THREE-DIMENSIONAL (3D) MODELLING IN A GEOGRAPHICAL DATABASE

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ABSTRACT

While most current Geographical Information Systems (GIS) are 2D GIS, more and more application domains as geology, civil and military engineering, etc, need 3D GIS. The particularities and capabilities of a 3D GIS are outlined in this paper. Although 3D GIS are not much developped, some studies have already been done, especially in order to represent 3D data for geological or architectural applications. Such 3D models are generally based upon Computer-Aided Design (CAD) models. The use of 3D CAD models for representing 3D in GIS is discussed in the paper. As 3D GIS represent a 3D map, not only the surface of the terrain but also the entities set on the relief, e.g. buildings, have to be represented. The proposed 3D model for a GIS consists of representing 3D entities set on a DTM. These entities are represented with different approximation levels, the first two being used as index and processes accelerators and the last offering an efficient visualization support. The first two levels are entities decomposition into minimal bounding rectangular parallelepipeds (MBRP). The last level is the external representation: it is a Boundary Representation (BR). Indexation is shortly discussed in the paper. An existing geographical database system is currently extended to support these 3D entities.

1. INTRODUCTION

Most current geographical database systems are restricted to 2D information. However, the third dimension appears to be more and more necessary for many application domains as geology, civil and military engineering, town planning, robotics, etc... These applications require not only the modelling and the visualization of 3D data but also the manipulation of these data. For example, in a town planning project, an impact study needs examining 3D visual intrusion caused by a building on the terrain and needs being able to move the building location. It is comparable with the design of a 3D scale model which would furthermore be easy to modify.

3D data involved in a GIS may be subsoil data or relief data or 3D geographical data that can either be human made entities like bridges or naturally occuring entities like forests. A 3D entity is a volumetric object which can be convex or concave and which may contain holes... A cube and a sphere are examples of 3D entities.

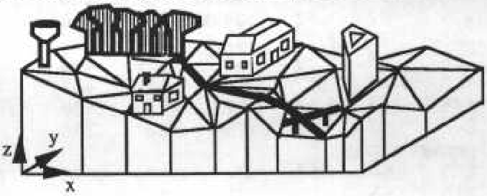

Figure 1: 3D Model for a GIS

Three-Dimensional GIS represent a 3D map. Consequently it is interesting to distinguish between the surface of the terrain and the entities set on the relief, e.g. buildings, bridges, forests, etc... In the proposed model, 3D geographical entities are set on a Digital Terrain Model (DTM) which describes the relief (See Fig. 1). As discussed later, a DTM alone is not sufficient to represent a 3D map.

It would also be possible to represent geological data by assuming geological layers are 3D entities limited by surfaces like DTM. A geological layer is then a polyhedron (the slice between DTM). In this paper we limit our scope on over the ground representation.

One the one hand, modelling the third dimension in a GIS provides the following main advantages. 3D is necessary to fully represent spatial reality and to remove some ambiguities, e.g. a road hanging above another one or a roman aqueduct with several levels. Those Geographical Information Systems (GIS) restricted to 2D usually rely on a special symbolization to represent a segment corresponding to two overlapping roads; furthermore, the user have to clearly indicate which road is above the other while this information is implicitly captured by the internal model in a 3D GIS. The elevation is lost in a 2D GIS while it may be important. 3D purpose is to represent as accurately as possible any three-dimensional shapes.

On the other hand, a 3D representation have the three following important disadvantages: (1) the volume of storage is generally very important, (2) geometrical computations are more complex with a 3D than with a 2D representation, and (3) 3D data capture is rather difficult. In particular, uncertainty about 3D data (as relief or subsoil data) is greater than about 2D data.

The paper is organized as follows: intermediate solutions between 2D and 3D are first reviewed, then the particularities and capabilities of a 3D GIS are presented. An overview of 3D CAD models is given in section 4. Our solution is then described. Section 8 corresponds to a short discussion on index. At last, we conclude.

2. 2.5D AND DTM

2.1 Digital Terrain Model (DTM) and 2.5D

Spatial data are 3D data. As a result, elevations are lost in a 2D map. Several intermediate solutions between 2D and 3D, as DTM and 2.5D may be defined. As many others, we define here 2.5D as a representation mode of a flat (or nearly flat) entity such that z= f(x, y) and f is a true function. This definition encompasses both usual 2.5D maps and DTM.

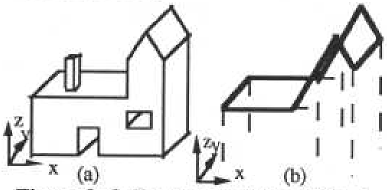

Figure 2: 2.5D representation (Fig. 2 (b) is a 2.5D representation of the entity plotted in (a))

With usual 2.5D maps, each geographical feature (or part of) such as a forest, a field or a road becomes an individual entity represented by its 2D-boundary augmented with a z-coordinate. While linear entities are well approximated, inner surface elevation variations are not captured. A non homogeneous approximation of the underlying terrain is provided whose accuracy depends on the local entity density.

With DTM the terrain itself is a single entity represented by a sample of points corresponding to a facetized surface with these points as nodes. Hence, a tesselation covering the whole terrain is achieved. Different tesselations (regular, irregular or hybrid) may be built with the same dataset. Usually an height is associed with each node of the tesselation. Other informations like the slope may be associated with a face of the mesh. Hence, we have a representation of the terrain offering an approximation of the heights. DTM extensions have been proposed where the function z= f(x, y) is assumed continuous and sufficiently differentiated: an example is given by spline function. However, in DTM specific entity boundaries are usually lost. A common solution to this problem is the combination of both a DTM and a 2.5D representation for a single map. (Miller 1958, Ebner 1992, Fritsch 1992)

Keeping "flat" entities in mind, another solution for 2.5D is possible: instead of augmenting a boundary-based 2D representation of an entity with elevation values, a 2.5D representation could be derived from the 3D representation of an entity given by its Boundary Representation (BR). Only upper facets of the BR are kept (as expected, we define upper facets as those visible - vertically - from above). Fig. 3 (b) is the 2.5D representation of the entity of Fig. 3 (a). Such a representation can be extended at low cost to capture the vertical thickness of entities such as most buildings.

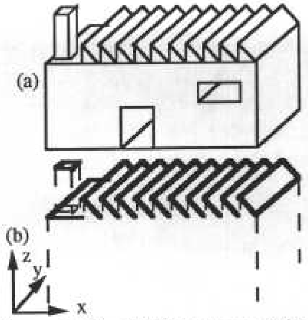

Figure 3: Our 2.5D representation

2.2 3D Versus 2.5D

A 2.5D GIS is able to represent two overlapping roads (assuming that a road has no thickness) but not an arch, nor a roman aqueduct with several levels (e.g. Fig. 4) because it can not represent entity thickness. In such a case, the user does not know if it is feasible to get between the columns of the arch. 2.5D models only give a rough approximation of an entity which may appear to be insufficient for applications like navigation. Likewise they can not represent holes in an entity. Complex 3D entities (e.g. Fig. 4), especially those with holes, openings, etc..., can only be represented with full 3D because of the need for an arbitrary number of elevations for a given (x, y) position.

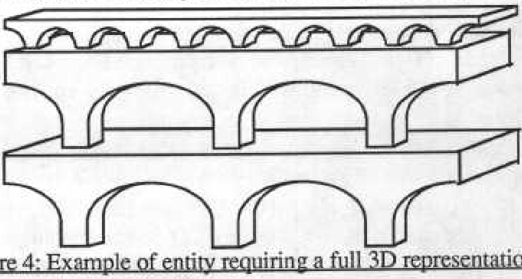
Figure 4: Example of entity requiring a full 3D representation

As representing a 3D map takes more than the representation of the surface of the earth and thus more than one z-coordinate for a given (x, y) position, a DTM or a 2.5D model are not sufficient. Reversely, as a full 3D representation of the terrain proves too costly and awkward to use, we have to model entities set on a representation of the surface of the earth given by a DTM.

3. 3D GIS

Although 3D GIS are not much developped, some studies on 3D GIS have already been done (Jones 1989, Raper 1989, Pigot 1991 and 1992), especially in order to represent 3D data for geological or architectural applications. Such 3D models are generally based upon Computer-Aided Design (CAD) models as the voxel model.

3.1 The Particularities of a 3D GIS

3D requirements of GIS differ from those of CAD systems on at least three accounts:

entity "flatness", data acquisition mode, and geographical entity shape.

Entity "flatness" arises because the data size in GIS is not the same as in CAD or in architecture in the sense that geographical data are drastically more expanded among the plane (e.g. 100 km x 100 km) than among the third dimension (the highest point in the world is the Everest which culminate at 8882 m above the sea level, and the highest mountain in the world is Mauna Kea (Hawaï) that reach 10203 m 4205 m of which are under the sea. In the case of buildings, their height do not top a few hundred meters).

In GIS, the data acquisition mode implies a descriptive approach; this is true not only for naturally occuring entities as forests and mountains but also for superstructures as buildings or bridges that can not be easily decomposed into simple (volumetric) primitives. Note that this is the same problem as trying to rebuild the CSG representation from a descriptive CAD representation such as voxel or octree.

Natural entities that are represented in a GIS (e.g. forest) seldom have a regular shape contrary to human made entities.

What is more, a 3D GIS does not deal uniquely with 3D volumes. The terrain itself is merely a 3D surface: unless the surface of the terrain is fictitiously closed by a "floor" and by upright "walls" joining the surface and the floor or is roughly approximated by contiguous parallelepipeds with differents heights, the relief can not be seen neither as a 3D closed entity, nor as a solid (in the sense of "solid" defined in (Requicha 1980)). The above-mentioned CAD models which are models for solids can not exactly represent the relief: a DTM is more appropriate for this purpose.

3.2 The Fundamental Capabilities of a 3D GIS

The operations of a 3D GIS can be sorted into four basic classes (Bak 1989):

(1) entity definition and modification - Boolean set operations (union, intersection and difference) and the geometric transformations (rotation, translation, scaling) have to be easily computed;

(2) visualization - which must be efficient;

(3) integral property computation - volume, surface area, distance or mass (it is specially important for geological applications);

(4) information retrieval - The system has to be able to do spatial and non-spatial queries and must therefore provide an efficient clustering method, an index in order to speed up searches.

4. 3D GIS VERSUS 3D CAD MODELS

4.1 An Overview of Main 3D CAD Models

CAD models can be sorted in two groups: volume representation and boundary representation (Foley 1990).

In the first group, an object is described as a combination of volume primitives (e.g. a cube). This group consists of the decomposition models which comes in two classes: either the model is a constructive one, either it is a descriptive one. Constructive Solid Geometry (CSG) and Primitive Instancing (PI) belong to the first class. In the second class there are the octrees, polytrees and their extensions and also the voxel model. As it is outlined by the name of this class, a constructive model is a model where an object is built by assembling components. On the contrary, a descriptive model do not aim to decompose the objects into its constituant parts but it decomposes the object space into a collection of contiguous nonintersecting solids; these solids are primitives (e.g. cubes) but not necessarily of the same type as the original object. They may not be object constituants. They are either partially or entirely occupied by the original object or empty.

In CSG, solids are built combining elementaries shapes as cylinders, cubes or spheres, by means of regularized Boolean set operators. Primitive Instancing (PI) differs from CSG in that PI primitives are only 3D shapes that are relevant to the application domain and are not limited to certain simple 3D solids. PI is an adequate representation if the set of entities that have to be modelled is well known and does not vary. It is often used for relatively complex entities, such as gears, that are tedious to define with a CSG model, yet are easily described by a few high-level parameters. If the entities to be represented are well adapted to the model, CSG and PI provide concise representations.

Cell-decomposition is an irregular decomposition of an entity into cells (e.g. a cube, a tetrahedron) and a representation of each cell in the decomposition. The voxel model (or spatial occupancy enumeration) is a special case of cell decomposition in which the solid is decomposed into identical cells arranged in a fixed grid.

In an octree (or its extensions: polytree, ...), the whole space is recursively divided into cubes giving a hierarchical structure. Leaves nodes are an accurate to a given degree cubical approximation of the entities constituents.

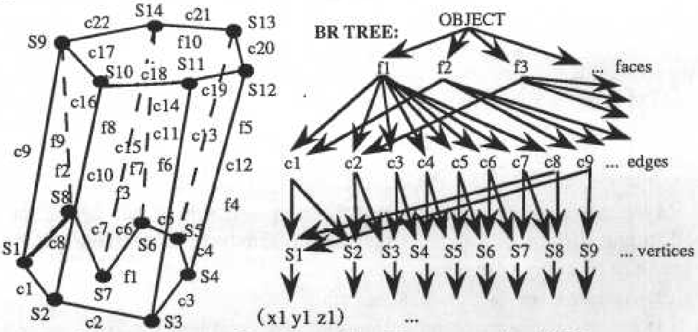

Figure 5: Example of Boundary Representation (BR)

In the second group, a solid is described by its bounding surface. These models are based on surfaces and have to include enough information in order to know how the surfaces join for defining closed volumes. Boundary Representation (BR) belongs to this group. BR represents a solid in terms of its bounding surfaces: solids are represented as union of faces, each of which is defined by its boundaries ie the vertices and edges bounding it. The model primitives are therefore: faces, edges and vertices. Every entity is represented by a constant depth tree. Such an entity representation is a polyhedral one because faces are planes and edges are straight lines: it can only be an approximation for non polyhedral entities. Fig. 5 illustrates a BR.

The most common CAD models are CSG and BR. For a complete survey of 3D modelling, we refer the reader to (Foley 1990) or (Cambray 1992).

4.2 Discussion about the Use of CAD Models for 3D GIS

In cartographic applications, the decomposition of an entity into faces, edges and vertices is a straight extension of the 2D case: BR seems natural. Unlike volume representation, BR model is easy to manipulate for the visualization but is restricted to polyhedral representations. BR is usually neither concise, nor efficient for computing boolean set operations or integral properties or for models validations.

CSG is a constructive model better fitted to the representation of entities built by men (by assembling simple constituents (sphere, cube, ...)), such as plumbing parts. Hence, a constructive (volume) representation such as CSG has to be eliminated.

The voxel model and spatial decomposition models are often used for geological applications. Such representations provide an easy way for computing integral properties or location problems or Boolean set operations. But these representations tend to be extensive on storage space and inexact for the purposes of representing entity shapes, particularly if they have curved surfaces.

More generally, integral properties or Boolean operations calculations can be easily achieved using a volume representation. Therefore, any storage-efficient descriptive (volume) representation can be selected. Storage efficiency disqualifies voxel mode. Note also that bounding volumes may be used for applications looking at entities as obstacles. Different representations can be chosen.

Unfortunately, there exist no CAD model perfectly adapted to 3D modelling in GIS.

5. A 3D MODEL FOR A GIS

In order to overcome CAD models shortcomings, a new solution (for representing 3D geographical entities) is described. In the proposed model, 3D geographical entities are set on a Digital Terrain Model (DTM). Every entity is represented with a multiresolution 3D model, i.e. having several approximation levels.

5.1 Multiresolution Model for Representing a Geographical Entity

The most important functions of a 3D GIS differ on their representation requirements. With a single representation mode a system can not be optimal for all applications. On the contrary, a multiresolution mode combines the advantages of several representations while limiting their disadvantages. To offer different approximation levels comes to the same thing as using for computing a given operation (e.g. indexation, Boolean operations) the most adapted representation for this operation.

Using different approximation levels provide for the selection of the appropriate level according to a given scale or priority in some computation. These levels are also used by the system for making rough approximations before the actual accurate, but expensive, computation is made.

It should be possible to gather together the same level representations of several nearby (semantically and spatially) entities: for example to gather together houses belonging to the same block. This operation is nicknamed "semantical zoom" in the sense that we obtain an increase effect: an entity (block) becomes several entities (houses), this gathering can be made because it is meaningfull: a block is a group of houses.

There are three approximation levels for each entity. The proposed model is an internal model. The external model is a set of faces. Indeed, the user provides a points set which gives a set of faces and allows then to compute the entity BR. We assume therefore that the BR is the most accurate entity representation. The system immediately computes the other levels from the provided BR. The user can only modify the BR, the modifications are then done on the other levels.

6. THE DIFFERENT APPROXIMATION LEVELS

6.1 The First Approximation Level

The highest level is the minimal bounding rectangular parallelepiped (MBRP) of the 3D entity shape. The MBRP is parallel to the axis of the referencing system of the world in order to ease the computations.

The MBRP of the geographical entity plotted in Fig. 6 (coloured with grey in Fig. 7) is shown in Fig. 7.

This level can be used as an index key because it is a bounding volume of the 3D entity which is the extension of the bounding rectangle in 2D GIS. This bounding volume allows an easy entity localization. It is then possible to cluster thanks to this approximated shape. This bounding volume may be also useful to speed up 3D entities operations (e.g. intersection): it allows a rough but quick approximation of the operation result. Indeed, it is much less expensive to compute Boolean operations with only one parallel to the axis MBRP than with any other representations (CSG, BR, octree,...). On the contrary, the result is only an approximation. As a matter of fact this level realize a filter that eliminates non adequate answers. For example, if the MBRP of two different entities do not intersect, then the two entities do not intersect: the answer is therefore given avoiding the intersection computation on two entity representations which may be extremely complex.

As far as semantical zoom is concerned, it is easy to calculate the MBRP of a group of entities semantically and spatially closely related. This allows a much more speeding up of spatial predicates.

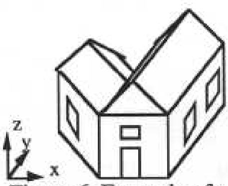

Figure 6: Example of a 3D geographical entity

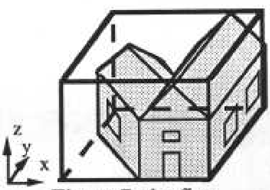

Figure 7: the first approximation level

This level is therefore useful for the system as an index and as processes accelerator, especially spatial predicates. The integral properties computation is very fast but the result is a very rough approximation.

6.2 The Second Approximation Level

The second level is an entity subdivision in at most n MBRP (n limited by system, e.g. n= 8). The MBRP are parallel to the axis of the referencing system of the world. Fig. 8 describes the eight MBRP (grey or shaded) of the second level decomposition of the entity plotted in Fig. 6.

The way chosen to calculate this representation level is equivalent to the octree construction. Indeed, the MBRP of the first level is recursively divided into eight parallelepipeds, then this decomposition is enhanced.

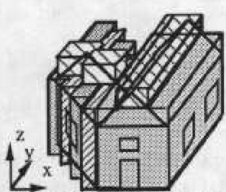

Figure 8: the second approximation level

This level provides a better approximation, not only of the processes but also, of an entity shape (which is usually sufficient for many applications). Computations remain fast. n is fixed by the system to ease computation and storage. The objective of this level is to speed up Boolean operations and integral properties computations while providing a relatively good approximation of the result. It realizes a more accurate filter than the one of the previous level because on the one hand, the global shape of the entity is better approximated than previously and, on the other hand, it is feasible to know which ones among the n parallelepipeds have, for example, an empty (resp. non empty) intersection with the parallelepipeds of an other entity: to enhance the response, it will only be necessary to consider the part of representation fitted with these parallelepipeds. Integral properties computations will be much better than with the first level.

This level is therefore useful for the system not only to speed up processes but also to quickly retrieve an entity part.

6.3 Remark about the First Levels

The first two levels may appear to be very much like a CSG representation but restricted to only one primitive (the parallelepiped) and only one operation: the gluing which is a restricted form of union. However, the parallepipeds considered in this model are not entity constituents: they are bounding parallelepipeds. The representations are therefore "true" in the sense that they include the entity. The operations on these representations provide approximations because a bounding parallelepiped is usually not fully filled by the entity. These operations do not generate errors because there is no part of the entity outside of the union of MBRP. The use of bounding volumes as a test to speed up processes (e.g. ray tracing) is a usual image synthesis method.

It would be possible to allow the user to choose the decomposition level. For instance, if the user wants three recursive decompositions of an entity, the three levels of the octree are computed for this entity; then, the decomposition is enhanced. This would be primarily used for visualization. Note that the corresponding accuracy may be as good as a user wishes to the price of additional complexity.

6.4 The Third Approximation Level

An additional third level is provided to ease the connection between our system and CAD softwares. This level is a BR because BR is the most common CAD model with CSG. As mentioned earlier BR has been prefered to CSG.

The chosen BR is a non manifold one in order to be able to uniformly consider 3D, 2D or 1D entities or 3D, 2D and 1D hybrid entities (Masuda 92). Such geographical entities do satisfy a generalization of the Euler-Poincaré formula (but not necessarily the usual one): they may have dangling faces or dangling edges.

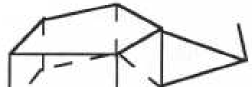

Figure 9: Example of 3D, 2D and 1D hybrid entity

This latter level provides almost perfect rendering and great accuracy in geometric computations. The entity BR is a good representation even if the entity is a polyhedral one. If we do not want to be restricted to polyhedra, we must consider a BR generalized to curves and curved surfaces.

7. CONCLUSIONS ABOUT THE MODEL

The main advantages of our model are the following ones:

• our model provides a spatial index (with the first level);

• operations are speeded up. As BR is the more accurate provided representation, the accurate computations have to be done with the BR. It is better to compute operations with the first two levels representations and particularly with the second one before the actual accurate, but expensive, computation is made (if the second level filter has not eliminated the entity). The benefits of computing with entity approximation not only lie in the fact that the system can filter the answers but also in the fact that it can straightly obtain, from the second level representation, the part of BR associated with one of the n parallelepipeds. Indeed, as far as the Boolean operations are concerned, the second level computation allows the system to determine what MBRP are really important for the computation, then the system only need to make the computation with the part(s) of BR associated to these parallelepiped(s). This enables the system to compute with only part of the BR. The operation complexity is then reduced. The two first level therefore allows efficient Boolean operations or integral properties computations;

• our model provides an efficient visualization thanks to BR level.

However, there are some disadvantages:

• accurate computations are expensive as BR is not efficient for Boolean or integral properties calculations. This disadvantage is reduced by the use of the speeding up techniques previously presented;

• entity shape may be roughly approximated by the first two levels as these levels provide a polyhedral representation and as the MBRP are parallel to the axis of the referencing system of the world and not of the entity. But, for the scale considered in a 3D GIS, a very accurate representation is not always necessary and is not realistic from a data capture point of view;

• BR is generally expensive from the storage volume point of view but the BR level is essential in the sense that the data provided to the system are set of faces (i.e. a representation mode close to BR). Furthermore, BR allows a relatively accurate representation of entities. On the other hand, the storage volume of the first two levels (n° 1 and n° 2) is limited and small.

8. INDEX

As we have already shown, spatial extent is larger among the plane than among the third dimension. Therefore, a 2D spatial index is sufficient for a 3D GIS. However, the first levels provide a 3D spatial index. Indeed, the MBRP of the first level provides a 2D spatial index (we only consider xmin, xmax, ymin, and ymax so that we have a bounding rectangle) and a rough elevation index (with zmin, zmax) which can ease the selection on the z axis. This elevation index is sufficient in the sense that, in the geographical world, (1) the spatial extent among the z axis is not very large, and (2) the number of objects which have the same planar location is rather small (a counter-example is the case of geological applications where there are many geological layers). The second approximation level allows to select a part of the 3D entity.

9. CONSISTENCY BETWEEN 2D AND 3D CARTOGRAPHICAL MODELS

Our model is compatible with 2D cartographic models. With 3D data, users can only consider 2D maps through a plane projection of a 3D map. It is therefore necessary for a 3D GIS to be able to display, manipulate and query a 2D map and to obtain again the 3D map as soon as it is needed.

A user may display and manipulate a 2D rendering of some map while keeping 3D correctness: for example, if there is visually an intersection between a river and a bridge on a 2D projection, the system properly answers that they do not intersect. A 2D intersection may correspond to a 3D null-intersection (i.e. a crossing: one entity is above the other). To this end, the process has to consider 3D data, even if a query is done on the 2D map.

Some 3D operators may be seen as refinement of 2D operators. Hence, the 3D extension of 2D intersection operator (cf. Fig. 10) is either a (true) 3D intersection, or a crossing.

figure 10: intersection

10. CONCLUSION

The particularities of a 3D GIS lies in the following points: entity "flatness", data acquisition mode, geographical entity shape, and the nature of data involved (relief, subsoil and 3D geographical data). Their capabilities are the definition and the modification of entities, visualization, computations, information retrieval. So that a 3D GIS has to meet these requirements.

This leads us to propose a model for 3D GIS which consists of representing 3D entities set on a DTM. These entities are represented with different approximation levels, the first two being used as index and processes accelerators and the last offering an efficient visualization support. The first two levels are volume representations: they are entities decompositions into MBRP. The last one is a surface representation: it is a BR. It is the external representation.

An existing database system based on an extended relational model and supporting built-in spatial data types, operations, and indexes, GéoSabrina, is currently extended to support these 3D geographical entities. GéoSabrina, which is developped at the MASI Laboratory, uniformly manages both semantical data and cartographical data (location and geometric shape) (Larue 93). New spatial data types are defined along with operations upon those types. Hence, two additional data types are defined: one for the pure 3D entities (ie 3D entities with no dangling faces, dangling edges nor isolated points) and the other for hybrid entities. Examples of operations upon these entities or/and the DTM are union and geometrical projection.

This model is also used to solve spatio-temporal problems (Yeh 93).

REFERENCES

Bak, P. R. G., and Mill, A. J. B. (1989), *Three dimensional representation in a Geoscientific resource management system for the minerals industry*, In: Three dimensional applications in Geographical Information Systems, Jonathan Raper (ed.), pp. 155-182. Publisher: Taylor & Francis.

de Cambray, B. (1992), *Modélisation 3D: état de l'art*, Technical report, MASI 92.71. Publishers: Institut Blaise Pascal & Laboratoire MASI, Paris.

Ebner, H. and Eder, K. (1992), *State-of-art in digital terrain modelling*, In: Proceedings of EGIS'92, Munich, pp. 681-690.

Foley, J. D., van Dam, A., Feiner, S., and Hughes, J. (1990), *Computer graphics, principles and practice*. Publisher: Addison-Wesley Systems Programming Series.

Fritsch, D. and Pfannenstein, A. (1992), *Conceptual models for efficient DTM integration into GIS*, In: Proceedings of EGIS'92, pp. 701-710.

Jones, C. B. (1989), *Data structures for three-dimensional spatial information systems in geology*, International Journal of Geographical Information Systems, 3 (1), pp. 15-31.

Larue, T., Pastre, D., and Viémont, Y. (1992), *Strong Integration of Spatial Domains and Operators in a Relational Database System*, SSD'93, LCNS n° 692, pp. 53-72.

Masuda, H. (1992), *Recent progress in 3D geometric modeling*, MICAD 92, Tutorial, Paris.

Miller, C. L. and Laflamme, R. A. (1958), *The Digital Terrain Model - Theory & Application*, Photogrammetric Engineering, XXIV (3), pp. 433-442.

Pigot, S. (1991), *Topological models for 3D spatial information systems*, In: Technical papers, ACSM-ASPRS Annual Convention, Baltimore, Auto-Carto 10, 6, pp. 368-392.

Pigot, S. (1992), *A topological model for 3D spatial information system*, In: P. Bresnahan, E. Corwin, and D. Cowen (ed.), Spatial Data Handling'92, Charleston, USA, 1, pp. 344-360.

Raper, J. and al. (1989), *Three dimensional applications in Geographical Information Systems*, Jonathan Raper (ed.). Publisher: Taylor & Francis.

Requicha, A. A. G. (1980), *Representations for rigid solids: theory, methods, and systems*, Computing Surveys, 12 (4), pp. 437-464.

Yeh T.S. and de Cambray B. (1993), *Time as a geometric dimension for modelling the evolution of entities: a 3D approach*, Second International Conference on Integrating Geographic Information Systems and Environmental Modelling, Breckenbridge, Colorado, USA, September 1993.