# AN EVALUATION OF FRACTAL SURFACE MEASUREMENT METHODS USING ICAMS (IMAGE CHARACTERIZATION AND MODELING SYSTEM)

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## ABSTRACT

With the fast pace of increase in spatial data anticipated in the EOS (Earth Observing System) era, it is necessary to develop efficient and innovative tools to handle these data. ICAMS (Image Characterization and Modeling System) is an integrated software module designed to provide specialized spatial analytical functions for visualizing and characterizing remote-sensing data. Fractal analysis is the main module in ICAMS. Although fractals have been studied extensively before, the question of which fractal measurement method should be used remains. This paper evaluates the three fractal surface measurement methods that have been implemented in ICAMS, including the isarithm, variogram, and triangular prism methods. Results from applying five simulated surfaces of known dimensions ($D = 2.1$, 2.3, 2.5, 2.7, and 2.9) to the three methods show that the isarithm method calculates the fractal dimensions fairly accurately for all surfaces. The variogram method, on the other hand, yields accurate results only for surfaces of low dimensions. For surfaces of higher dimensions, the variogram method is unstable. The triangular prism method produces inaccurate results for almost all the surfaces, and its usefulness is questionable. More in-depth evaluation, however, is needed to verify the present findings.

## INTRODUCTION

We are currently working on the development of a software module called ICAMS (Image Characterization and Modeling Systems). ICAMS is designed to run on Intergraph-MGE and Arc/Info platforms to provide specialized spatial analytical functions for characterizing remote-sensing images. The main functions in ICAMS include fractal analysis, variogram analysis, spatial autocorrelation analysis, texture analysis, land/water and vegetated/non-vegetated boundary delineation, temperature calculation, and scale analysis.

The development of ICAMS has been driven by the need to provide scientists efficient and innovative spatial analytical tools for characterizing and visualizing large-scale spatial data such as remote-sensing imagery. As spatial data become increasingly available, the need for useful analytical tools to analyze these various forms of spatial data becomes more pressing. The NASA's Earth Observing System (EOS) to be launched in the late 1990's is one example data source that will provide useful data to the scientific community. The fast pace of increase in digital data posts an immediate problem, that is, how such an enormous amount of data can be handled and analyzed efficiently. Clearly, advances in global as well as local environmental modeling must need both components: data; and the analytical tool to handle the data. An overview of the theoretical background of and the practical need for developing ICAMS, as well as its system design and functionality, can be found in Quattrochi, et al. (1997).

Along with the need for more efficient and innovative spatial analytical techniques is the need for more fundamental research on the applicability and reliability of such techniques. Through the employment of an integrated software package such as ICAMS, it would be easier to carry out the evaluation tasks, and by making the software available to the wider scientific community, a variety of applications and evaluations can be made. These advantages will be realized especially in ICAMS, as most of the implemented specialized functions have seldom been applied to landscape characterization using remote-sensing imagery, though they were considered to have great potential in characterizing landscape patterns for global environmental studies (Woodcock and Strahler 1987).

This paper focuses on the use of the fractal module in ICAMS. In particular, we examine the three fractal surface measurement methods that have been implemented in the software, including the isarithm, variogram, and triangular prism methods. A series of hypothetical fractional Brownian motion (fBm) surfaces with known fractal dimensions were first generated. These surfaces were applied to the three algorithms in ICAMS on the Intergraph-MGE platform to compute their fractal dimensions. The comparison between the known and the computed fractal dimensions provides an assessment of the reliability and effectiveness of the three most commonly used fractal surface measurement methods for characterizing and measuring landscape patterns.

The evaluation results will be useful to further improvement of the fractal measurement methods and possible modifications of the algorithms in ICAMS. A host of related research questions utilizing fractals can be examined. For example, do different environmental /ecological landscapes and processes (e.g. coastlines, vegetation boundaries, wetlands) have their unique fractal dimensions? Can the fractal dimension be used as a means to identify regions with different properties, and ultimately be used as a part of metadata? Or, what is the significance of changes in fractal dimension, either in time or space?

## METHODS AND DATA

Fractal analysis has been suggested as a useful technique for characterizing remote sensing images as well as identifying the effects of scale changes on the properties of images (De Cola 1989 & 1993; Lam 1990; Lam and Quattrochi 1992). A major impediment in applying fractals is that there are very few algorithms readily available for researchers to use and experiment, and for those who can access or directly construct their own programs, the frustration is that the results from applying differing algorithms often contradict each other. A thorough evaluation of the various measurement techniques is necessary before they can be used to reliably characterize and compare the various types of landscapes.

The three fractal surface measurement methods that have been implemented in ICAMS, the isarithm, variogram, and triangular prism methods, have been applied to real data and documented in detail in various studies (e.g., Lam and De Cola 1993; Jaggi et al., 1993). However, they have never been systematically evaluated using controlled, synthetic surfaces. The use of controlled surfaces in testing these algorithms, such as the fractional Brownian motion (fBm) surfaces used in this study, should provide a standard to compare with and therefore helps in revealing the major characteristics and differences among the methods. For the ease of interpretation, the following provides a brief description of the three methods as implemented in ICAMS.

The isarithm method, sometimes also called the walking-divider method, utilizes the isarithms of the surface as a means in determining the fractal dimension $D$ of the surface, where $D_{surface} = D_{isarithm} + 1$. The algorithm was evolved from Goodchild (1980), Shelberg, et al. (1982), and Lam and De Cola (1993). In addition to the data matrix with the numbers of rows and columns specified (note that the number of rows does not have to be the same as the number of columns), the isarithm method in ICAMS requires the following parameter input by the user: the number of steps or walks, the isarithmic interval, and the direction from which the operation proceeds (either row, column, or both).

For each isarithmic value and each step size, the algorithm first classifies each pixel below the isarithmic value as white and each above this value as black. It then compares each neighboring pixel along the rows or columns and examines if the pairs are both black or both white. If they are of different colors, then there is an isarithm lying between the two neighboring pixels. The length of each isarithm line is approximated by the total number of boundary pixels. It is possible for a given step size that there are no boundary pixels. In this case, the isarithm line is excluded in the calculation. The total number of boundary pixels for each step size is plotted against step size in log-log form, also called the fractal plot, and a linear regression is performed. The regression slope $b$ is used to determine the fractal dimension of the isarithm line, where $D = 2 - b$. The final $D$ of the surface is the average of the D values for those isarithms for which $R^2 \geq 0.9$. Figure 1 shows a typical output from the isarithm method in ICAMS on the Intergraph-MGE platform.

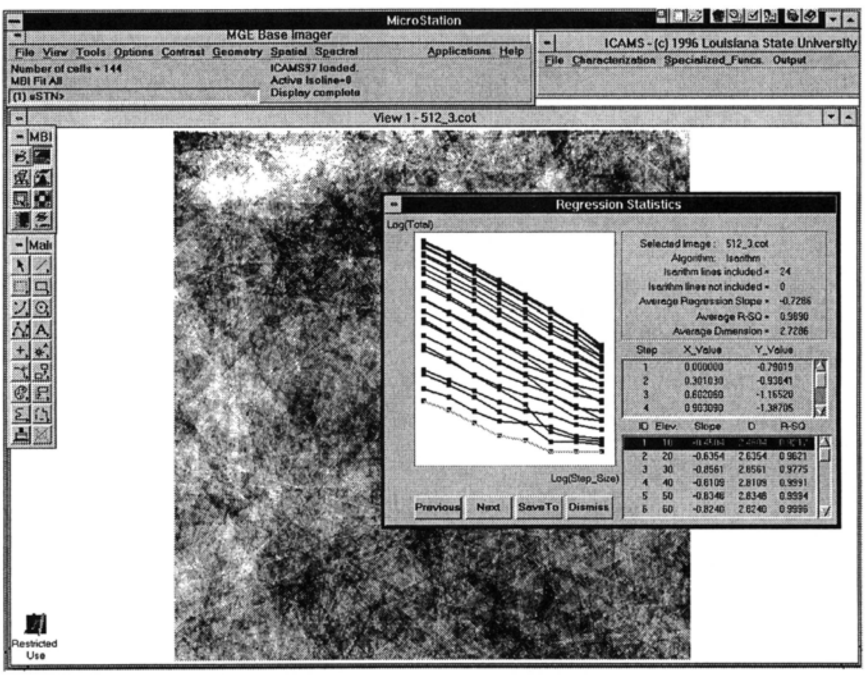


Figure 1. An output from the isarithm method. The background image is a simulated surface with $D = 2.7$ (see discussion below).

In the variogram method, the variogram function, which describes how variance in surface height varies with distance, is used for estimating the fractal dimension. The only difference between the traditional variogram and the variogram used in fractal estimation is that distance and variance are portrayed in double-log form. The slope of the linear regression performed between these two variables is then used to determine the fractal dimension, where in this case, $D = 3 - (b/2)$. Mark and Aronson (1984) pioneered the use of the variogram method. Detailed discussion of the method can also be found in Lam and De Cola (1993) and Jaggi, et al. (1993). In ICAMS, the variogram method requires the following parameter input: the number of distance groups for computing the variance, the sampling interval for determining the number of points used in the calculation, and the sampling method (regular or stratified random). Sampling only a subset of points for calculation is necessary especially for large data sets such as remote-sensing imagery, as the computational intensity will increase dramatically with increasing number of data points. Figure 2 shows an output from the variogram method.

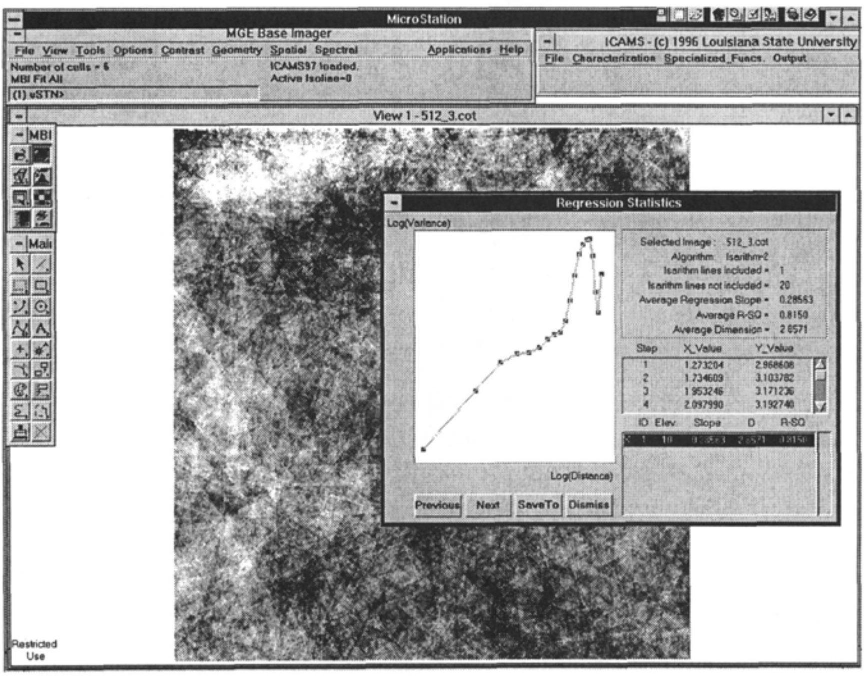


Figure 2. An output from the variogram method using the same simulated surface as Figure 1.

The triangular prism method compares the surface areas of the triangular prisms with the pixel area (step size squared) in log-log form (Clarke 1986; Jaggi et al. 1993). For each step size, the triangular prisms are constructed by connecting the heights of the four corners of the pixel to its center, with the center height being the average of its four corners. The areas of these surfaces can be calculated by using trigonometric formulae. The fractal dimension is calculated by performing a regression on the surface areas and pixel areas, where $D = 2 - b$. Figure 3 is an example output from the triangular prism method.

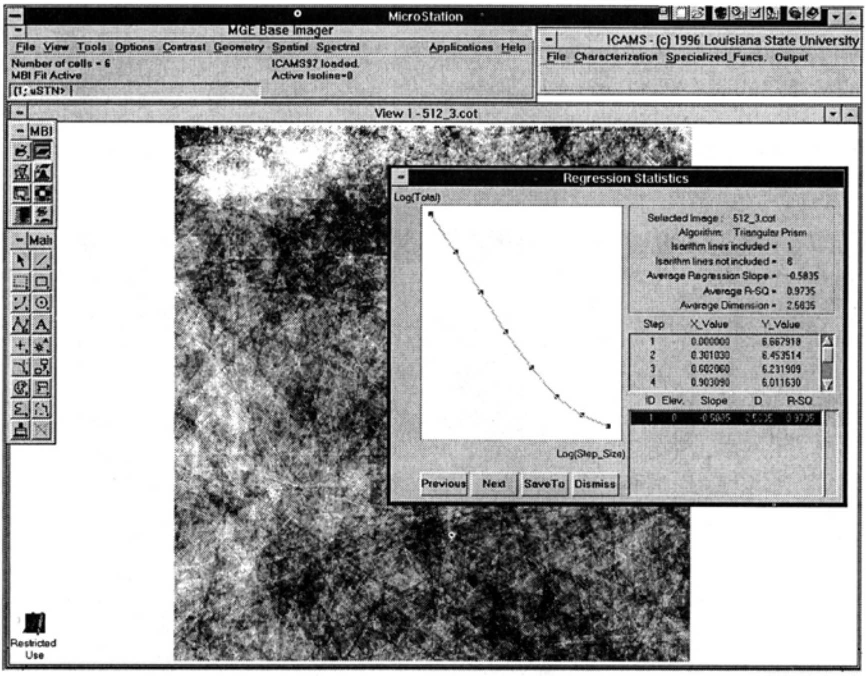


Figure 3. An output from the triangular prism method using the same simulated surface ($D = 2.7$).

To test the three fractal surface measurement methods, we use the shear displacement method to generate a series of hypothetical surfaces with varying degrees of complexity (i.e., fractal dimension) (Goodchild 1980; Lam and De Cola 1993). The method starts with a surface of zero altitude represented by a matrix of square grids. A succession of random lines across the surface is generated, and the surface is displaced vertically along each random line to form a cliff. The process is repeated until several cliffs are created between adjacent sample points. The amount of displacement is controlled by the variogram parameter $H$ in such a way that the variance between two points is proportional to their distance scaled by $H$. $H$ describes the persistence of the surface and has values between 0 and 1, and the fractal dimensions of the simulated surfaces can be determined by $D = 3 - H$. The value $H = 0.5$ ($D = 2.5$) results in a Brownian surface.

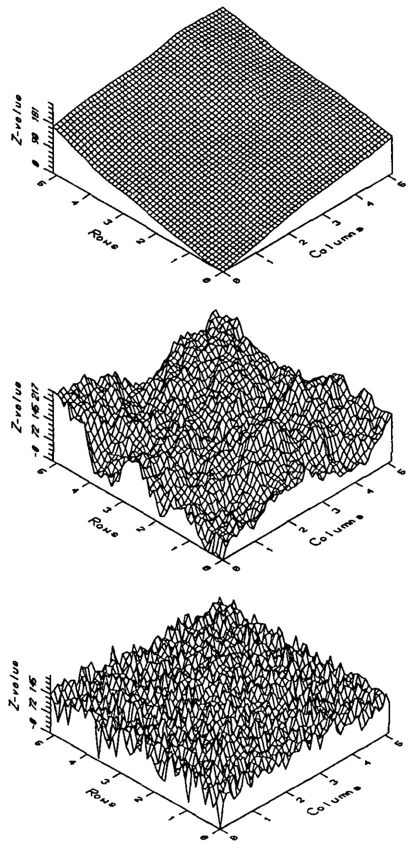


Figure 4. Three simulated surfaces from top to bottom, $D = 2.1$, 2.5, and 2.9.

Five surfaces with $H = 0.1$, 0.3, 0.5, 0.7, and 0.9 were generated for this study. Each surface has 512 x 512 rows and columns and was generated with 3000 cuts and the same seed value for the random number generator. Figure 4 displays three of the simulated surfaces ($H = 0.1$, 0.5, 0.9 or $D = 2.9$, 2.5, 2.1). The Z values of all simulated surfaces have been normalized so that their minimum and maximum $Z$ values are 0 and 255. These surfaces are input to ICAMS for fractal calculation. For the isarithm method, the parameter input are 8 step sizes with an isarithmic interval of 10. In the variogram method, number of distance groups were fixed at 20, with a sampling interval of 10 using the stratified random sampling method. The only parameter in triangular prism is the number of steps, which was also fixed at 8.

## RESULTS AND DISCUSSION

The results from applying the five hypothetical surfaces to ICAMS are summarized in Table 1. Table 1 shows that as *D* increases (or *H* decreases), the standard deviations of the surface values decrease. The inverse relationship between *D* and standard deviation is notable, because *D* is considered as a measure of spatial complexity and standard deviation a measure of non-spatial variation.

Table 1: Summary of results for the five simulated surfaces. $R^2$ values are in parentheses. The isarithmic algorithm includes Row, Col. and Both and all have $R^2 > 0.90$ and therefore $R^2$ are not listed.

| D | H | Mean | SD | Row | Col. | Both | Variogram | Tiangular |
|---|---|---|---|---|---|---|---|---|
| 2.9 | 0.1 | 110 | 26 | 2.93 | 2.99 | 2.96 | 2.85 (0.57) | 2.73 (0.97) |
| 2.7 | 0.3 | 113 | 30 | 2.73 | 2.90 | 2.79 | 2.88 (0.63) | 2.58 (0.97) |
| 2.5 | 0.5 | 118 | 48 | 2.53 | 2.57 | 2.54 | 2.59 (0.84) | 2.31 (0.98) |
| 2.3 | 0.7 | 112 | 67 | 2.27 | 2.13 | 2.21 | 2.21 (0.99) | 2.10 (0.98) |
| 2.1 | 0.9 | 121 | 75 | 2.14 | Nil | 2.05 | 2.09 (0.99) | 2.10 (0.86) |

The isarithm method in ICAMS generally performs very well for all five surfaces, with the computed fractal dimension agreeing with the dimension values used in simulating these surfaces. There are some discrepancies in resultant dimension values when using different orientations (row, column, and both). Such difference may be attributed to individual surface characteristics, where some surfaces may have more features with distinct orientations, such as roads, canels, or agricultural fields. In fact, the availability of an orientation option in this method could help in disclosing these individual surface characteristics that are otherwise not obvious.

The variogram method yields accurate results for surfaces of low fractal dimensions, but its performance becomes unstable with increasing dimensionality. Perfect fit ($R^2 = 0.99$) occurs in surfaces of $D = 2.1$ and 2.3, which are also the dimensionality of most real-world topographic surfaces. For surfaces of higher dimensions, the variograms do not behave linearly in the log-log plot. The user would have to determine through eye-balling only a range of points that looks reasonably linear to be included in the regression. For example, for the surface of $D = 2.7$ (Figure 2), if only the first 9 points are included in the regression, then *D* becomes 2.92 with a $R^2 = 0.94$, which is different from the result in Table 1 when all 20 points are included in the regression (D = 2.88, $R^2 = 0.63$).

The performance of the triangular prism method is disappointing, with the computed dimensions being consistently lower than the known dimension. Similar findings have also been reported in Jaggi, et al. (1993).

Based on the results from this analysis, we may conclude that the variogram method may not be a good measurement method for most remote-sensing imagery, as they tend to yield much higher dimensions than topographic surfaces. The variogram method, however, would be a useful method for computing fractal surfaces of low dimensions. Our findings on the reliable performance of the isarithm method, however, are contrary to those of Klinkenberg and Goodchild (1992), where the divider methods were reported to have extremely disappointing performance due to their inability to discriminate visibly different surfaces. More studies are needed to verify the initial findings.

## CONCLUSION

The three fractal surface measurements methods implemented in ICAMS, including the isarithm, variogram, and triangular prism methods, were evaluated using five simulated surfaces of varying degrees of complexity. The results show that the isarithm method yields accurate and reliable results for all surfaces, whereas the variogram method is only accurate for surfaces of low dimensions such as topographic surfaces. The use of variogram method for remote-sensing imagery is questionable, as the images are generally of much higher dimensions than topographic surfaces. The triangular prism method is the most inaccurate as it does not yield similar fractal dimension values. We will in the near future perform more evaluation to confirm the results from this study.

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