CONTOURING ALGORITHMS

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INTRODUCTION

Computer algorithms for contouring spatial data differ in subtle ways that affect the appearance of maps which they produce. Most operate by estimating the value of the surface at a regular gridwork of points across the map area. Contour lines are then drawn through this grid by mathematical interpolation.

The grid intersections are calculated by some form of weighted average of nearby data points. "Nearby" points are selected according to different search constraints. In some algorithms, surface dips are projected from the control points to the grid nodes being estimated. The effectiveness of different combinations of weighting function, search patterns, and projection techniques varies with complexity of the surface being mapped and the amount of control; some widely used programs may produce maps which are seriously misleading. Despite claims to the contrary, it is impossible at the present time to cite a single contouring method which is superior under all conditions.

If a contouring system is very flexible, many errors may be avoided by selecting the best combination of parameters and methods for a particular map and specific objective. This requires, however, an evaluation of the performance of different mapping techniques under different conditions. Results of an empirical analysis of contour maps of a subsurface horizon are presented as an example of such a comparison.

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Computer programs for drawing isoline or contour maps from scattered data points can be categorized into three general groups. These three basic approaches are embellished almost endlessly, however, making direct comparisons between different programs difficult. Comparative studies are important because conflicting claims have been made about the relative merits of the various alternatives as incorporated in commercial software packages. Few user groups have the financial or technological resources to develop their own computer contouring programs; most much purchase one of the packages marketed by hardware suppliers or vendors of proprietary software. This paper presents empirical comparisons of performance between some of the more common features incorporated in the most widely used class of contouring algorithms. These comparisons were made using SURFACE II, a general-purpose contouring package written by R. J. Sampson (1975 a & b) for the Kansas Geological Survey. The program is highly modular, and the various components can be combined to emulate most commercial contouring procedures.

The most obvious computer contouring approach is incorporated in various triangulation procedures, which simulate the process of manual contouring. Lines are projected from each data point to the nearest three points, dividing the map area into triangles. The points where contour lines cross the triangles are established by linear interpolation down the sides of the triangles. The final step is to connect points of intersection that have equal value to form contours. Essentially this process represents the surface as a "geodesic dome" composed of flat triangular plates (IBM, 1965).

Modifications include fitting curved rather than flat plates to the triangular areas, subdividing the basic triangles into finer subtriangles of similar form, and restricting the manner in which control points are connected so the resulting triangles are as nearly equilateral as possible.

The principal advantages of this procedure are the directness of the methodology and the fact that all control points must lie on the contoured surface. The principal drawbacks are the non-uniqueness of the triangular mesh, which can result in different patterns of contour lines for the same data, and the extreme slowness of the procedure as compared to gridding routines.

An alternative methodology includes the global fit procedures, in which a complex mathematical function of the geographic coordinates is fitted to the control point values. Polynomial trend surfaces and double Fourier surfaces are examples. Basically, global fit methods are an extension of statistical regression procedures into two-dimensional space. The equation which represents the surface is usually calculated so the sum of the squared deviations of the surface from the control point values is a minimum. Trend surface procedures are widely discussed in the geologic literature; a general treatment is included in Davis (1973). Modifications include two-stage procedures for fitting small "trend surfaces" to local areas of residuals from the global trend surface (McIntyre, and others, 1968).

The advantage of global fitting as a contouring procedure is its extreme computational speed. Its disadvantage is that it provides a very poor map of the data, as it is impossible to represent the detail in most mapped variables with any single, tractable equation.

Local Fit methods estimate values at the nodes of a regular grid across the map from a weighted average of the control points nearest each grid node. Contours are laced through the grid work by linear interpolation between the nodes to find the points of intersection of the contour levels with the grid lines. Points of common elevation are then connected to form the contour lines.

Estimating the regular grid of values is called "gridding" and consists of two steps. First, the nearest neighbors must be found. The simplest procedure is to take the n nearest points to the grid node being estimated. With certain distribution of control points, this may result in unconstrained estimates of the surface, if all the nearest points lie on one side of the node to be estimated. Constraints may be introduced to insure some equitable radial distribution of the nearest neighbors used. These include a quadrant search, where n points must be found in each of four quadrants around the estimated points, and the octant search which carries the concept of radial constraint one step further (Walters, 1969; Batcha and Reese, 1964).

The second step is the estimation of grid values from control points that have been located in the first step. The estimates may simply be weighted averages, where the control points are weighted by a function of their distance D from the grid node. The most commonly used functions decline with distance at least as rapidly as $1/D^2$ and some decline as rapidly as $1/D^6$ (J.A.B. Palmer, 1973, personal communication).

A more elaborate procedure which is widely used in commercial contouring software divides the estimating process into two phases. During the first phase, the dip of the surface at each control point is found by fitting a weighted least-squared plane to the surrounding control points. In the second phase, these dips are projected from the control points that surround a grid node to that location. An estimate of the surface at the node is then made as a weighted average of these projections (Osborn, 1967; Jones, 1971).

Most commercial contouring programs are combinations of different weighting functions, search procedures, and a great variety of other modifications on a local fit procedure. The superiority of specific combinations is loudly proclaimed by their proponents, but the relative merits of the more elaborate procedures are questionable. It should be noted that, in addition to drawing isoline maps, commercial contouring packages usually have the ability to construct block diagrams, isopach maps, and maps of other transforms of the surface. Although these embellishments may greatly add to the cost of a particular package, and may be very important in their own right, they do not affect the relative performance of the contouring procedure.

The primary advantages of the local fit method derive from the intermediate gridding step; this allows storage of the mathematical representation of the surface as an array in the computer. Storage is minimized and the process of drawing contour lines is speeded. Two or more variables can be compared (by isopaching or other methods) even if they are measured at different locations, because the grids, rather than the control points, are compared. However, the gridding step also is the cause of most of the drawbacks of the local fit method, especially the distressing tendency for contour lines to sometimes pass on the wrong side of control points in areas of low dip, or when excessive smoothing of contour lines is performed (Walden, 1972).

An empirical analysis was performed to evaluate the differences between some of the more widely used variants of the local fit procedure. A geologic subsurface structure map was constructed in an area of moderate to dense well control. The test data set includes all wells drilled to the top of the Pennsylvanian Lansing-Kansas City Group in Graham County, Kansas, to the end of 1974. This includes approximately 3,000 wells in an area 30 x 30 miles square. Surface values are in feet above sea level, so statistics of errors are also given in feet. The fidelity of the computer-drawn map to the original control points was found by back-calculating to each well location from surrounding grid nodes. The root mean squared (RMS) error of these differences is a measure of the scatter in values of the contoured surface around the true surface. A large RMS value indicates the procedure is ineffective, or inaccurate at the data points. The skewness of these differences is a measure of bias, or tendency for the surface representation to consistently fall above or below the correct values.

Figure 1 is a plot of RMS error and skewness of control-point errors for maps constructed by an algorithm which used various numbers of nearest neighbors, selected without constraints on the search pattern. Grid nodes were estimated by averaging the control points found, after weighting by a function which declines at the rate of $1/D^4$. Both RMS error and skewness increase as the number of nearest neighbors increases. Other plots for weighted projection methods and octant or quadrant search patterns are essentially identical to this illustration.

The influence of different weighting functions on control-point error is shown in Figure 2. Errors were found by back-calculating from a surface created using eight nearest neighbors, no search constraint, and no projection of dips. A function which is heavily influenced by nearby control points creates a surface representation which has the smallest RMS error and skewness. In contrast, a surface created using a slowly declining weighting function is smoothly undulating and has many of the averaging properties of a global fit surface. Search constraints and projection of dips do not significantly alter the degree to which the map representation honors the original control points.

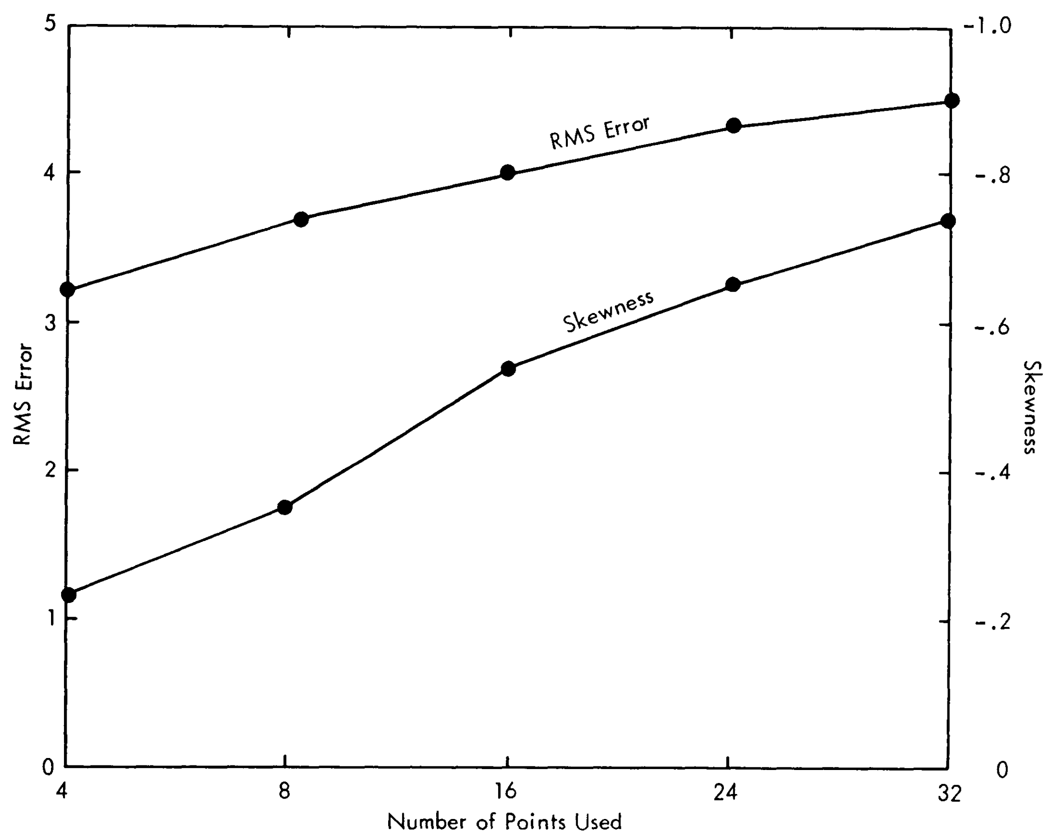

Figure 1: Error at control points for nearest neighbor searches without projection of dips at control points. RMS error is given in feet.

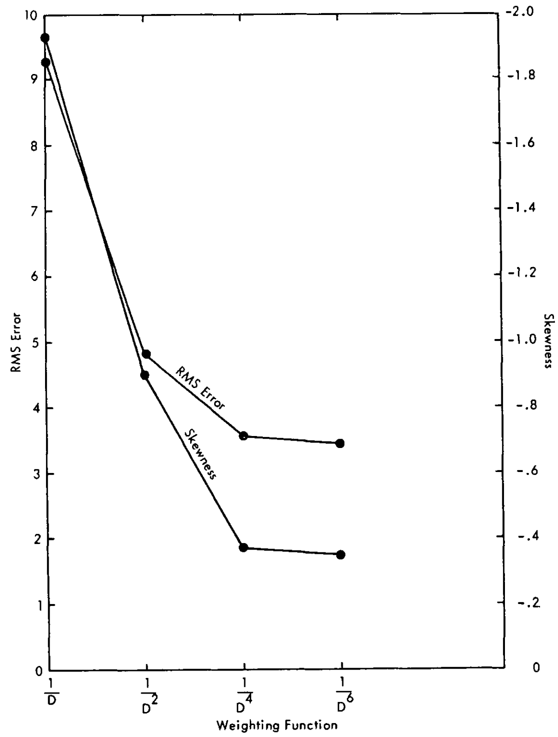

Figure 2. Errors at control points for nearest neighbor searches using eight points, without projection of surface dips at the control points, for different weighting functions. RMS error is given in feet.

The primary objective in many contour mapping exercises is, however, not to represent the available data as accurately as possible, but to estimate with minimum error values of the surface at locations where no control is available. In petroleum exploration, for example, structural contour maps are used to predict the locations of potential targets such as closed positive structures or anticlines prior to drilling. The ability of various algorithms to produce accurate estimates at locations where no control exists was checked by an empirical test using the same set of subsurface data. The well data were first divided into two subsets, one containing approximately 700 wells drilled prior to 1952, the other containing about 2,700 wells drilled after that year. The set of early wells was used to generate structural contour maps which were checked by comparison with the structural elevations at the "blind," post-1952 locations.

Figure 3 summarizes the estimation errors made by various combinations of search patterns and numbers of control points used in the estimation process. The algorithm weights control points according to a function which drops off at the rate of approximately $1/D^4$, with or without dip projection. Bias is expressed by the mean error, which is a measure of the average tendency for the algorithm to underestimate or overestimate. RMS error is a measure of the inefficiency of the estimating procedure.

Ideally, a contouring algorithm should have both a low bias (i.e., it should be accurate) and a low RMS error (i.e., it should be precise). There is little difference in precision between the various combinations considered, but there are large differences in the amount of bias. Methods using large numbers of nearest neighbors have less bias, because the estimate has the character of a statistical average and the law of large numbers is operating. Use of dip projections significantly increases the bias, especially in areas where

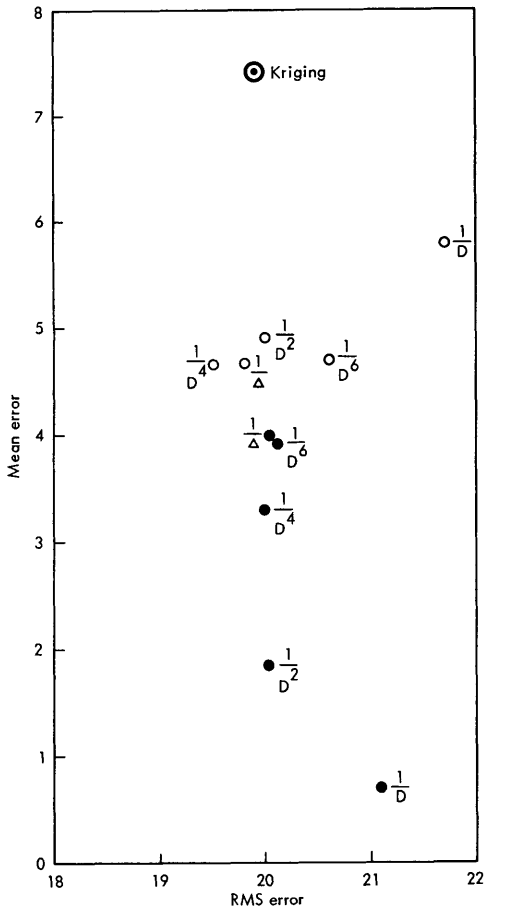

Figure 3: Errors at estimated points for different search patterns and numbers of points used in the gridding algorithm. Estimations are calculated as standardized averages of control points weighted inversely to the square of distance. Squares represent nearest neighbor search, circles are quadrant search, and triangles are octant search patterns. Solid symbols are methods without projection, open symbols are methods with projection of dips.

control density is low and local gradients may be high.

An example of the effect of different weighting functions on estimation error is shown in Figure 4 for an algorithm which uses an octant search constraint. Weighting functions which drop off slowly have the lowest bias but the highest RMS error. Again, this is because they are assuming the characteristics of a global averaging process. Relationships in this plot

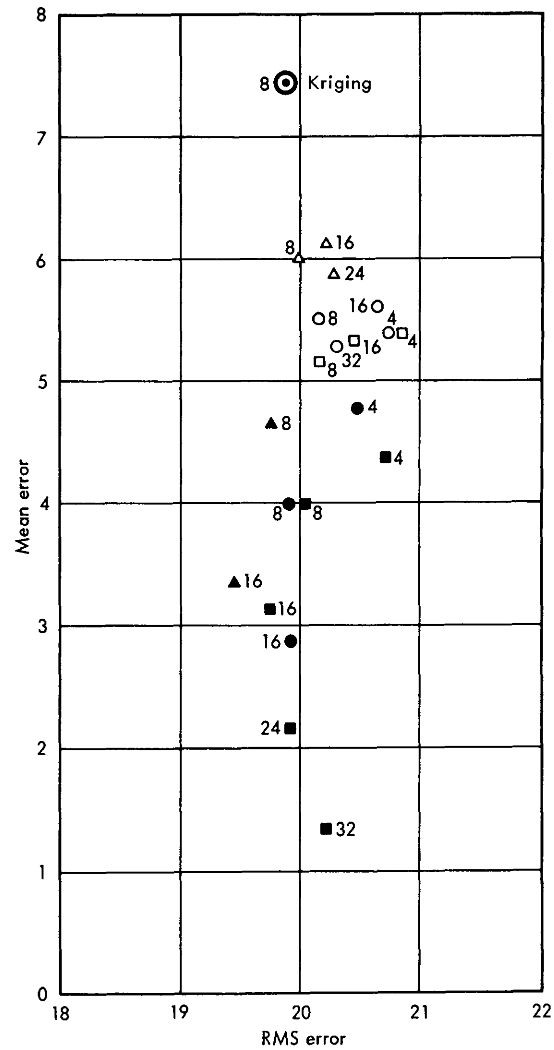

Figure 4: Errors at estimated points for different weighting functions, using an octant search with two points per octant. 1/Δ represents standardized inverse distance-squared weighting. Solid symbols are for procedures without projection of surface dips, open symbols are with projection.

are typical of those for other combinations of search constraint and number of control points used, although the scales may be shifted somewhat.

The distressing (although not surprising) conclusion from this empirical study is that the different objectives of the contouring procedures considered are not mutually obtainable. An algorithm which faithfully honors the original control points should utilize a weighting function which drops off extremely rapidly with distance, and which uses only a few nearest neighbors. However, such an algorithm will produce poor predictions or estimates at locations where no control is available. The best estimating procedure might be one that used 16 or 24 control points in each calculation of a grid node, and which weighted distant points relatively heavily. This, of course, would provide a poor reproduction of the original control points.

Since it apparently is not possible to specify a combination of features that will lead to a map that is "best" in both a representational and predictive sense, the selection of features should be based on the specific purpose for which an individual map is made. This requires a contouring package which contains a variety of alternative procedures under the control of the user. There is, in addition, a third selection criterion, based on a practical constraint: Figure 5 shows increases in computation times related to the number of nearest neighbors used by the algorithm. For routine or production contour mapping, the cost criterion may be equally as important as fidelity or predictive ability.

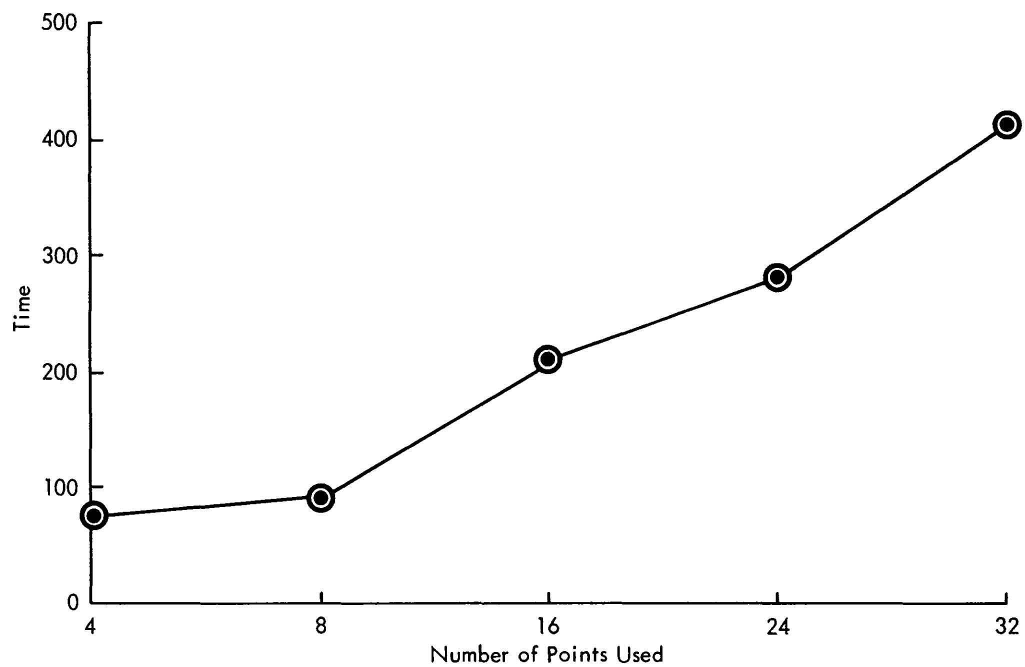

Figure 5: Run times for gridding phase of SURFACE II, in hundredths of seconds of Honeywell 635 CPU time, for nearest neighbor searches without projection of dips at control points. Dip projection increases run times by a constant factor of (no. data points)/(no. data points + no. grid nodes).

REFERENCES

1. Batcha, J. P., and J. R. Reese (1964), "Surface Determination and Automatic Contouring for Mineral Exploration, Extraction, and Processing," Colorado School Mines Quarterly, vol. 59, pp. 1-4.

2. Davis, J. C., (1973), Statistics and Data in Geology, New York: John Wiley and Sons.

3. IBM (1965), Numerical Surface Techniques and Contour Map Plotting: IBM Data Processing Application, White Plains, New York, 36 p.

4. Jones, R. L., 1971, A Generalized Digital Contouring Program: NASA Langley Research Center, Hampton, Virginia, NASA TN D-6022, 78 p.

5. McIntyre, D. B., D. D. Pollard, and R. Smith (1968), Computer Programs for Automatic Contouring, Kansas Geological Survey, Computer Contrib. No. 23, The University of Kansas, Lawrence, Kansas.

6. Osborn, R. T. (1967), An Automated Procedure for Producing Contour Charts, U.S. Naval Oceanographic Office, IM No. 67-4 (Available from National Technical Information Service as AD807617).

7. Sampson, R. J. (1975a), "The SURFACE II Graphics System," in Davis, J. C. and M. J. McCullagh, editors, Display and Analysis of Spatial Data, London: John Wiley and Sons, Ltd., pp. 244-266.

8. Sampson, R. J. (1975b), SURFACE II Graphics System, Kansas Geological Survey, Series in Spatial Analysis, No. 1, The University of Kansas, Lawrence, Kansas.

9. Walden, A. R. (1972) Quantitative Comparison of Automatic Contouring Algorithms, Kansas Geological Survey, K.O.X. Technical Report, The University of Kansas, Lawrence, Kansas.

10. Walters, R.F. (1969), "Contouring by Machine: A User's Guide: Bulletin of American Association Petroleum Geologists, volume 35, No. 11, pp. 2324-2340.