made computationally efficient, and it will be noted is an extension of the first algorithm listed.

The random number generator used in programming the algorithms was also tested, since like all such implementations it is truly a pseudo-random number generator (Ripley, 1986). The generator included with Turbo Pascal 5.5 was used here. The runs test was used to check for serial autocorrelation, the chi-squared test was used to check for a uniform distribution, and serial autocorrelation was tested for all lags to check for cycling in the generator. The generator performed satisfactorally for all cases, when number sequences up to 10,000 long were tested (corresponding to the 100 by 100 array used in the generation of autocorrelation).

MEASURING UNCERTAINTY

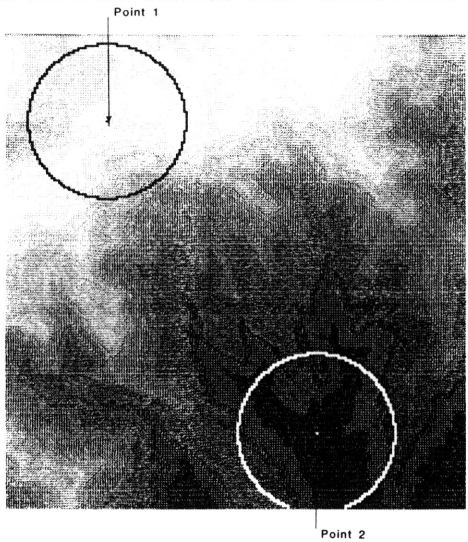
Each realization of the random process then resulted in a new DEM. The viewshed was calculated for each, and for any one view point, all the viewsheds in which the noise term had the same RMSE and spatial autocorrelation were summed to yield a summation image which describes the uncertainty of the viewshed, or the fuzzy viewshed. Since the viewshed is reported as a raster image coded as 0 or 1, the maximum value in the fuzzy image is 20, the number of realizations. It is possible to define the viewshed with a particular likelihood (probability). Thus, with 20 realizations, cells with value 19 in the resulting image, have probability, $p = 19/20 = 0.95$ of being within the viewshed, and, similarly, those with value 10 have probability, $p = 10/20 = 0.5$.

THE STUDY AREA

A 200 x 200 cell subset of the USGS Prentiss, NC, 7.5 minute DEM was acquired covering the Coweeta Experimental Watershed (Fig. 1). This DEM has been the subject of considerable research on DEM products (Band, 1986; Lammers and Band, 1990). Within the area of the DEM two test viewing locations (viewpoints) were arbitrarily

Figure 1

The Digital elevation model of the Coweeta Experimental Watershed, N.C. The two test locations are shown, and the 1 km zone around each indicated.



identified, one near an interfluve (Point 1), and one in a valley bottom (Point 2). All viewsheds calculated in the research reported here were only to within 1km of the viewpoint, and from an elevation of 2m above the viewpoint (corresponding to approximately the near-view, and the eye level of an individual, respectively).

The DEM was read into a format compatible with Idrisi (Eastman, 1989), a PC based package for Geographic analysis, and all further processing was done with either Idrisi modules, or implementations of the above algorithms written by the author in Turbo Pascal version 5.5. The VIEWSHED module of the Idrisi package is crucial to the research reported here, and so some simple test situations were established to examine the veracity of the viewable area calculated by that module. In every test, the module performed satisfactorally. The module operates on a DEM of any size, by using random access files, but at great expense in processing time. Only examining locations within 1 km of the viewpoint also made the processing time required for the research realistic.

RESULTS

Tables 1 and 2 report the frequencies of occurrence of values in the fuzzy viewsheds derived from the noisy DEMs, and those fuzzy viewsheds are shown in Figures 2 and 3. For each set of viewsheds with a specific spatial autocorrelation in the noise, and for a particular viewpoint, the tables record in the first column the frequencies of cells which are outside the viewshed in the original DEM, but inside those in simulated elevation models, the second column records those that are in both viewsheds, and the last column records the sum of the first two. The results all refer to applications of noise with variable spatial autocorrelation and with RMSE = 7, the value specified for this DEM. Table 1 and Figure 2 show results for Point 1, while Table 2 and Figure 3 show results for Point 2.

DISCUSSION

It is apparent in both Tables 1 and 2 that when there is no spatial autocorrelation in the noise, there are very low frequencies of cells with high cell counts in the viewsheds of either test viewpoint. 8 and 9 cells occur within all 20 of the viewsheds of the two points (i.e. the nearest neighboring cells plus 1 in 1 case), and in the case of Point 2 only 16 cells have cell counts of 18 or greater. The viewshed of the higher, ridge-top location (Point 1) appears to be more stable, however, with higher frequencies of cells with count greater than 10 (giving $p > 0.5$ of being within the viewshed), 706 as opposed to 443 for Point 2.

TABLE 1

Frequencies of occurrence of values between 1 and 20 in the image resulting from summing all noisy viewsheds, for Point 1 where the autocorrelation in the noise varies from 0 to 0.9. All points within 1 km of the viewpoint are included.

Cell	I=0			I=0.7			I=0.9		
	Out	In	Sum	Out	In	Sum	Out	In	Sum
Count	View			View			View		
0	1456	1	1457	1425	1	1426	1574	3	1577
1	192		192	226	3	229	204	2	206
2	142	9	151	123	1	124	100	3	103
3	122	18	140	117	3	120	68	6	74
4	92	31	123	81	9	90	72	16	88
5	76	45	121	90	14	104	47	15	62
6	60	60	120	63	30	93	35	29	64
7	37	78	115	47	49	96	35	32	67
8	21	97	118	23	66	89	34	35	69
9	15	123	138	21	87	108	31	49	80
10	3	120	123	4	73	77	14	48	62
11	6	94	100	6	73	79	5	57	62
12	4	74	78	3	78	81	7	61	68
13	3	93	96		104	104	3	63	66
14		80	80		84	84		73	73
15		75	75		85	85		90	90
16		90	90		93	93		82	82
17		81	81		86	86		110	110
18		73	73		123	123		92	92
19		24	24		142	142		101	101
20		9	9		71	71		308	308

The distribution of cell count frequencies becomes progressively less skewed towards the low frequencies as the autocorrelation in the noise increases. Indeed, in the case of Point 1, the distribution becomes strongly bimodal when I = 0.9. When the noise perturbing the DEM has high autocorrelation, the frequency of cells with high counts increases, so that as the value of I for the noise increases the number of cells with count 20 increases dramatically for both Point 1 (9, 71, and 308 for I = 0, 0.7 and 0.9), and Point 2 (8, 38, and 112). There are, however, only slight, but probably useful, rearrangements of frequencies in many of the other cell counts, and an increase in the number of cells with only a count of 1 can be noted in the case of Point 1. At Point 2, the number of cells with count 1 is reduced by nearly a third, but the number of cells with count 20 does not increase by nearly as much as in the results for Point 1. There is, however, an evening of frequencies corresponding to counts from 5 to 20, which is not observed in the results for Point 1.

As the spatial autocorrelation increases the number of cells that are identified as not even possibly being within the viewshed, but within the search distance of the viewpoint, does increase with autocorrelation, but the change is not continuous in both cases. From Point 1, the counts are 1456, 1425 and 1574 when I = 0, 0.7, and 0.9 respectively, and at Point 2, the values are 1109, 918, and 897. Furthermore, the number of cells that may be within the viewshed (>0 in the fuzzy viewshed) but were not in the viewshed in the original DEM, increases with autocorrelation at Point 2, (822, 1013, and 1034 for I = 0, 0.7, and 0.9 respectively), but at Point 1 the reverse is true (733, 804, and 655 respectively). The upper frequencies of cells outside the original viewshed changes very little either between or within viewpoints (frequencies of 12 to 15 can be noted).

TABLE 2

Frequencies of occurrence of values between 1 and 20 in the image resulting from summing all noisy viewsheds, for Point 2 where the autocorrelation in the noise varies from 0 to 0.9. All points within 1 km of the viewpoint are included.

	I=0			I=0.7			I=0.9		
Cell Count	Out View	In View	Sum	Out View	In View	Sum	Out View	In View	Sum
0	1109	16	1125	918	3	921	897	1	898
1	319	33	352	265	12	278	255	2	257
2	179	89	268	205	21	226	177	6	183
3	122	90	212	143	33	176	137	13	150
4	73	119	192	100	46	146	124	19	143
5	46	122	168	83	63	146	88	29	117
6	47	132	179	64	81	145	60	39	99
7	14	152	166	42	76	118	64	59	123
8	8	137	145	45	85	130	45	66	111
9	5	135	140	27	97	124	39	89	128
10	1	85	86	25	101	126	25	113	138
11	5	99	104	10	109	119	9	88	97
12	1	82	83	4	116	120	4	100	104
13	1	78	79		107	107	3	115	118
14		64	64		106	106	4	113	117
15	1	42	42		120	120		119	119
16		36	36		97	97		111	111
17		18	18		93	93		109	109
18		7	7		91	91		125	125
19		1	1		49	49		117	117
20		8	8		38	38		112	112

SPATIAL ARRANGEMENT OF UNCERTAINTY

The spatial distribution of these fuzzy values are shown in Figures 2 and 3, together with the viewshed in the original DEM, the elevation map of the immediate area, and a viewshed image derived from the fuzzy image where I

Figure 2

DEM and Viewsheds of Point 1: A) the elevations in the near-view; B) the viewshed image from the original DEM; fuzzy viewsheds where C) I=0, D) I=0.7, and E) I=0.9; and F) an image showing the viewshed where p>=0.5 from E.

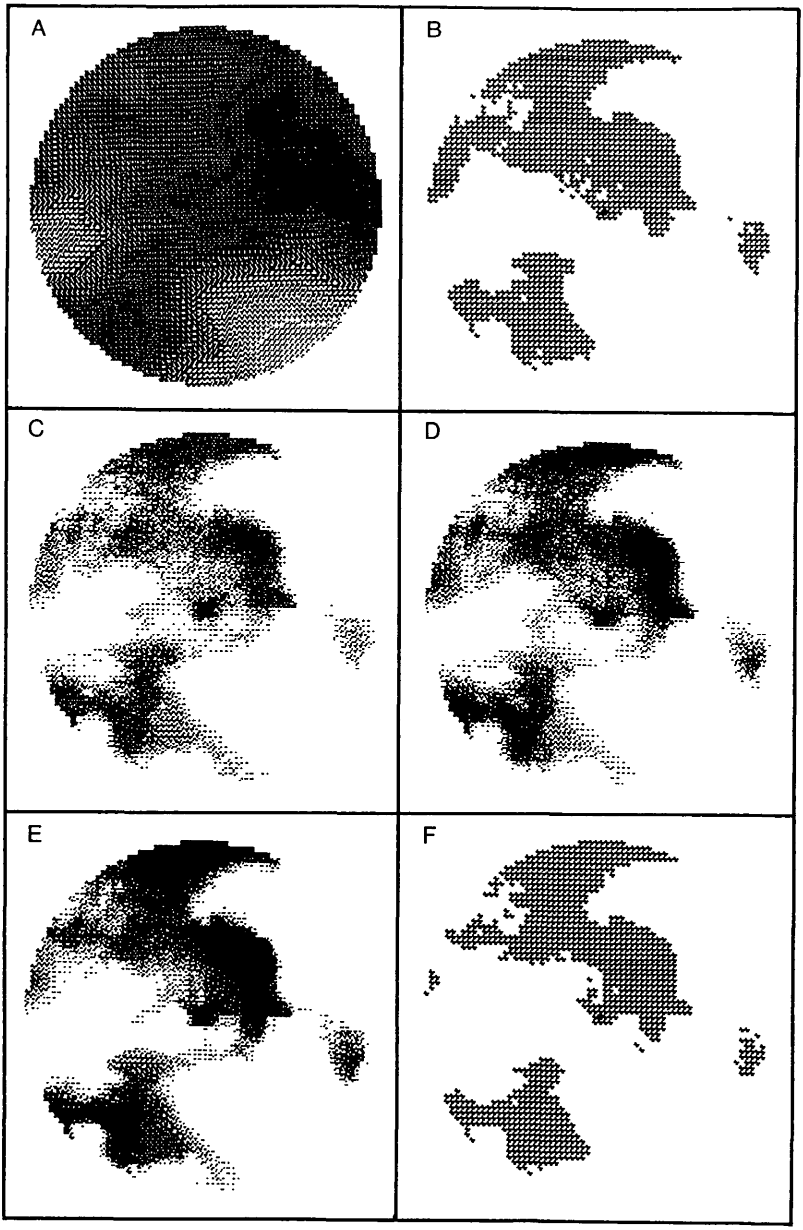
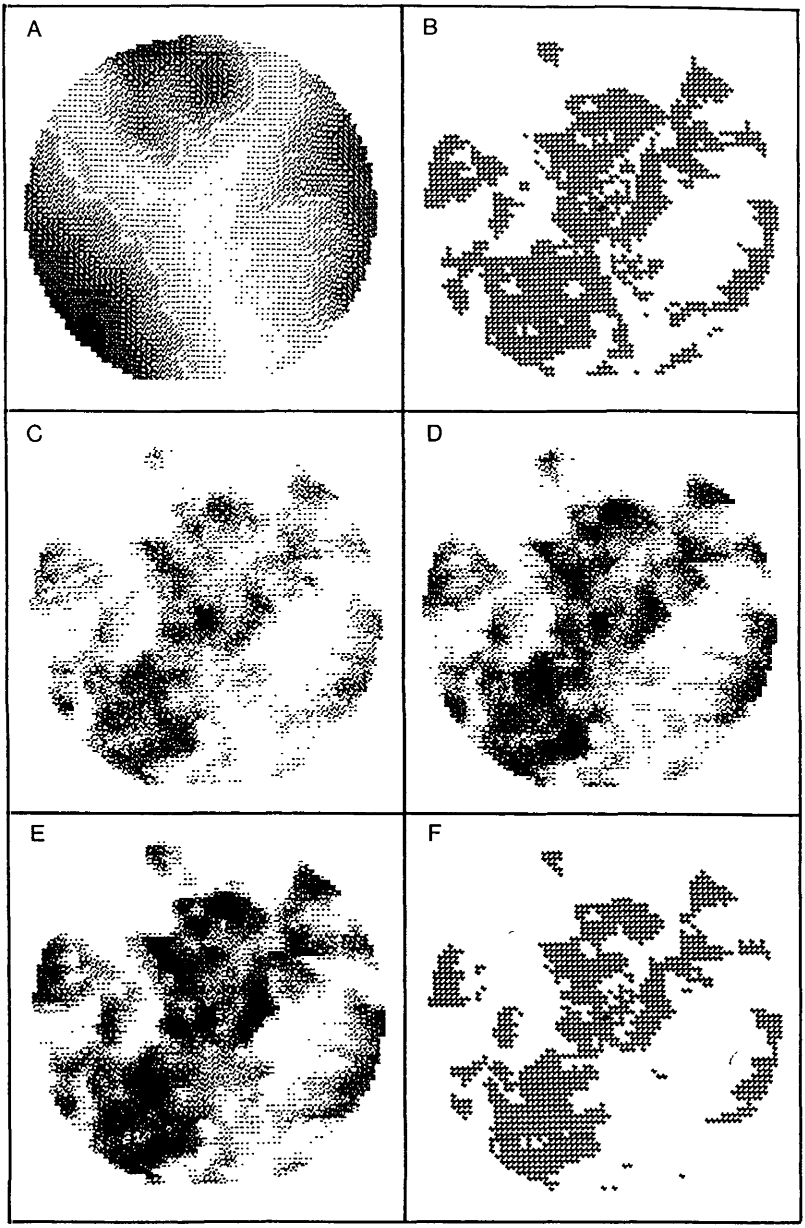


Figure 3

DEM and Viewsheds of Point 2: A) the elevations in the near-view; B) the viewshed image from the original DEM; fuzzy viewsheds where C) I=0, D) I=0.7, and E) I=0.9; and F) an image showing the viewshed where p>=0.5 from E.



in the error = 0.9, and value in the fuzzy viewshed >= 10 (probability of being within the viewshed >= 0.5). In those figures therefore, the spatial arrangements of the tabulations presented and discussed above can be seen.

In both areas, the application of increasing autocorrelation to the noise progressively increases the certainty of similar areas being within the viewshed. Thus the nucleus of the zones with high probability identifiable in those viewsheds where the noise term had I = 0.9 are identifiable in images where the noise had I = 0.

The ridge-top location, Point 1 (Fig. 2), has a higher frequency of high counts in the image. The areas of high likelihood of belonging to the viewshed are more contiguous for this location than for Point 2 (see Fig. 3); most of the high likelihood values are in three blocks of land, one immediately to the northeast of the viewpoint, one to the north, and the other to the southwest. From Point 2 (Figure 3), the areas of greater certainty are by contrast highly disjoint, although one large area does exist to the southwest.

Particularly, it should be noted that in neither test location is it possible to identify those areas that are of high likelihood in the fuzzy images from properties of the viewshed as calculated in the original DEM (Fig 2b, and 3b). For example, elevations both above and below the viewpoint may contain both high and low certainty.

CONCLUSION AND CONTINUING WORK

Firstly, it is possible to observe that no absolute certainty can be placed on the viewshed. Depending on the spatial autocorrelation that is applied to the noise term, it is apparent that the likelihood of cells being in the viewshed, can be very low. Indeed, with the assumption of spatial independence (where I = 0, the only assumption that is acceptable given the method of calculation and publication of the USGS error statement) very little if any certainty can be placed upon the standard viewshed calculated. Fortunately, perhaps, the viewshed from an elevated location seems to be more reliable than one in a depression, but work presented here is only exploratory.

Although the method used here is too computationally intensive for widespread implementation, it does yield alternative fuzzy viewsheds from a particular viewpoint. In this paper alternative fuzzy viewsheds derived from simulated DEMs with variable spatial autocorrelation are discussed. The algorithms can already accommodate variable RMSE, and can be recoded to accommodate variability in other parameters.

This paper points to further work in three main areas, the effect of the error, the source of it, and terrain control on the stability of the viewshed. In the first of these, it is necessary to develop a method to predict the fuzzy viewshed, which derives results similar to those generated by simulation, but which is more computationally efficient, and so possible to use in regular GIS operations. To achieve this it is necessary to explore further, probably by simulation, the relationships between error structures in DEMs, and fuzziness in viewsheds. The effects in the middle and far view should also be explored. In the area of error sources, considerable need exists for more information on the structure of the error in the DEM, and the relationship between error derived from digitization (the only error studied here), and that derived from original map compilation. Finally, aspects of relative elevation, other relief properties, and further aspects of terrain on patterns of fuzziness and the nature of error need to be explored.

ACKNOWLEDGEMENT

I wish to thank Larry Band, University of Toronto, who supplied the digital elevation data when it was needed, and Ron Eastman, Clarke University, for allowing me to examine the code of the Idrisi Viewshed module. John Felleman, Mike Goodchild, Dave Mark, and Dan Griffith all commented on aspects of the research, and Audrey Clarke assisted and advised on figure preparation. The work was conducted on a 386 PC, and I also wish to thank all those students who also wished to use it for their patience.

REFERENCES

Anderson, D.P., 1982, Hidden line elimination in projected grid surfaces: ACM Transactions on Graphics, Vol.1, pp.274-291.

Aronoff, S., 1989, Geographic Information Systems: A Management Perspective, WDL Publications, Ottawa.

Band, L.E., 1986, Topographic partition of watersheds with digital elevation models: Water Resources Research, Vol.22, pp.15-24.

Besag, J. and P.J.Diggle, 1977, Simple Monte Carlo tests for spatial patterns: Applied Statistics, Vol.26, pp.327.

Chrisman, N.R., 1989, Modeling error in overlaid categorical maps, in M.F.Goodchild and S.Gopal (Eds.), The Accuracy of Spatial Databases, Taylor and Francis, London, pp.21-34.

Clarke, K.C., 1990, Analytical and Computer Cartography, Prentice Hall, Englewood Cliffs, NJ.

DeFloriani, L., Falcidieno, B., and Pienovi, C., 1986, A visibility-based model for terrain features, in Proceedings of the Second International Symposium on Spatial Data Handling, pp. 235-250.

Eastman, R., 1989, IDRISI: A Grid-based Geographic Analysis System, Version 3.1, Graduate School of Geography, Clark University, Worcester, Mass.

Felleman, J., and Griffin, C., 1990, The role of error in GIS-based viewshed determination - A problem analysis, US Forest Service, North Central Experiment Station, Agreement No. 23-88-27.

Fisher, P.F., 1990, Simulation of Error in Digital Elevation Models, Papers and Proceedings of the 13th Applied Geography Conference.

Goodchild, M.F., 1980, Algorithm 9: Simulation of autocorrelation for aggregate data, Environment and Planning A, Vol.12, pp.1073-1081.

Goodchild, M.F., 1986, Spatial Autocorrelation, CATMOG 47, Geo Books, Norwich, UK.

Griffith, D.A., 1987, Spatial Autocorrelation: A Primer, Association of American Geographers, Washington, DC.

Lammers, R.B. and L.E.Band, 1990, Automating object representation of drainage basins, Computers and Geosciences, Vol.16, pp.787-810.

MacDougall, E.B., 1975, The accuracy of map overlays: Landscape Planning, Vol. 2, pp.23-30.

Maffini, G., Arno, M., and Bitterlich, W., 1989, Observations and comments on the generation and treatment of error in digital GIS data, in M.F.Goodchild and S.Gopal (Eds.), The Accuracy of Spatial Databases, Taylor and Francis, London, pp.55-67.

Newcomer,, J.A., and Szajgin,J., 1984, Accumulation of thematic map errors in digital overlay analysis: The American Cartographer, Vol. 11, pp.58-62.

Openshaw, S., M.Charlton, C.Wymer and A.Craft, 1987, A Mark 1 Geographical Analysis Machine for the automated analysis of point data sets: International Journal of Geographical Information Systems, Vol. 4, pp.335-358.

Ripley, B.D., 1987, Stochastic Simulation, John Wiley, New York.

Sutherland, I.E., Sproull, R.F., and Scumaker, R.A., 1974, A characterization of ten hidden-surface algorithms, Computing Surveys, Vol. 6, pp.1-55.

Thompson, M.M., 1988, Maps for America, Third Edition, USGS, Reston, Va.

USGS, 1987, Digital elevation models, Data Users Guide 5, USGS, Reston Va.

Veregin, H., 1989, Error modeling for the map overlay operation,iIn M.F.Goodchild and S.Gopal (Eds.), The Accuracy of Spatial Databases, Taylor and Francis, London, pp.3-18.