MAP EDITING USING A TOPOLOGICAL ACCESS SYSTEM

Marvin White
U.S. Bureau of the Census

NEIGHBORHOODS

A file structure and access system in which the boundary and coboundary operators E_j^i, described by Corbett in "Topological Principals in Cartography," are implemented affords the user excellent editing capabilities. With these operators, one may construct the fundamental neighborhoods of any point, line segment, or area. These neighborhoods are the smallest definable areas, in terms of the topology, completely surrounding the object under study. Some neighborhoods and their computations are shown in Figures 1, 2, and 3.

Every record in a geographic file which refers to a particular object, e.g., a line segment, falls within the fundamental closed neighborhood of that object. Thus, analysis of errors and the effects of changes are limited to that neighborhood. This is a great advantage, particularly for interactive editing, since the editor need only examine the few records in the neighborhood of the object under study. For line segments this is generally fewer than a dozen records. Furthermore, the editor may be able to restrict his attention to even fewer records due to graph theoretical constraints discussed below.

GRAPHS AND THE KIRCHOFF ANALYSIS

Every set of segments, e.g., the neighborhoods illustrated above, generates two graphs corresponding to the two ordered pairs described by Corbett. The 0-cells (points) and their ordered incidence relations give the primal graph while the 2-cells (areas) and their ordered incidence relations (adjacency relations) give the dual graph. These two graphs are duals because they are embedded in an orientable two-dimensional manifold. This powerful constraint provides the basis for several consistency edits. The usual DIME block and node bounding edits verify the consistency of representation of the dual graphs. These tests determine only whether there is a loop in the primal graph of a block or in the dual graph of a vertex.

There is available a more general and informative analysis based on a theorem by Kirchoff.1/ This analysis yields three numbers for each component of the graph: the number of chains between essential points; the number of cycles; and the number of acyclic chains. This analysis characterized the homeomorphically irreducible graph, i.e., the simplest topologically equivalent graph, to the given graph.2/ Figure 4 shows such an analysis.

Only certain codes are possible for particular features. The dual of the coboundary of a vertex must be a single component and a set of loops, i.e., there must be zero acyclic chains. Figure 5 shows the dual graph around vertex 45. If there were an error in encoding, the chain would be open or have a tail. If vertex 45 were replicated elsewhere on the map, there would be more than one component. Figures 6 and 7 show such cases. This test will reveal all topological inconsistencies an coding, except replicated block numbers, if applied to every node.

To detect replicated block numbers we examine the graph code for the boundary of the fundamental neighborhood of the point. That boundary must be a set of loops, usually only one, otherwise there is an error. More than one loop may occur if there are lakes inside the boundary. A replicated block appears as an additional component. We can also test for consistency of the coordinates assigned to vertices using the boundary of the neighborhood. The test is whether the point falls within its neighborhood. If not, there is an error in the coordinates of the central point or some boundary point. Figures 8 and 9 show the series of edits for point 226, whose coordinates are erroneous.

Errors that result in acyclic chains, e.g., missing segments, are always in the coboundary of the endpoints of the acyclic chain. The error detected in Figure 10, an open chain ending at vertices 1004 and 1003, would also be uncovered in the coboundary edit around each of those vertices. The editor may now restrict his attention to only these two points, which are automatically annotated.

ORIENTATION AND RETRIEVAL

In the Arithmicon system, 1-cells (segments) are specified by a pair of ordered pairs: (A,B), (C,D). The interpretation of that pair of pairs is A = from vertex, B = to vertex, C = left block and D = right block, so that an observer standing on A -- the from vertex, facing B -- the to vertex, would find C -- the left block, on his left and D -- the right block, on his right. This representation, which includes orientation, i.e., distinquishable left and right, is possible because the graphs mentioned earlier are embedded in an orientable two-dimensional manifold.

There are two possible orientations of a single 1-cell: (A,B), (C,D) and (B,A), (D,C). The second orientation places the observer at B facing A with D on the left and C on the right. Both orientations refer to the same 1-cell. To provide a unique representation of a 1-cell, (A,B), (C,D), the following convention is observed in Arithmicon: Choose the orientation in which $A < B$ unless $A = B$, then choose the orientation in which $C < D$. If both $A = B$ and $C = D$, the 1-cell is not orientable and already has a unique representation.3/

Oriented retrieval is accomplished in Arithmicon by assigning a + 1 coefficient to the retrieval pointer for conventional orientation and - 1 coefficient for the opposite orientation. Coefficients for successive retrievals are added. This provides a convenient indicator, even before the actual retrieval, of which 1-cells are interior to the requested region and which are not. Interior segments will have coefficient zero, since they will be retrieved twice -- once with a + 1 coefficient and once with a - 1 coefficient. Boundary segments will have a + 1 or a - 1 coefficient since the retrieved region will be on the left side or right side of the boundary segment, but not on both sides.

FUTURE RESEARCH

Every field in a DIME file generates a graph and that graph may be analyzed using the Kirchoff algorithim. Street names, which we are now implementing on Arithmicon, may be examined at the end points of components for misspellings. ZIP codes and address ranges present more difficult editing problems, since they are not as constrained as street names, but with the Kirchoff analysis, we will be able to limit the search for possible errors to likely candidates.

REFERENCES

1. For Kirchoff's theorem see Berge, The Theory of Graphs and Its Applications, Dunod, Paris, 1958 (English Translation by A. Doig, Methuen, London, 1962).

2. For a discussion of homeomorphically irreducible graphs, see Berge, Ibid.

3. A non-orientable 1-cell, (A,A); (C,C), would be a loop from a back to a interior to C.

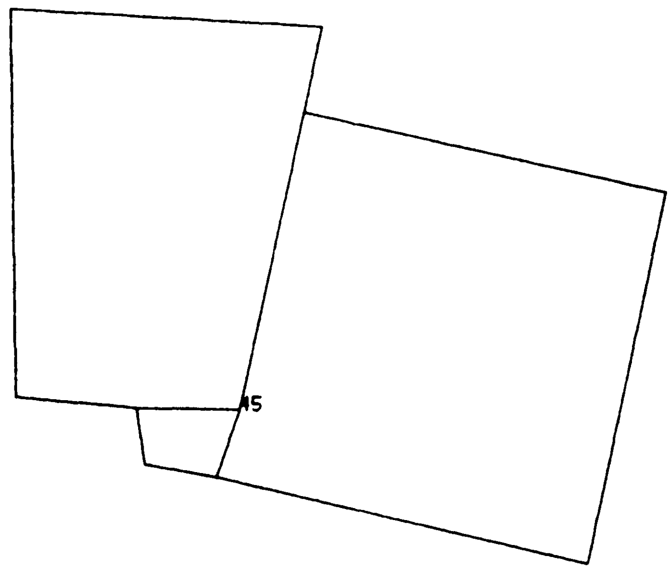

Figure 1. The closed neighborhood of vertex 45.

$$E_1^2 E_2^1 E_1^0 (45)$$

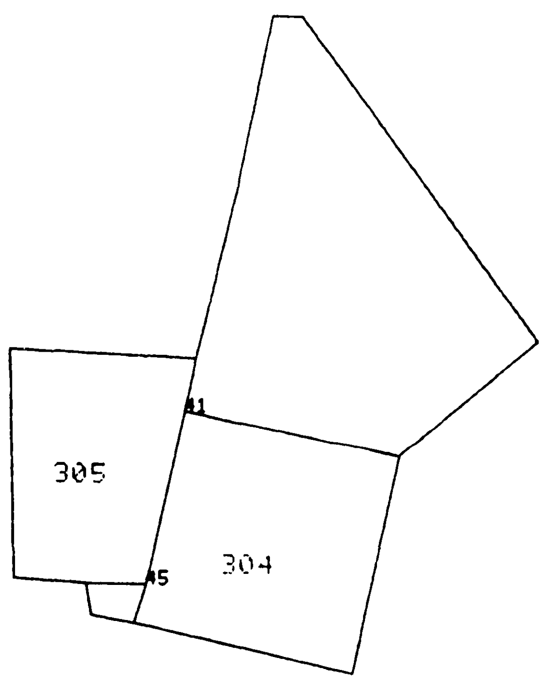

Figure 2. The closed neighborhood of segment

$$(41,45);(304,305) = E_1^2 E_2^1 E_1^0 E_0^1 (41,45);(304,305)$$

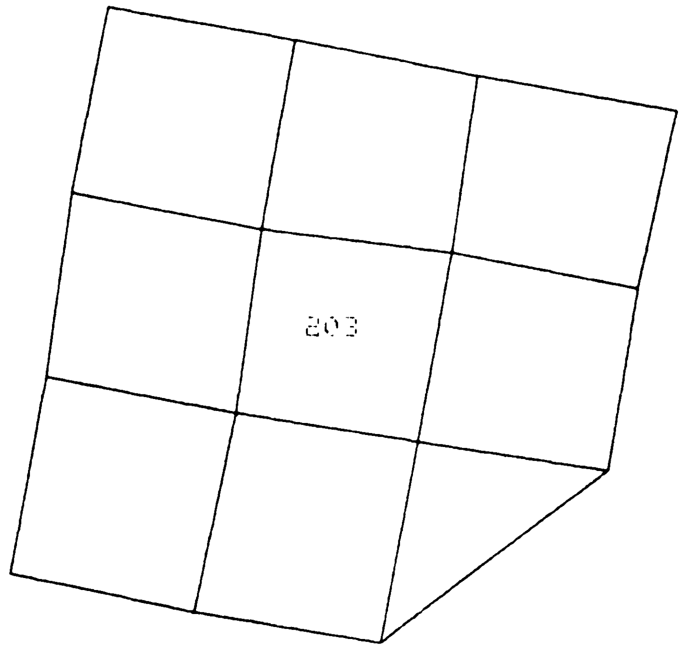

Figure 3. The fundamental neighborhood of block 203.

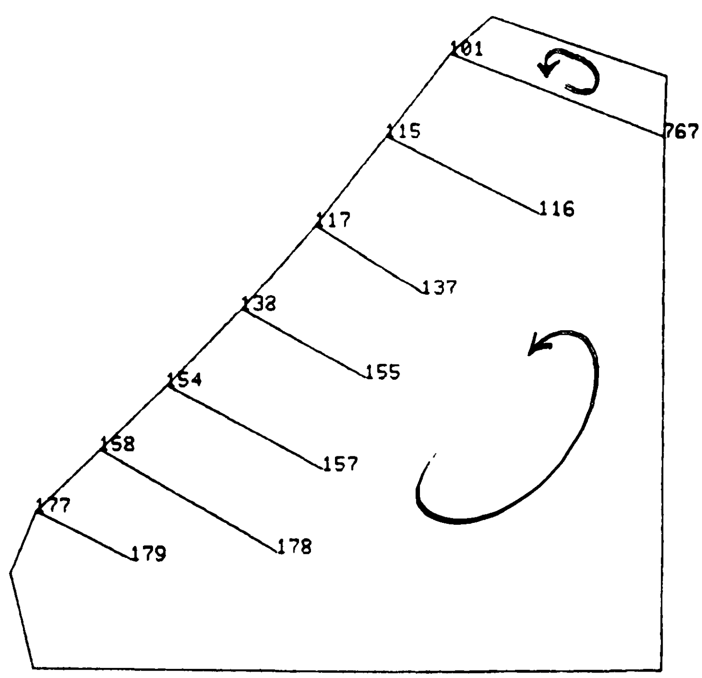

Figure 4. Kirchoff analysis of block 107: 1 component, 15 essential arcs, 2 cycles, 6 acyclic arcs and 14 essential points. Essential points are labelled. Cycles are indicated by arrows.

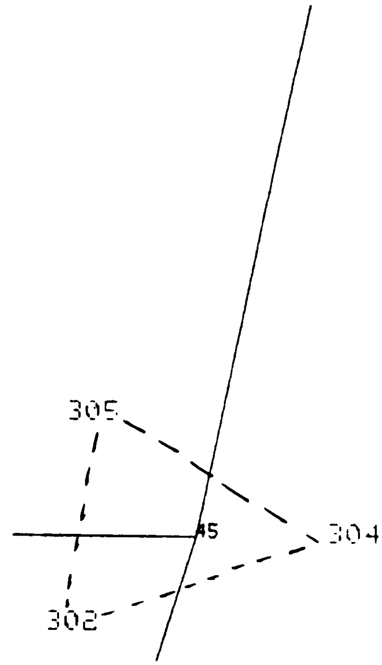

Figure 5. The dual graph around vertex 45 is a single cycle with one arbitrary essential point.

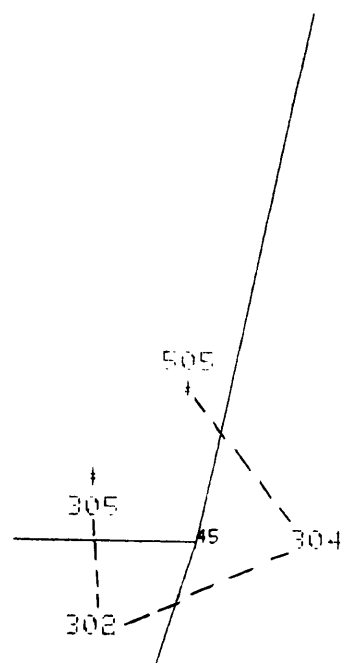

Figure 6. The dual graph around vertex 45 is a single acyclic arc with 2 essential points, which are labelled with asterisks (*).

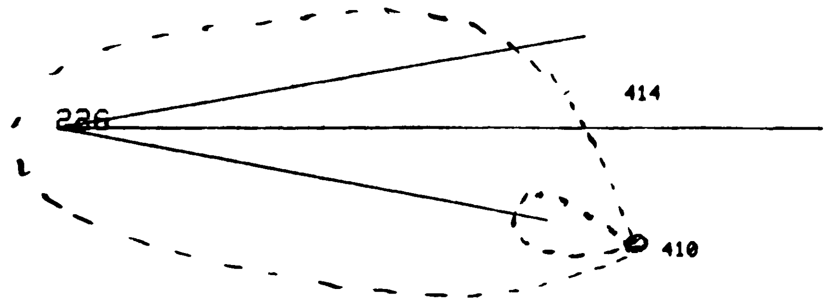

Figure 7. The dual graph around vertex 226: 2 cycles, 0 acyclic arcs, 1 essential point (410). This graph is permissible.

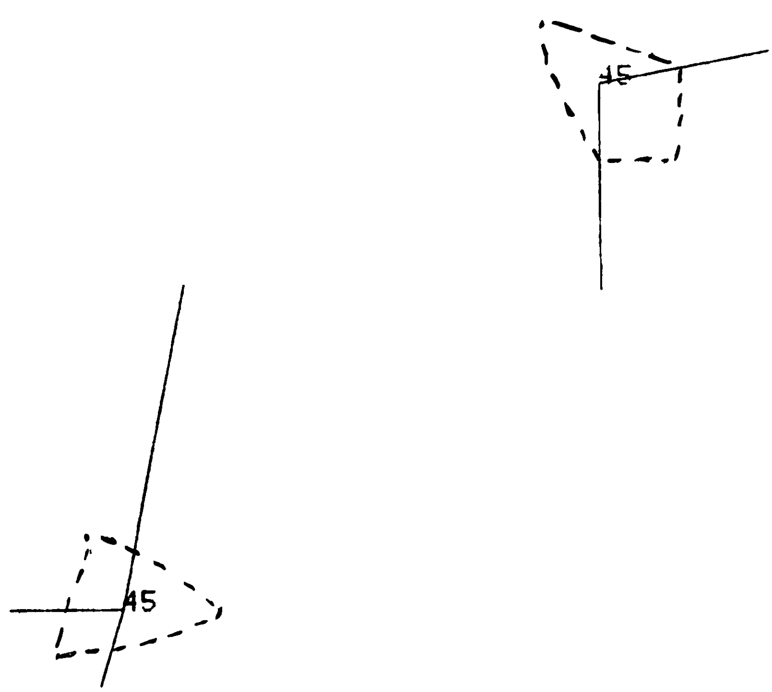

Figure 8. The dual graph around vertex 45 consists of 2 components, due to replication of vertex number 45.

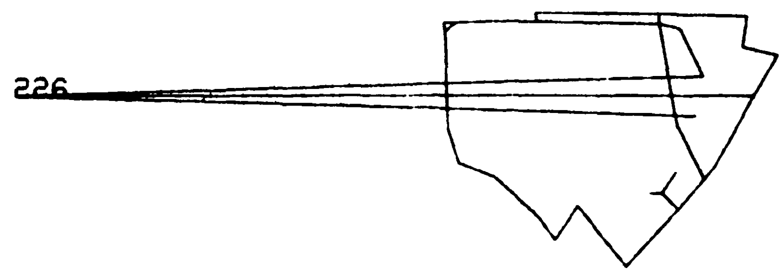

Figure 9. Vertex 226 lies outside its fundamental neighborhood.

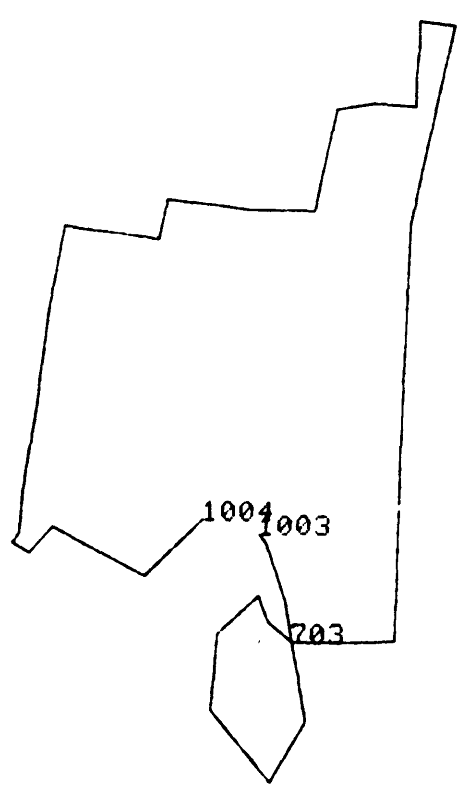

Figure 10. The boundary of the neighborhood of a vertex has acyclic arcs. The essential points are labelled.