

TOWARD A NETWORK MAP ALGEBRA

Marc P. Armstrong
Department of Geography and
Program in Applied Mathematical
and Computational Sciences
The University of Iowa
Iowa City, IA 52242
marc-armstrong@uiowa.edu

Paul J. Densham
Centre for Advanced Spatial
Analysis
Department of Geography
University College London
26 Bedford Way
London WC1H 0AP
pdensham@geog.ucl.ac.uk

ABSTRACT

When decision-making groups work together to solve location-selection problems they often experience difficulty in understanding the elements of alternative solutions to problems that are held in common by different stakeholders. Because location selection problems are often addressed using decision support tools that represent the supply of services, and demand for it, on a street network, we have developed an approach, called network map algebra, that can operate on collections of alternatives in order to synthesize, summarize and differentiate between entire solutions or parts of them. This network map algebra uses abstracted representations of solutions in the form of vectors and matrices and requires a set of transformations between these abstracted structures and the conventional structures used by vector-based GIS software.

INTRODUCTION

Groups of decision-makers often meet to address complex, ill-structured problems. Although individuals working in isolation normally lack the breadth of experience required to solve such problems, when they work as a group they can pool their expert knowledge, explore their unique perspectives and viewpoints on a problem, and work to resolve the conflicts that result from their differing backgrounds, experiences and opinions. Since complex problems of the type that are routinely encountered in the formulation of public policy often have important spatial components, several issues must be resolved before groups can most effectively use existing geographic information handling and analysis tools to support their decision-making efforts. One important problem is that current software systems do not provide the kinds of geographic tools that are required to support group interaction, visualization and consensus-building activities. The purpose of this paper is to develop the structures and operations needed to generate maps that synthesize the results of a set of location selection scenarios created by either an individual, or two or more group members. Because location-selection algorithms often use networks to provide the spatial structure of movement between supply and demand locations, we place a particular emphasis on the development of a conceptual framework for a network map algebra (NMA) which facilitates the production of maps that are designed to assist group decision-making.

BACKGROUND AND DERIVATION

Within the current GIS paradigm, thematic maps are created by assigning symbols to geometric objects that reflect their attributes; for example, a polygon such as a census tract might be shaded to depict average income or a city might be symbolized with a circle proportional to its population. These methods rely on a set of visual variables that are used to guide the choice of appropriate symbols (MacEachren, 1995) and are now well established in GIS and desktop mapping software. In contrast, many methods of locational analysis operate on the attributes of objects and an abstracted form of their topological relations (Densham and Armstrong, 1993). Figure 1 illustrates a multi-step transformation, performed routinely by GIS software, that converts a simple map into a tabular representation that can be used to support analytical NMA operations. In the first step the map is recast into a discrete set of topological nodes. These nodes are uniquely identified and a simple table (ID and locational coordinates) is derived from this representation.

To generate map solutions to locational analyses, topological relations in the form of a list of allocations of demand locations to facilities, and attributes in the form of a tabular structure that specifies the volume of demand that each facility serves, are created during analysis and must be reconciled with the geometrical representations of objects in a geographical database (demand locations, facilities, and the underlying transportation network). This process of reconciliation can be accomplished through a series of transformations between maps, abstract data structures, operations on these structures and transformations back to mapped representations. In this section we describe this transformational view (see also Tobler, 1979; Clarke, 1995).

Networks are constructed from nodes, points, links and chains. These constitute a set of cartographic primitives that can be analyzed independently or assembled into higher-level structures based on results from analytical operations. For example, a shortest path is computed through a network based on topological connections and distances along chains between nodes. The shortest path between two nodes that lie on different sides of a city, therefore, is a kind of compound object that is built from chains, nodes and their respective topological connections. Likewise, a scenario result from a locational model is reported as a tabular set of allocations of demand locations to one or more supply sites. Thus, for any given supply site the links between it and demand locations, when related to geometrical representations stored in a geographic database (e.g. TIGER files) constitute a compound object that describes the particular geographical characteristics of one possible solution to a location-selection problem. Typically, tabular structures that result from analyses are linked to geometrical representations to facilitate the creation of geographical visualizations (GVIS: MacEachren, 1995) that support locational decision-making (Armstrong *et al.*, 1992).

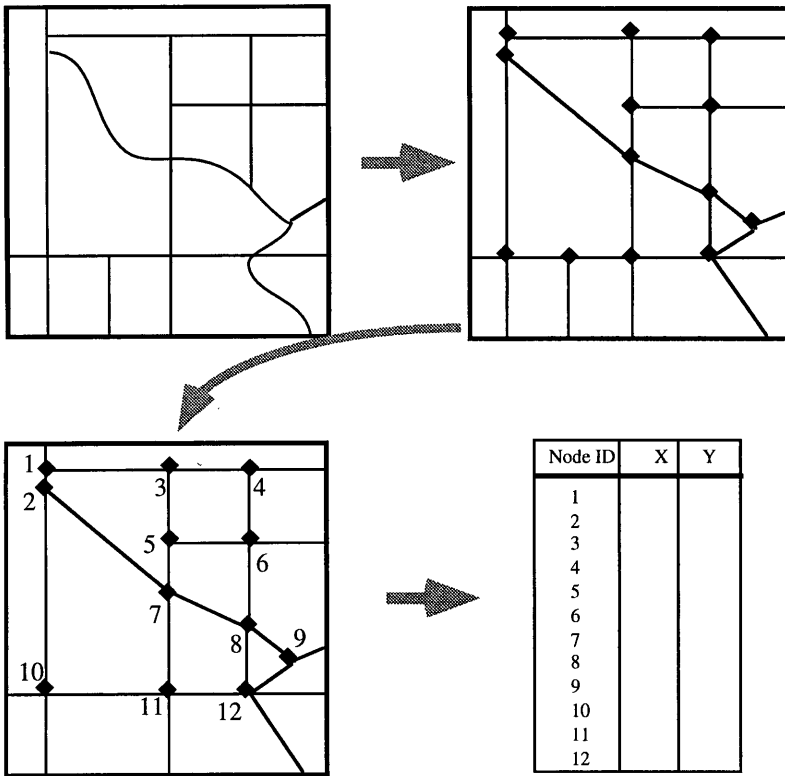


Figure 1. Map abstraction to topological nodes.

A network map algebra must be designed to operate on the structures, vectors and matrices that are derived from the topological relations produced by locational models. In a solution to a locational problem, for example, each supply location (facility) is topologically linked to a set of demand locations. Figure 2 illustrates the process through which data are used to create a simple GIS or to provide input to locational modeling software, such as LADSS (Densham, 1992). The results of such models are often produced in tabular form and specify the assignment of demand to supply locations while satisfying some user-specified optimization criteria such as minimization of average distance between demand and supply locations. This tabular representation is then used with appropriate geographical information to produce maps that show, for example, the relationship between supply and demand locations.

In Figure 3 a tabular structure that specifies the identity of a set of nodes can be used to generate a crude map as well as raw data that is input to a location model. The model then computes a set of allocations, based on user-specified criteria, of demand to supply locations has been written in tabular form. However, an additional transformation, also shown in Figure 3, and one that is of particular interest in a NMA context, is a simple binary representation of these relationships between supply and demand locations.

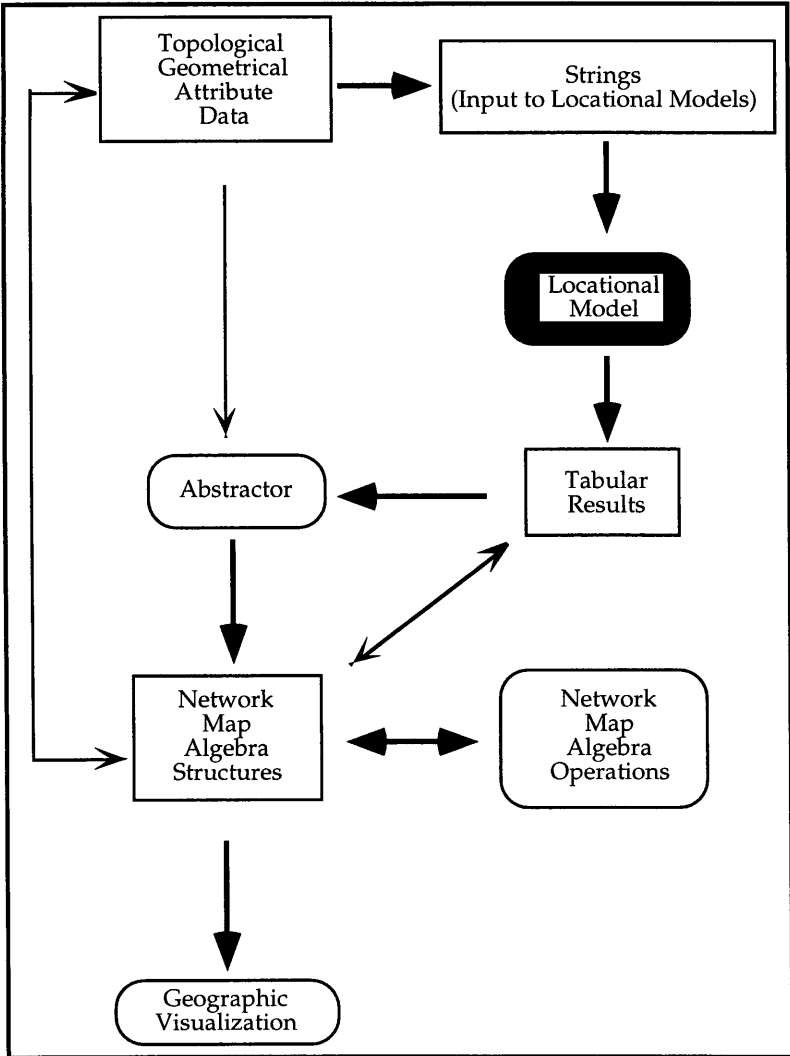


Figure 2. Transformational flows that support the use of network map algebra. Heavy arrows indicate primary flows while lighter arrows indicate flows that are used less frequently. Rectangles indicate data; rounded rectangles indicate processes.

Several types of these transformed representations of solutions are used in NMA operations. Figure 4 shows the derivation of *supply vectors* for a set of three solutions. In this case each node that is specified as a supply site in a solution is denoted with a “1”. Blanks are treated as zeros. More complex structures also can be derived to distill relationships between demand and supply locations. In Figure 5 links between supply and demand locations for the same three solutions are represented as binary *allocation of demand* matrices (in this

case a “1” represents the allocation of a demand location to a facility). In this way, several characteristics of each solution can be reduced to a matrix form that is then subjected to further analysis using NMA operations. Other types of analytical structures also can be generated (e.g. demand vectors, network shortest paths and second-order neighbors) with the resulting transformations used with NMA operations to move between alternative representations of a problem and its solutions as required by users as they either visualize or analyze their data.

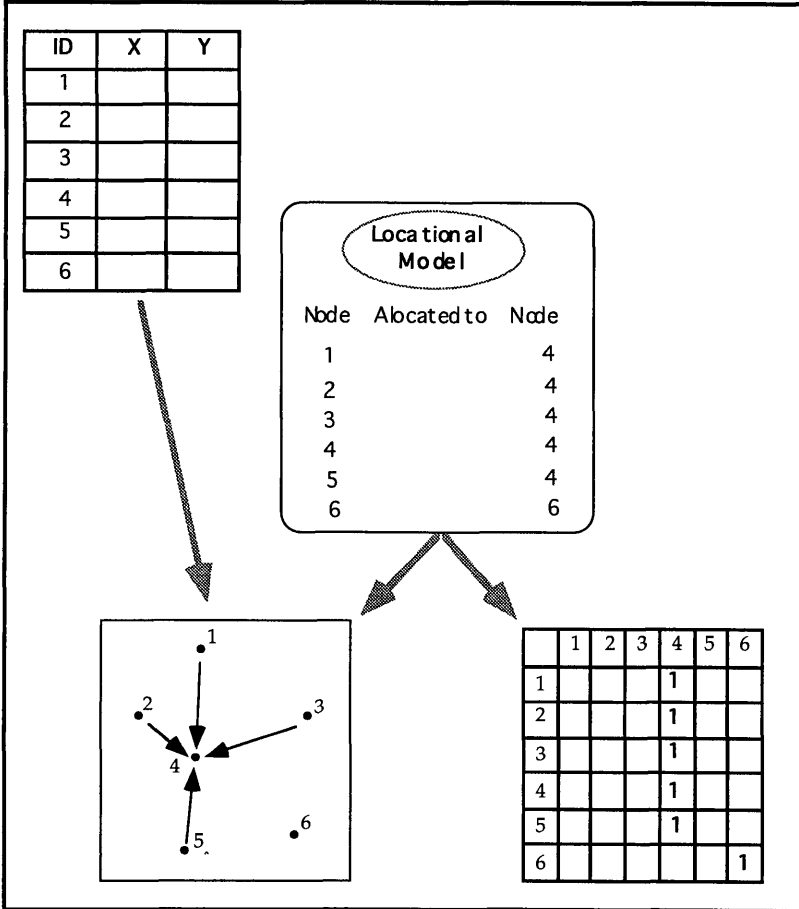


Figure 3. Transformational view of the location-selection process. Solid arrows indicate allocation of demand to a supply location.

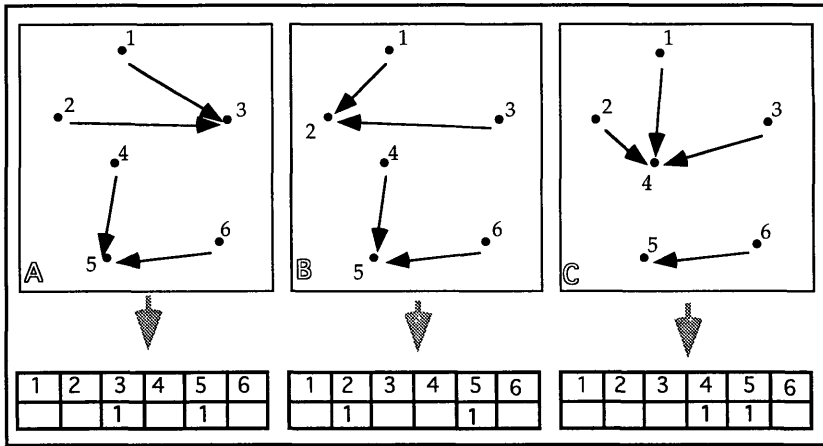


Figure 4. Binary supply vectors

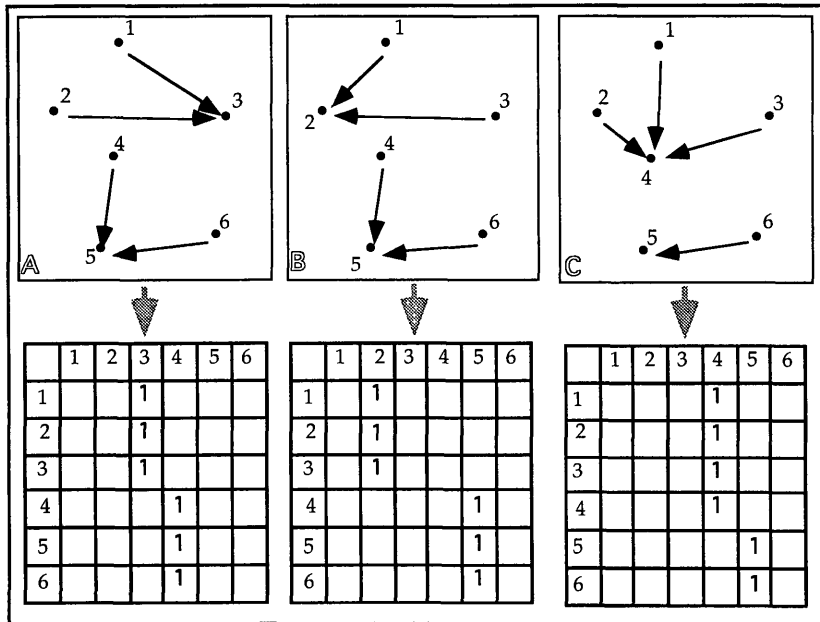


Figure 5. Creating allocation of demand matrices.

ILLUSTRATION

In this paper the primary focus is placed on nodal NMA operations. These are in many ways analogous to Tomlin's focal operations as described in several specifications of map algebra and cartographic modeling that have been

developed for polygon and grid data (see for example, Burrough, 1986; Tomlin, 1990; 1991). In particular we describe how the network map algebra is used to generate two types of maps (facility frequency and allocation consistency) that are designed explicitly to support group decision-making. For example, maps that synthesize the characteristics of a collection of scenarios can be created by applying operations such as “*add solution_5 to solution_7 to produce sum_9*”. The resulting maps enable decision-makers to identify, for example, locations that occur frequently, and are thus robust with respect to a range of objective and subjective criteria, and to highlight those areas over which there is disagreement. This enables group members to work towards the resolution of conflicts and supports the process of consensus building.

Facility Frequency Maps

Facility frequency maps depict those nodes that are selected as facility locations in two or more solutions. In the two-scenario case, a pair of supply vectors is summed, using a local operation on each node, to yield a third vector from which a map is produced. Figure 6 shows how several alternatives, which might be generated by a single user or by different stakeholders in a multiple-participant public policy problem, could be subjected to network map algebra analysis to determine those places in which agreement occurs among the participants about the location of a facility. This summation of supply vectors, yields information about the common elements that exist across the set of solutions and can be linked to a geometrical representation to produce a facility frequency map that is used to support discussion about the solutions generated. Note also that the link between the NMA-generated structures and the data from which they are derived enables the creation of other map types that depict, for example, the volume of demand served at each facility.

Allocation Consistency Maps

Allocation consistency maps depict how often a demand location is allocated to the same facility across a range of scenarios or, conversely, how stable a facility’s service area is under a range of allocation criteria. Focal operations are also used to produce these maps. Clearly, in many cases the results will be somewhat chaotic with mapped allocations visually crossing each other, thus leading to considerable confusion in any map created directly from the raw allocations. However if a measure of saliency is used, the resulting maps will be far more simplified. As shown in Figure 7 a simple threshold operation on the resulting summed allocation of demand matrix reduces the number of allocations that are symbolized, thus leading to a less cluttered appearance.

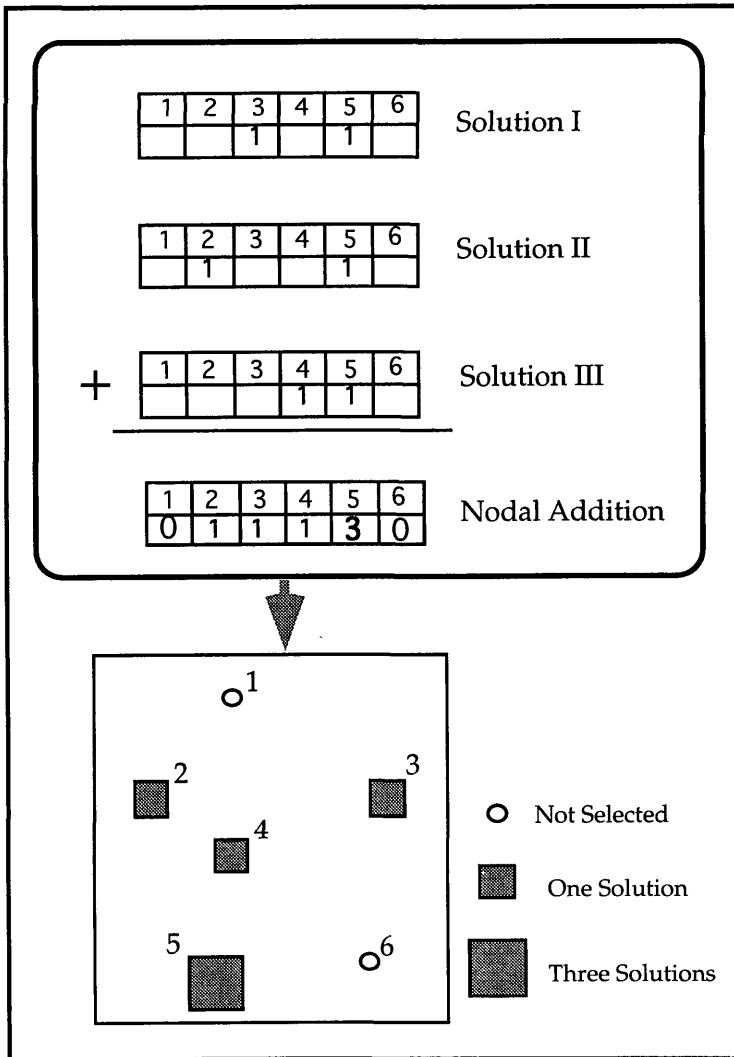


Figure 6. Creation of a facility frequency map using nodal network map algebra operations.

Nodal NMA operations can also be used to produce link-based maps. A network spider map, for instance, depicts maps through a network that link each demand location with its allocated supply location. To build such maps, a matrix-based shortest path algorithm (Arlinghaus *et al.*, 1990) is supplied with origins from the solution's binary supply vector and the requisite destinations from the column in the allocation of demand matrix. When the algorithm identifies the shortest path between facilities and demand locations the relevant links in the network are output and can then be symbolized.

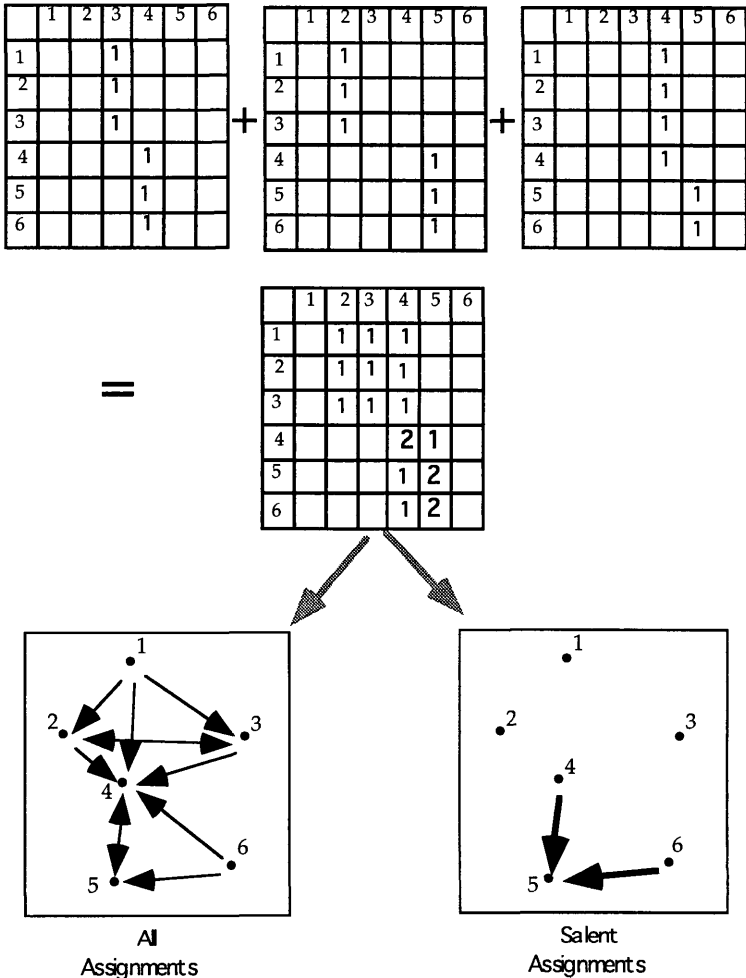


Figure 7. Creation of allocation consistency maps using network map algebra operations.

CONCLUDING DISCUSSION

We have described the rudiments of a network map algebra that supports the process of synthesizing solutions to network-based location-selection problems. This algebra relies on the transformation of scenarios into a set of matrix representations that are then manipulated by operators. These abstracted realizations of solutions to location-selection problems can be passed among users as tokens that are used to generate maps. The entire detailed network (e.g. TIGER file) need not be passed each time a solution is developed. Rather, the network representation is static and the structures are used to symbolize results, thus resulting in a substantial reduction in network traffic.

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