MAP GENERALIZATION:
THEORY, PRACTICE AND ECONOMICS

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INTRODUCTION

The theory of map generalization is discussed with particular reference to the processes of simplification and classification. Both of these processes are used extensively in computer-assisted cartography; however, a comprehensive, flexible software package has not yet been organized from the myriad of algorithms available. Further, systematic use of algorithms, based on the reasons for generalizing in cartography, is not common practice. When viewed in respect to the reasons for generalization, the costs of using a theoretically based algorithm can easily be absorbed in most computer-assisted cartographic situations.

Robinson and Sale in the third edition of Elements of Cartography define cartographic generalization as being composed of four elements. These elements or processes are simplication, classification, symbolization, and induction. They do not claim that these four processes are independent of one another, only that they represent facets of the total process of cartographic generalization which cartographers can conveniently isolate and discuss. The definitions of these four processes by Robinson and Sale are not rigorous, but they are the best available.

Non-rigorous definitions are usually ignored when the cartographer interacts with automated systems to produce a map. Therefore, it is not out-of-line to suggest that a non-rigorous definition of generalization and its elements is of little or no use for the purpose of making a map with computer assistance. One can use non-rigorous definitions only to discuss the map after a software manipulation sequence has been selected and/or the map plotted. What is basically needed is to define generalization so that the reason or purpose for making the map can aid the cartographer in selecting the most appropriate software sequence for map production given the structure of stored data files and the available software manipulation sequences. To date, this luxury is not available.

The intent of this paper is to offer more rigorous definitions for the purpose of classifying the processes involved in map generalization, to show how these processes have been and/or may be implemented in practice, and hopefully to give some insight into the economics of generalization algorithms relative to the total map preparation costs.

THEORY

It is possible to redefine and categorize the four processes of generalization mentioned above as transformations possessing or not possessing certain properties. The transformational properties considered are two: the property of being one-to-one and the property of onto. A function $f: A \rightarrow B$ is said to be one-to-one if distinct elements in A have distinct images, (i.e., if $f(a) = f(a') => a = a'$). A function $f: A \rightarrow B$ is said to be onto if every $b \epsilon B$ is the image of some $a \epsilon A$, (i.e., if $b \epsilon B => a \epsilon A$ for which $f(a) = b$). 2/ The property of one-to-one is the easier to understand. If there exists a data file A and a derivative data file B resulting from a transformation of file A, so long as each single entry selected from file A results in a single entry in file B, the property of one-to-one is preserved. The onto property is a little harder to illustrate. It requires that every entry in file B have a referent entry in file A. In other words, file B has no element that has no referent in file A.

The possession or lack thereof of these two properties gives four possible categories of transformations: 1) transformations that are both one-to-one and onto (bijective), 2) transformations that are neither one-to-one nor onto, 3) transformations that are one-to-one and not onto, and finally 4) transformations that are onto but not one-to-one.

If one examines closely how a cartographer generalizes, it appears reasonable to refer to a transformation that is one-to-one and onto as approximating the process of simplification, a transformation that possesses neither property as induction, a transformation that is one-to-one but not onto as symbolization and finally a transformation that is onto but not one-to-one approximates classification. If cartographers then use these transformation properties to refer to the four processes of generalization, any algorithm can be classified as to which generalization process or combination of processes it approximates. Thereafter discussion can follow based on the knowledge that everyone involved understands what each other means by a simplification process or a classification process.

To use these more rigorous definitions of the processes of generalization can help the cartographer. But before this can be illustrated, the term "map" must also be defined. Simply stated, a map is a communication channel. Its primary function is to serve as a vehicle to communicate information from one cognitive realm to another cognitive realm. To start the mapping process, an idea about what the cartographer wishes to communicate initiates a selection from his cognitive realm of information to be communicated. (See Figure 1.) This initial selection is both one-to-one and onto and is not part of map generalization. Only after this selection process can map generalization take place, and furthermore, the processes of map generalization can only take place under certain additional constraints.

Ideally all of the following constraints are present: purpose of map, map format, scale, intended audience, data quality, and graphic limitations. Practically, of course, there is an economic constraint; and in fact, all of the above constraints are usually based in economic terms.

After the selection process, generalization of the selected information can take place to form the cartographer's conception of the map (illustrated as set A" in Figure 1). The processes of simplification and classification may be performed sequentially several times and in any order until the cartographer has arrived at a conception of the map, $A" = C"UD$. Once the cartographer has a conception of the

map, symbols can be assigned and thus the third generalization process, symbolization, becomes operative.
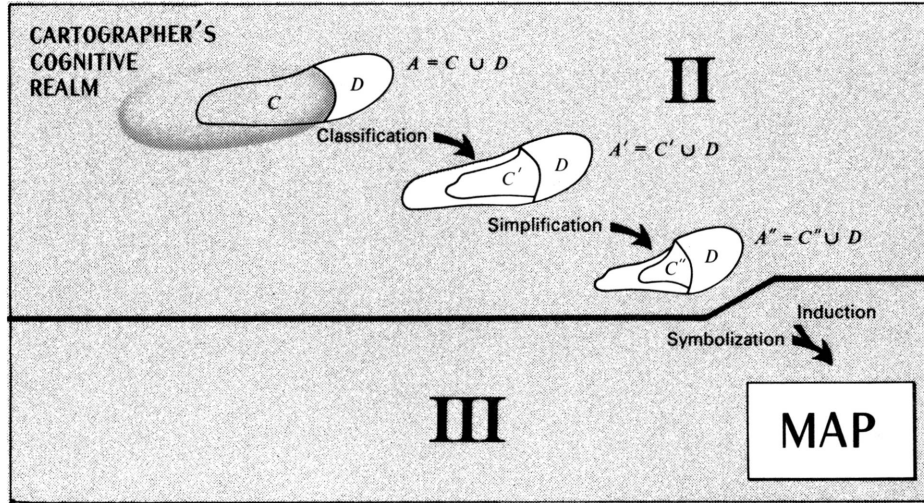

Figure 1. The mapping process

The cartographer may reiterate back to more simplification and/or classification transformations if a given symbolism appears inappropriate, but in automated cartography such reiteration may become costly. In fact, the automated case of map production is only efficient where the cartographer has so specified the conception of the map that each generalization process need be performed only once. The final process of inductive generalization accompanies symbolization and the cartographer has little or no control over it. But if the three processes of simplification, classification and symbolization are done well, induction will probably not hinder the communicative effectiveness of the map. Rather inductive generalization usually enhances a map's effectiveness.

According to this line of reasoning then, in theory, map generalization is a series of processes defined by the properties of their transformations applied to a selected set of information with the explicit purpose of efficiently conveying that information via a communication channel called a map to a map reader or a map-reading audience. How is this instituted into cartographic practice and, more specifically, into computer-assisted cartographic practice?

PRACTICE

In computer-assisted cartography, two types of computer-readable data storage currently predominate: 1) data stored as strings of coordinates (mostly the result of digitizing), and 2) data stored as picture elements (remotely sensed or scanned). While all four generalization processes can operate on both types of stored data, this paper will concentrate solely on the processes of simplification and classification while ignoring symbolization and induction.

SIMPLIFICATION

Two cases can be identified in the simplification of data stored as strings of coordinates: 1) point simplification and 2) feature simplification. Point sim- plification refers to the simplification of a string of coordinate points defining a line or the outline of an area. Such simplification takes place by eliminating all points but a chosen few that are deemed more important for phenomonological character retention. The feature case of simplification occurs when many small items of the same class are present in an area. Certain of the items are retained while others are omitted. The essential character is retained while detail is elim- inated. In addition to these two cases of simplification, the simplification of data stored as picture elements can consist of smoothing techniques including sur- face fitting, and/or enhancement procedures.

Phenomena exist in reality as points, lines, areas or volume. It has been shown that by rigorous definition the cartographer conceives of these for mapping purposes as really only three categories: point, line and area/volume. 3/ The car- tographer must conceive of all data as existing for mapping purposes in one of these three categories. Theoretically, when discussing data stored as strings of coordi- nates, each of these three categories can occur in each of the two cases, point and feature simplification; however, if the features themselves are points, there need be no difference in the simplification algorithm between the two cases.

Figures 2 and 3 illustrate the point case for line data and the feature case for area/volume data respectively. In Figure 2, the map of Portugal illustrates
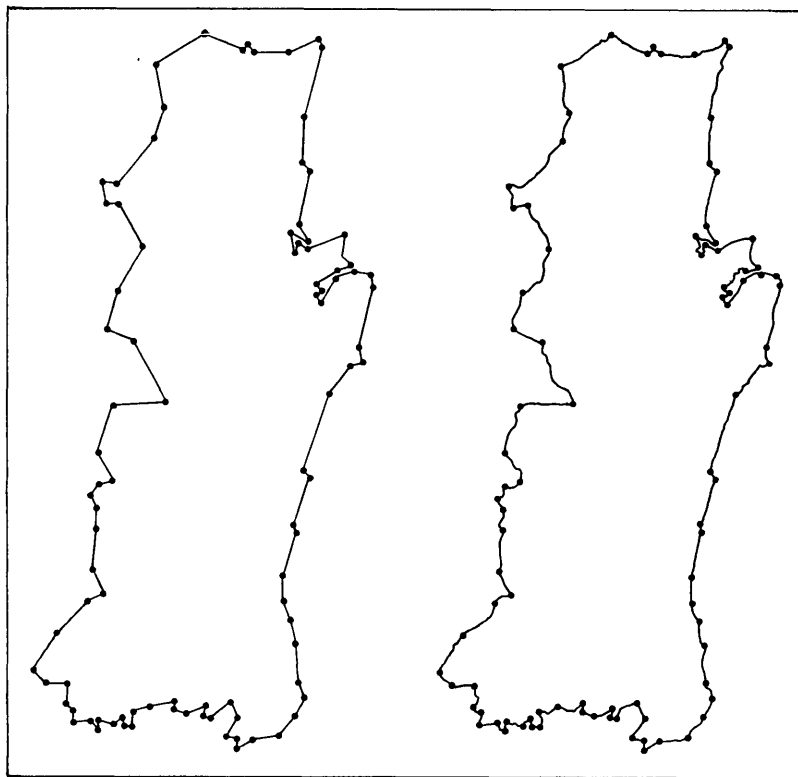

Figure 2. Simplification of lines from selected points

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that simplification of lines results from the one-to-one transformation of selected points onto a new map. The points denoted on both sides of Figure 2 are exactly the same. These selected and transferred points from the left map are connected by straight line segments and are shown on the right map.

Simplification by feature elimination is illustrated in Figure 3, which shows forest cover areas of Sheboygan County in Wisconsin. In feature elimination simplification, either a feature is shown in its entirety or it is omitted. In Figure 3, the smaller areas present on the map on the left are eliminated on the map on the right. Obviously it is possible to combine point and feature elimination simplification by eliminating features and by point simplifying the outlines of the uneliminated features.

Figure 3. Simplification by feature elimination

For data stored as picture elements, simplification using a smoothing operator is illustrated in Figure 4. The picture elements are evaluated one at a time and retained as is or modified; thus the transformation retains its properties of one-to-one and onto.

Simplification can be applied along either of two dimensions. Either a reduction in the scale dimension (see Figures 7 and 8), i.e., reduction from an original scale to a smaller scale representation, or a simplification along some constant scale dimension (see Figures 5 and 6), i.e., a detailed representation versus a simplified representation at the same scale. Figures 2, 3, and 4 are all illustrations of this latter type. Along either of these two dimensions, the process of simplification can be applied to each data category in both point and feature elimination cases and/or using smoothing operators. Usually generalization in practice calls for an application of the simplification process primarily along the dimension of scale reduction and secondarily along the dimension of constant scale. It also usually involves, for stored data strings, both the simplification of points and feature elimination.
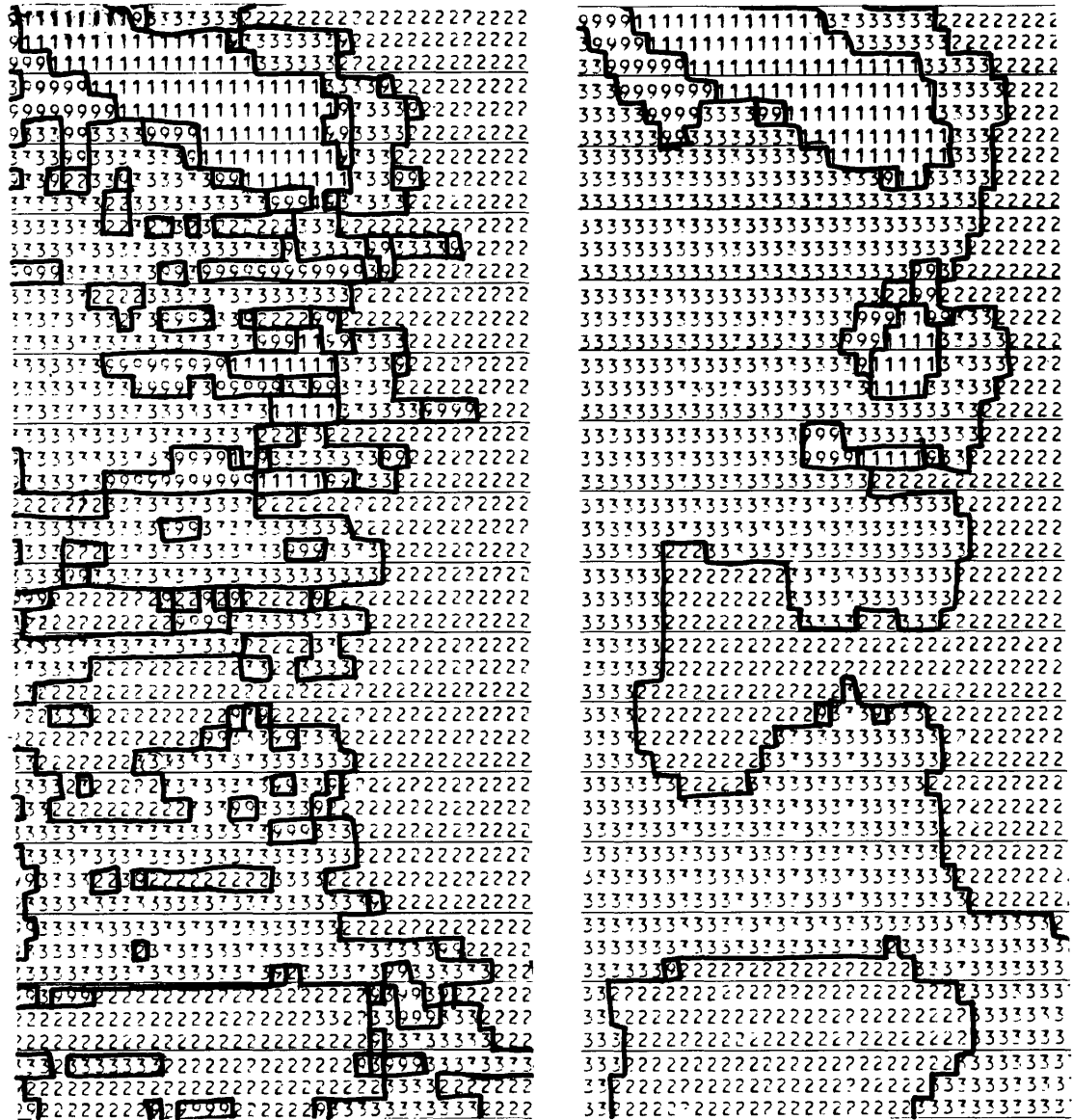
Figure 4. Simplification using a smoothing operator

Many algorithms currently exist to aid the cartographer in computer-assisted simplification. Currently in computer-assisted cartography data reduction is sometimes referred to as a separate process in constructing and editing files from the generalization process. Data reduction is not different from the simplification process and, therefore, should be considered part of cartographic generalization, not just data preparation. As long ago as 1964, Tobler outlined several possibilities for the simplification of lines composed of a series of points. 4/ For 1964, Tobler's work was a very useful contribution, and his conclusion at that time was that none of his tested methods resulted in generalizations that were appreciably better than a generalization done by allowing the output hardware to eliminate all points that were closer together than its plotting resolution. Since 1964, however, common plotter resolutions have become smaller, so that the efficiency of allowing the output hardware resolution to determine which points are plotted can again be questioned.
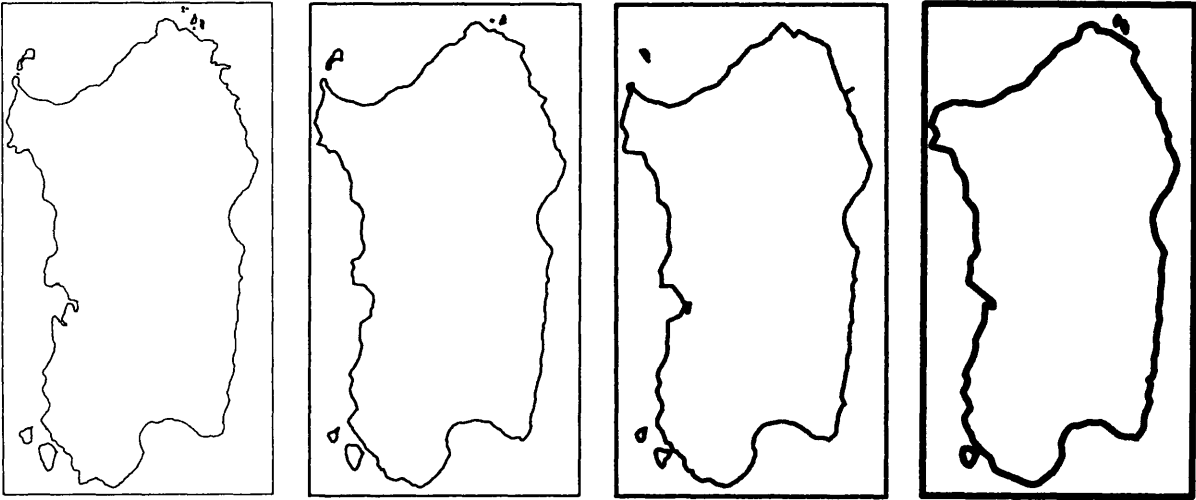
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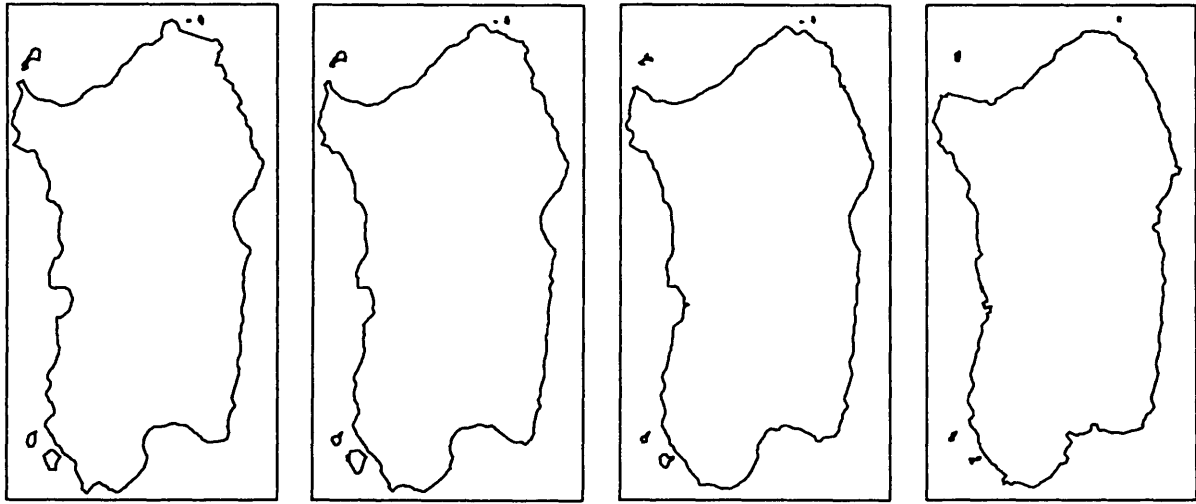
Map 3a.　.01　　　Map 3b.　.30　　　Map 3c.　.60　　　Map 3d.　.90

Figure 5.　EFFECTS OF GENERALIZATION LEVEL
(from Brophy, 1973)

Map 4a.　.012"　　　Map 4b.　.020"　　　Map 4c.　.032"　　　Map 4d.　.047"

Figure 6.　EFFECTS OF LINE WEIGHT
(from Brophy, 1973)

Later during the 1960's, several other point or feature elimination simplification methods were proposed. 5/ Likewise, smoothing operators which can be applied to picture elements during a simplification algorithm have been proposed for spatial smoothing and filtering. 6/ The computer application of these algorithms to data matrices is straightforward.

Two methods of simplification applicable to data strings were readily available at the University of Wisconsin to run a comparison, and the results are illustrative and should be of interest to cartographers. The methods were programmed by Brophy and Douglas respectively. 7/ Brophy's algorithm appears to have a firmer base in cartographic theory.
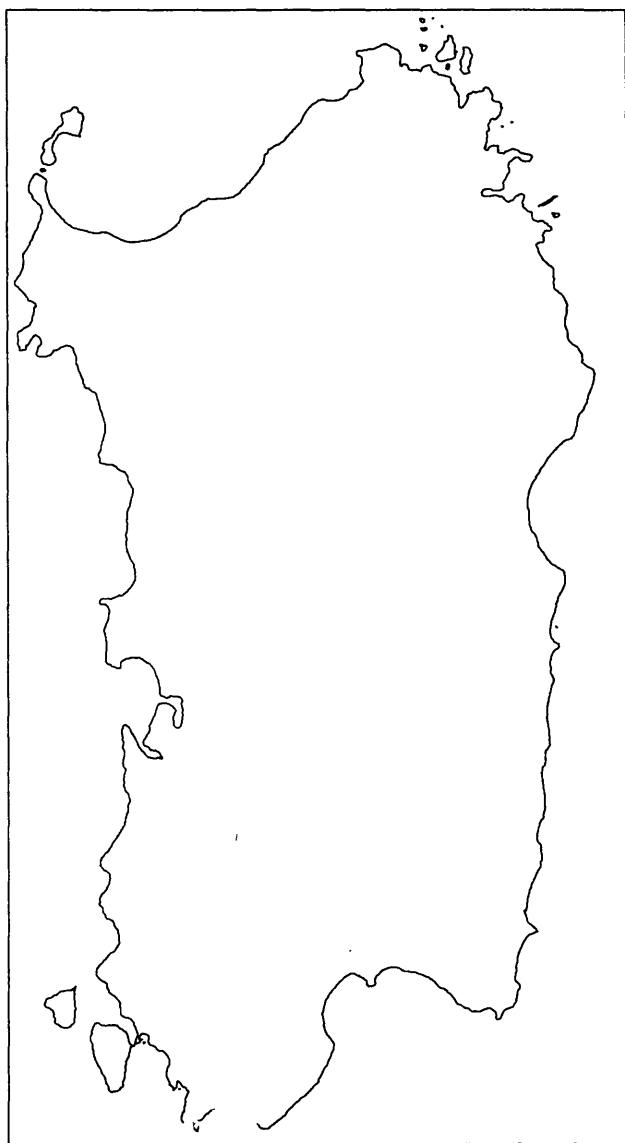
Brophy programmed a simplification routine for the point case of lines that requires classification first. The method consists of five components and a detailed explanation is readily available in the literature. 8/ Although this algorithm sounds complex, it runs rather efficiently and gives reasonable results. Figure 5 is taken from Brophy's work and shows from left to right increased generalization levels at a constant scale and line width. Figure 6 illustrates increased line widths at a constant level of generalization and scale. Figures 7 and 8 show constant levels of generalization and line width at increased scale reduction. The total algorithmic package allows the cartographer to specify: a) the amount of generalization, defined on a $0<x<1$ scale, b) the line width (assuming more detail can be presented at a constant scale with a finer line width), and c) the amount of scale reduction.

Figure 7. 1:125M Generalization .20 Feature Elimination (from Brophy, 1973)
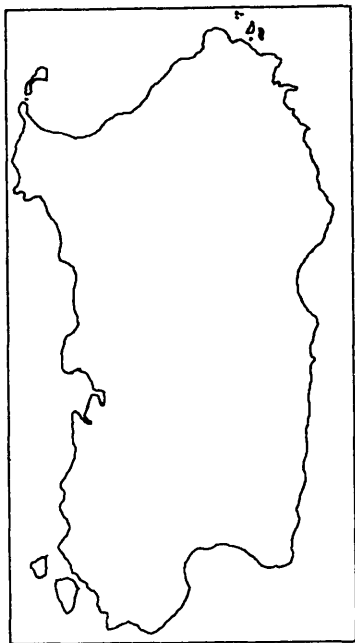
Map 2b.　1:3M

Map 2c.　1:8M

Map 2d.　1:16M

Map 2e.　1:32M

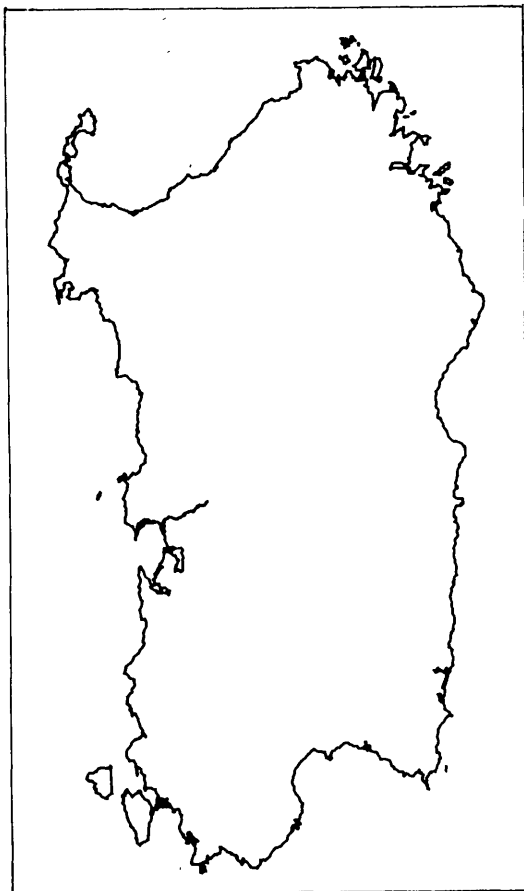Figure 8.　EFFECTS OF SCALE
(from Brophy, 1973)

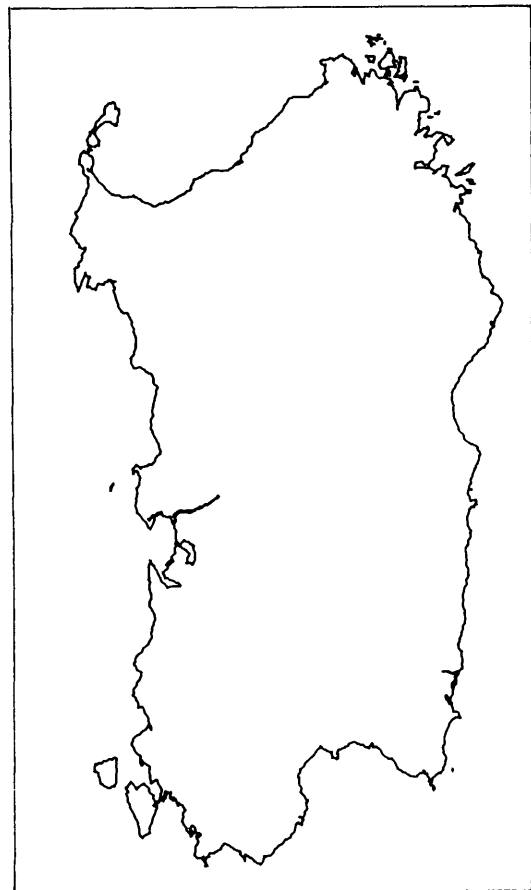Figure 9.　A plot of the original file,
15,000 points　　scale 1:2,125,000

Figure 10.　A plot of a reduced file,
1000 points　　scale 1:2,125,000

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The second algorithm, included in a software package developed by Peucker with the particular subroutine of interest written by Douglas in 1972, offers a simplification routine that considers the elimination of points based on the degree of offsetting of these points from lines connecting separated points along the curve.9/ A case study comparing those algorithms can be seen by observing Figures 9, 10 and 11.

Figure 9 shows a plot of the original Brophy data file of Sardinia, 15,000 points in all (plotter resolution generalized). Figure 10 shows a plot of every 15th point of Brophy's data file (1,000 points in all). Figure 11 shows the results of Douglas's generalization of these files. These are comparable to Brophy's plot shown as Map 2c in Figure 8. Brophy's system would appear to offer more options to the cartographer for the point case of simplification, however, economics will probably dictate which algorithm will be used. Douglas's algorithm has no provision for feature elimination, thus the reduced line work on the west side of Sardinia closes on itself and results in heavy dark lines. Brophy's algorithm removes such features altogether.
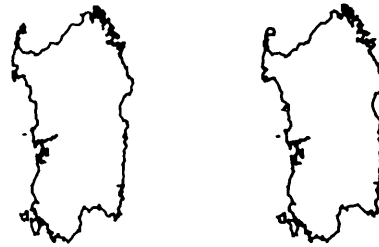

Figure 11. Figures 9 and 10 reduced to a scale of 1:8,000,000 using the Douglas algorithm

The rather unique case of feature elimination of points, however, can be treated just like the point elimination in lines as discussed above. For example, consider a dot map where one dot represents 50 people. Using simplification, a map where one dot represents 250 people would be as shown on the right in Figure 12. Simply eliminating four out of every five dots, allowing one of the original dots to remain, is an example of simplification.
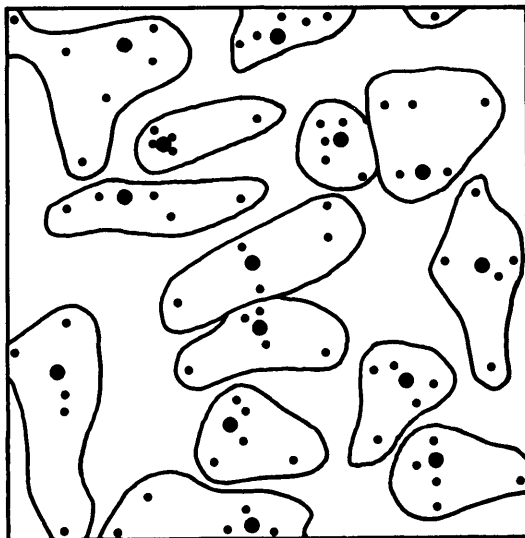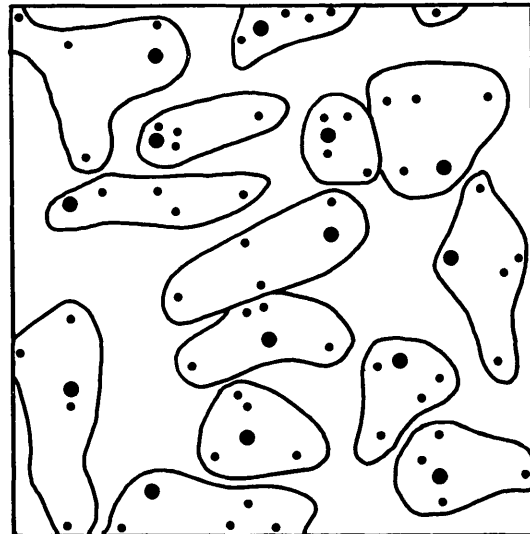
classification   simplification

Figure 12. Examples of classification and simplification of a dot pattern

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As mentioned earlier, most surface fitting and smoothing routines are simplification techniques. Moving averages represent one such class. The number of automated algorithms available to accomplish this are legion. Surface approximations by mathematical equation are simplification routines, provided the region approximated is exactly coincident with the region to be mapped. If extrapolation takes place using a surface approximating equation, the process has become one of induction for the area of extrapolation.

Finally, feature elimination routines can only be done easily in simplification if rankings have been recorded in the header information. Simplification is then accomplished merely by calling for all streams of rank 5 or above, or all roads of rank 2 or above, etc. Without rank assignation, there are no simple feature elimination schemes available. All available schemes would require operator intervention or size calculation for elimination.

What commonly seems to be lacking in most of these simplification algorithms is a convenient means for determining the correct input parameters. For example, smoothing is often applied iteratively, but at the end of what iteration does one stop? Statistical criteria can be established but a visual subjective decision, efficient only in the interactive mode, is usually relied upon. Brophy gives us flexibility in terms of specifying these parameters, but where are the guidelines for using the parameters? Cartographers to date have rarely relied on a theory to specify the weights that one might wish to apply to a smoothing algorithm. These are areas in need of research. The same arguments can be raised about surface approximation techniques. Is 95% explanation sufficient? The point is: Given our map purpose, what can we deduce from cartographic theory about answers to these questions? Answers to these questions should be our goal.

CLASSIFICATION

If one clustered 5 dots in Figure 12 and assigned a dot to the centroid of the dot cluster, the process would be classification. (See the left side of Figure 12). Two cases of classification routines are important for cartographers. (A) the agglomeration of units, and (B) the selection of class limits. Again, the categories of point, line and area/volume data can occur in both cases. Ancillary problems include the allocation of unit areas to a type or class, e.g., if an area is 40% cropland, 30% forest and 30% water covered, is it classed as cropland or as mixed since no category covers over 50% of the area? A similar problem is the categorization of picture elements.

The agglomeration of like units takes place most often by point clustering techniques from tables of locations of data points in storage. The agglomeration of lines is rare and the agglomeration of areas is part of one type of dasymetric mapping as well as applicable to data stored as picture elements. The automation of point clustering techniques is possible and at least one software system can purportedly agglomerate irregular areas as in dasymetric mapping.[10]

The second case of classification, namely, class interval selection, has perhaps received more than its fair share of space in the cartographic literature. These routines obviously work only on ordinally or higher-scaled data. In general, in class interval selection, the cartographer has a theoretical base from which to work.

Since the cartographic goal is the communication of information via a map, at least three theoretical bases can be justified in class interval classification schemes: 1) the use of critical values, 2) the satisfaction of a set of statistical criteria, and 3) standardized schemes which purportedly enhance map comparisons. The first basis, critical values, is externally imposed on the cartographer. For example, if the Department of Health, Education and Welfare desires that all families with annual incomes under $10,000 qualify for a given Federal program and those with incomes $7,500 - $10,000 qualify for 25% benefits, those from $5,000 - $7,500 for 50% benefits and those under $5,000 for full benefits, the cartographer's class limit classification scheme presents no problem.

Examples of the second theoretical basis, the satisfaction of a set of statistical criteria, have also been outlined and programmed. When no outside criteria are imposed, cartographic theory could dictate that the aim of class interval classification is to define a set of class limits such that a member classified into any one class is more like the members of that class than the members of any other class. In other words, the theoretical goal of classification is a maximization of the homogeneity within the classes and a maximization of the heteogeneity between classes. These are statistical criteria, and Jenks has proposed iterative algorithms to solve this problem. In 1967 he proposed the equal average deviation scheme and the relative equal average deviation scheme.[11] Both satisfy a theoretical goal that can be deduced from cartographic theory.

Finally, when considering the comparison of maps the appropriate class interval scheme is less certain. Muehrcke and Muller [12] have studied the problem. Armstrong suggests the use of the mean and standard deviations for class interval selection when comparisons will be necessary both between maps and over time.[13] Much work remains to be done in this area, however. Olson and Monmonier [14] have separately studied map complexity as it relates to class interval determination. Standard routines have not been agreed upon. In any event, computer assistance will be both necessary and relatively easy.

In concluding this section the problem with simplification and classification schemes, i.e., the problems in automating cartographic generalization, would appear to be solved for the majority of the cases encountered. The need is to have a flexible efficient software package that has all of these routines available and to have cartographers sufficiently educated to allow them to make a judicious choice for their mapping requirements. A few areas still demand software development, however, and cartographers need to be more thoroughly educated before they can fully utilize computer-assisted generalization as it should be used from a theoretical point of view; that is, objective generalization before the making of the map.

ECONOMICS

The final part of this paper quickly touches on the subject of the economics of computer-assisted cartography.

What are the economics of computer-assisted generalization? Some algorithms vary directly with the number of input data points, others vary exponentially. Simplification which is one-to-one is often assumed to vary linearly with the number of points. This tends to be true except for some applications of surface approximations. Classification schemes are often not linear, rather they tend to vary

exponentially with the number of points, and most hybrid algorithms, i.e., those algorithms having both classification and simplification components, are therefore exponential.

Returning to the examples of the point generalization of Sardinia discussed above, a cost breakdown for the 1:8 000 000 plots utilizing both software routines is as follows:

	Brophy		Douglas
Reading original data file (15,000 pts)	14.23		14.23
Reducing to 1:8 000 000 (15,000 pts)	3.22		5.67
Reducing to 1:8 000 000 (1,000 pts)	N/A		0.88
Reduction of 15,000 pts to 1,000 pts		5.47	
Reading reduced file (1,000 pts)		1.09	

All data in seconds of CPU time

Does this have important economic considerations for cartographers? In any generalization scheme there is the fixed cost of reading the original data. Obviously the above data support the hypothesis that the costs here can be overly large due to the storage of too much data. In fact, it may be possible to more than absorb the cost of a theoretically-based generalization scheme by reducing to reasonable size the amount of stored data. Experience at the University of Wisconsin Cartographic Laboratory has been that our digitizers capture far too much data for our intended uses. Therefore, a one-time reduction of the data in storage to eliminate unneeded and unuseable detail often allows long run savings in excess of the cost of adding objective generalization schemes, as the above illustration demonstrates.

Cartographers need theoretically-based generalization algorithms that are of use at the planning stage of map preparation not only in the analysis of the map after it is made. The additional cost of utilizing such algorithms in computer-assisted cartography may well be less than the cost of current operating expenses due to the fact that presently in computer-assisted cartography, cartographers are commonly storing and processing too much data, allowing hardware resolution to determine output, and in so doing relinquishing direct control over map generalization.

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