STATISTICAL ACCURACY AND MAP ENHANCEMENT IN CHOROPLETHIC MAPPING

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INTRODUCTION

Many cartographers have been concerned with the difficult problem of conciliating map accuracy and legibility in portraying spatial distributions. Geographic distributions are seldom simple enough to be readily perceived and understood. For the sake of cartographic communication, a certain amount of generalization is usually required. The original complicated distribution is converted into a simpler form. The new cartographic image, however, contains less information than the original data set; and there is no guarantee that the information lost is irrelevant. This last observation has led Waldo Tobler to challenge the idea of class generalization in choroplethic mapping. 1/ Class generalization is accomplished by partitioning the range of the data set values into intervals, and by representing all map points with values in a given interval by a same gray tone. This type of choropleth generalization was traditionally motivated by 1) the difficulty of finding shaded screens whose visual intensity was exactly proportional to the original data intensity, and 2) the reader's inability to distinguish and visually separate more than a limited number of different shades. 2/ The first obstacle can now be technically overcome by programming an automatic line plotter to create a continuum of gray tones with virtually any light intensity. 3/ The gray tone values simulate the statistical values of the enumeration units. Thus, choropleth maps can be used to portray accurately every value of a given spatial distribution. Tobler's solution, however, has been strongly criticised on the ground of map legibility. 4/ It was argued that the resulting increase of information displayed by choropleth maps without class intervals may obliterate the reader's ability to perceive the map distribution. Aside from the complexity problem, the poor enhancement of unquantized choropleth patterns presents a major drawback. The purpose of this paper was to investigate the effect of quantization on one particular aspect of map enhancement -- map contrast. Contrast in blackness was computed within and between a series of quantized and unquantized choropleth patterns. Finally, the contradictory relationship between statistical accuracy and pattern enhancement was discussed and solutions conciliating both variables were described.

DEFINITION OF CONTRAST

Choropleth maps usually show subsets of regions that appear light or dark. The lightness or darkness of each region can be described by a picture function $Z(x,y)$ proportional to the light intensity impinging the map at the points (x,y) of the region. Contrast within a choropleth map was measured by the variation of darkness between adjacent regions, whereas contrast between different maps was measured by the variation of the overall darkness of the map bodies.

CONTRAST WITHIN MAP

The outline of choropleth regions are the common boundary of enumeration units whose Z values are different. Thus the contrast between adjacent regions can be found by measuring the absolute difference of darkness between the corresponding enumeration units:

$$c_{12} = |Z(x_1, y_1) - Z(x_2, y_2)| \qquad (1)$$

where (x_1, y_1) and (x_2, y_2) are map points on two adjacent regions. The overall contrast within maps was computed by the formula:

$$C_w = \sum_i^n \sum_i^n c_{ij} \qquad (2)$$

where n = number of enumeration units.

$c_{ij} = |Z_i - Z_j|$, absolute difference of Z values between units i and j if units i and j are adjacent, and $c_{ij} = 0$ when otherwise. In this study, the above formula was standardized by defining the measure of contrast as a ratio between the observed contrast within a map and the maximum contrast if the absolute difference $|Z_i - Z_j|$ was maximum for every i and j.

CONTRAST BETWEEN MAPS

In a multimap comparison situation, some maps may appear visually darker than others. In this study, contrast between two maps, say k and l, was defined as a function of the absolute difference of blackness between the corresponding map bodies:

$$C_b = |B_k - B_l| \qquad (3)$$

where blackness of map k (B_k) is the average percentage of the paper in the map body that is covered with ink. Map blackness was computed as follows:

$$B = \sum_i^n s_i b_i / \sum_i^n s_i \qquad (4)$$

where n = number of enumeration units; s_i = area of unit i; and b_i = blackness of unit i (in percentage).

THE EXPERIMENT

Seventeen geographic variables depicting various aspects of the geography of France were mapped on an automatic line plotter using Tobler's Calcomp Choropleth Mapping Program. 5/ Area, number of enumeration units (22), scale, and symbolization were held constant.

Symbolization in itself is a complex problem. Shaded patterns can vary in color, style, texture, orientation and value. In this study, shaded patterns were held constant except for value. The assignment of value intensity to the choropleth classes was purposely conventional. The lowest and highest classes were represented by solid white and black color, respectively. The other classes were symbolized by shaded patterns whose visual intensity was exactly proportional to the class ranks. Although this type of symbolization does not appear statistically very sound, it is the most traditionally used. 6/ The purpose of this experiment was precisely to study the effect of quantization on pattern contrast in the context of conventional choropleth mapping.

Each geographic variable was represented by a series of quantized and unquantized choropleth maps. The number of classes on the quantized maps varied from two to ten. The class interval system for quantizing the data was derived from Jenks and Caspall. 7/ Several maps with and without class generalization were shown in Figures 1 and 2. Considerable variations of contrast within or between the maps could be observed depending on the geographical variables represented and the level of generalization. Study was made of these variations to show the relationship between map contrast and choropleth generalization. Map contrast was related to the broader concept of map enhancement, and solutions were proposed trading off statistical accuracy and map simplification.

CHOROPLETH GENERALIZATION AND CONTRAST WITHIN MAPS

Contrast within map (formula 2) was measured on every quantized and unquantized map. The mean contrast was then computed for each set of seventeen maps (Table I). Results showed the following:

1. Quantized maps were usually more contrasted than unquantized ones. Visual examination of the choropleth patterns confirmed this result(Figures 1 and 2). The grouping of the data into classes emphasized the regions containing abrupt dark-light transitions, and de-emphasized regions of close homogeneous density. This combined smoothing and sharpening process had important implications. Sharp contrast at the edges of choropleth regions produced a figure-ground effect. This was especially true for the maps with few classes. Distributional characteristics that were concealed in the original data set, such as tight regions or contrasted areas were emerging by compression of the interval scaled data into ordinal scaled map values.
2. Contrast increased slowly as the number of classes decreased. The slow increase may be explained by the counter effect smoothing has on contrast. This fact was illustrated by the two-class maps, which in some instances were much less contrasted than the three- or four-class ones (map number 16). In this case, the smoothing effect of pattern simplification and regionalization was more important than the sharpening of the remaining region edges.

CHOROPLETH GENERALIZATION AND CONTRAST BETWEEN MAPS

Contrast between maps was defined as a function of the difference of blackness (formula 3 and 4). Within a given map set, contrast can be measured by computing the deviation of blackness of each individual with respect to the mean blackness of

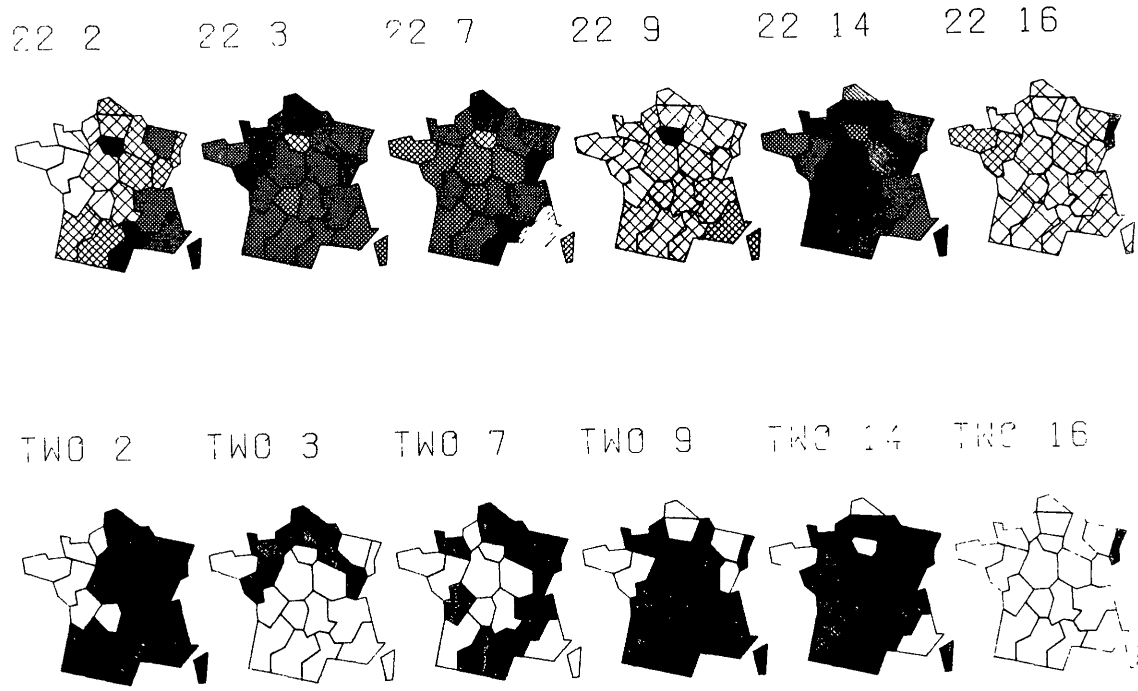

Figure 1. A Sample of Unquantized Maps and Two-Class Maps. The visual intensity of the shaded patterns on the unquantized maps are exactly proportional to the data intensity.

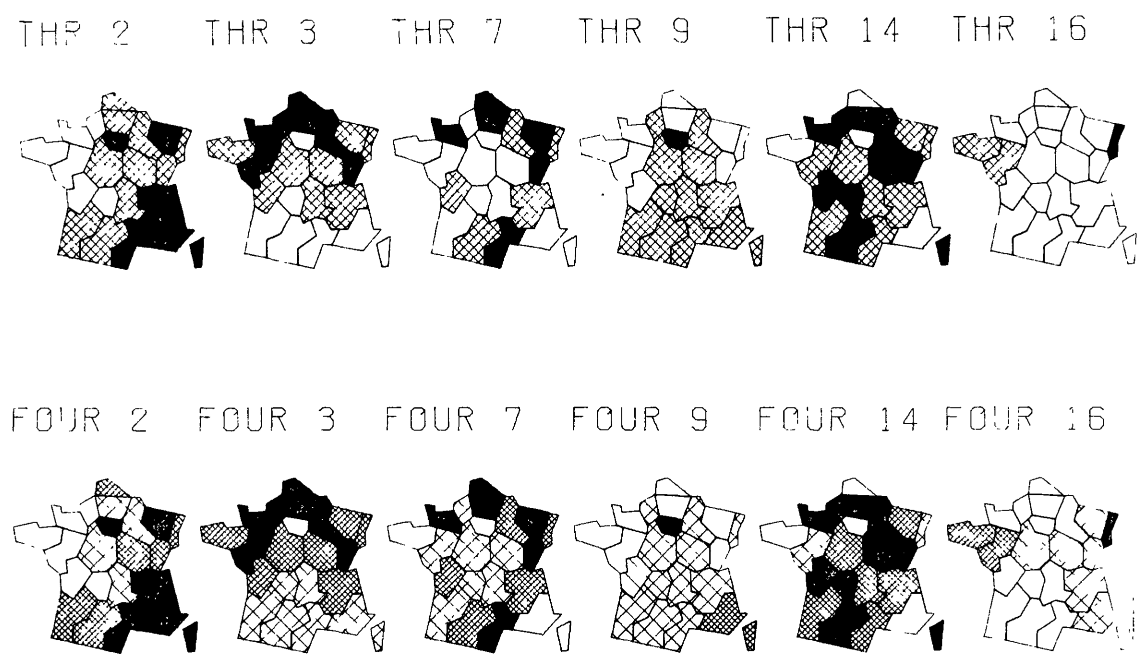

Figure 1. (continuation) A Sample of Three- and Four-Class Maps. The visual intensity of the shaded patterns are exactly proportional to the class ranks.

Figure 2. A Sample of Five- and Seven-Class Maps. The visual intensity of the shaded patterns are exactly proportional to the class ranks.

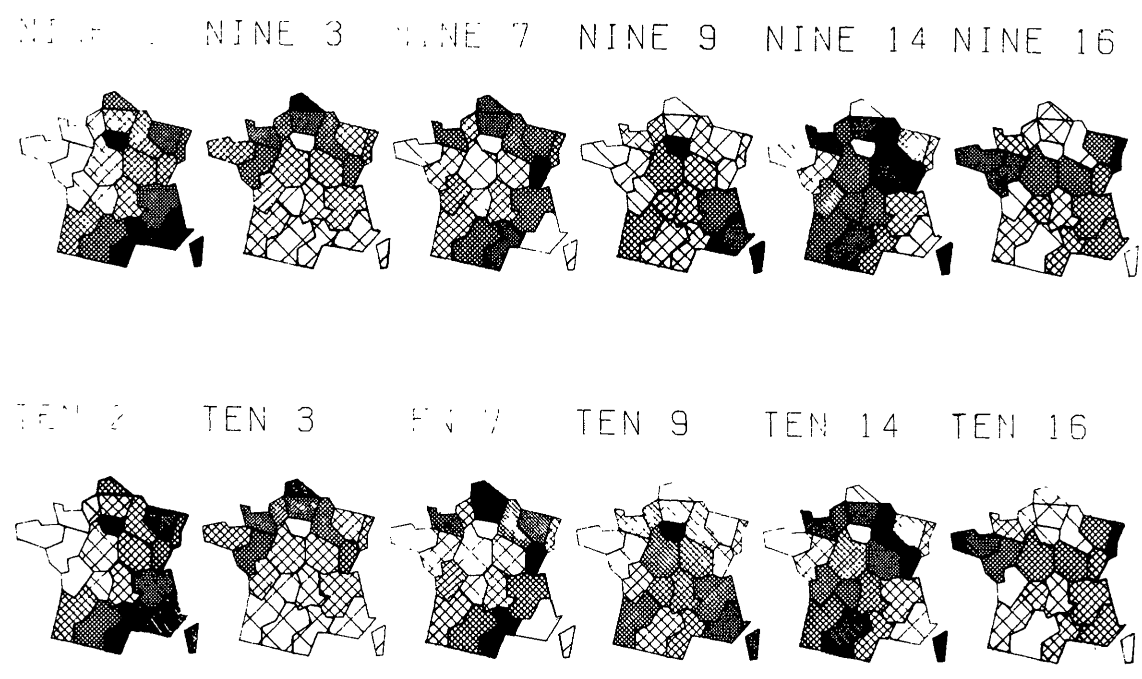

Figure 2. (continuation) A Sample of Nine- and Ten-Class Maps. The visual intensity of the shaded patterns are exactly proportional.

TABLE I. CONTRAST WITHIN MAPS FOR THE QUANTIZED AND UNQUANTIZED CHOROPLETH PATTERNS

Number of Classes:	Quantized Maps 2	3	4	5	6	7	8	9	10	Unquantized Maps
Map Number										
1	0.34	0.25	0.26	0.31	0.28	0.27	0.24	0.27	0.28	0.29
2	0.25	0.35	0.37	0.37	0.36	0.32	0.32	0.33	0.23	0.36
3	0.21	0.30	0.23	0.23	0.18	0.17	0.22	0.20	0.20	0.08
4	0.33	0.34	0.35	0.29	0.25	0.26	0.25	0.27	0.28	0.14
5	0.25	0.25	0.24	0.23	0.25	0.22	0.23	0.21	0.20	0.29
6	0.47	0.45	0.38	0.28	0.25	0.27	0.30	0.29	0.27	0.15
7	0.49	0.42	0.34	0.27	0.30	0.25	0.27	0.29	0.33	0.13
8	0.28	0.34	0.32	0.27	0.25	0.26	0.27	0.24	0.25	0.19
9	0.25	0.15	0.16	0.22	0.18	0.23	0.23	0.21	0.23	0.10
10	0.39	0.37	0.30	0.31	0.32	0.34	0.33	0.31	0.28	0.08
11	0.19	0.33	0.23	0.26	0.26	0.26	0.23	0.26	0.25	0.18
12	0.33	0.25	0.26	0.21	0.26	0.24	0.25	0.26	0.24	0.17
13	0.11	0.23	0.27	0.27	0.28	0.24	0.25	0.22	0.22	0.05
14	0.22	0.40	0.37	0.29	0.32	0.31	0.28	0.26	0.30	0.06
15	0.31	0.33	0.27	0.30	0.27	0.25	0.26	0.29	0.25	0.15
16	0.02	0.07	0.19	0.24	0.27	0.23	0.24	0.26	0.24	0.14
17	0.38	0.22	0.23	0.17	0.23	0.24	0.22	0.23	0.23	0.08
Mean Contrast	0.28	0.30	0.31	0.29	0.27	0.26	0.26	0.25	0.25	0.15

the set. Blackness, mean blackness, and standard deviation of blackness were computed for the maps with and without quantization (Table II). Results were as follows:

1. Quantized maps were often lighter than unquantized ones. In other words, the overall data intensity tended to be underestimated in the generalized maps.

2. Difference of blackness between quantized maps decreased as the number of classes increased. Except for the two-class maps, these differences did not appear as sharp as those existing between the unquantized maps. Thus, the effect of quantization on contrast between the maps was mixed. First, the grouping of the data into numerous

classes tended to smooth out the blackness differences between the maps. Second, these differences were recovered as the number of classes decreased.

3. Contrast was not always consistent. For instance, in the set of two-class maps, map number 9 was darker than number 3; whereas this situation was reversed in the set of three-class maps.

TABLE II. BLACKNESS, MEAN BLACKNESS AND STANDARD DEVIATION OF BLACKNESS FOR THE QUANTIZED AND UNQUANTIZED CHOROPLETH PATTERNS

Number of Classes:	Quantized Maps 2	3	4	5	6	7	8	9	10	Unquantized Maps
Blackness (%) of Map Number										
1	0.59	0.72	0.53	0.52	0.56	0.48	0.54	0.53	0.53	0.70
2	0.79	0.53	0.53	0.61	0.53	0.59	0.58	0.59	0.59	0.50
3	0.38	0.57	0.68	0.74	0.60	0.52	0.58	0.55	0.52	0.86
4	0.24	0.51	0.50	0.59	0.48	0.54	0.48	0.48	0.45	0.71
5	0.42	0.28	0.28	0.39	0.42	0.50	0.47	0.42	0.47	0.60
6	0.61	0.50	0.36	0.52	0.57	0.51	0.51	0.47	0.50	0.70
7	0.61	0.49	0.61	0.69	0.71	0.59	0.57	0.59	0.60	0.82
8	0.36	0.35	0.45	0.37	0.33	0.33	0.41	0.36	0.47	0.27
9	0.68	0.27	0.26	0.31	0.44	0.41	0.38	0.44	0.50	0.41
10	0.28	0.40	0.55	0.45	0.46	0.43	0.48	0.52	0.56	0.91
11	0.25	0.52	0.38	0.47	0.53	0.52	0.58	0.59	0.59	0.68
12	0.41	0.26	0.47	0.36	0.44	0.51	0.47	0.46	0.51	0.63
13	0.89	0.74	0.56	0.45	0.51	0.59	0.60	0.53	0.49	0.93
14	0.82	0.63	0.66	0.73	0.68	0.69	0.72	0.65	0.62	0.94
15	0.66	0.64	0.71	0.63	0.53	0.59	0.61	0.59	0.64	0.75
16	0.02	0.09	0.22	0.32	0.32	0.42	0.49	0.51	0.46	0.32
17	0.33	0.58	0.47	0.34	0.36	0.40	0.36	0.35	0.38	0.69
Mean Blackness	0.49	0.48	0.48	0.50	0.50	0.51	0.52	0.51	0.52	0.67
Standard Deviation of Blackness	0.24	0.18	0.15	0.15	0.11	0.09	0.01	0.08	0.07	0.20

Inconsistency in darkness can be explained by the distribution of the data sets. Quantization of distributions statistically close to uniformity usually provides a misleading image of the data intensity. This is especially true in the case of a distribution of large values and small range. In the mapping process, the relative differences between individual observations are exaggerated and the map does not appear as dark as it should be. Map number 3 provided an example of this case (Figure 3). Another source of darkness inconsistency is the presence of an erratic value isolated from the core of a distribution otherwise uniform. Map number 9 illustrated this situation (Figure 4).

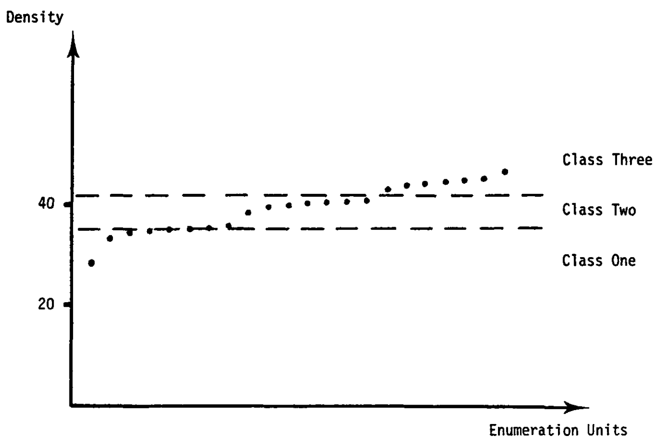

Figure 3. Data Set of Map Number 3. In a three-class partition, fifteen enumeration units were represented in white or medium gray (class one and two). This representation did not simulate accurately the overall data intensity. Whereas the visual intensity of the shaded patterns ranged from white (0%) to black (100%), the corresponding data intensity ranged from 63% (smallest value of the data set) to 100% (largest value of the data set).

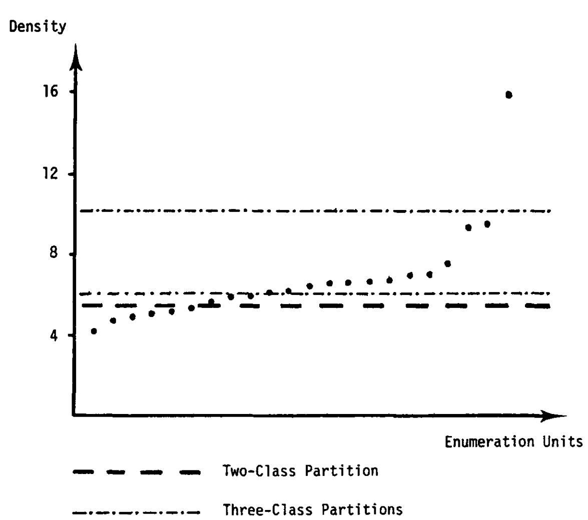

Figure 4. Data Set of Map Number 9. Except for one value, most of the observations were uniformly distributed. Such erratic value had strong unstabilizing effects on the apparent blackness of the map. In the two-class map, sixteen enumeration units belonged to the higher class and were represented in black. Only one enumeration unit remained in the higher class from the three-class partition, which caused the three-class map to appear much lighter.

CHOROPLETH GENERALIZATION AND STATISTICAL ACCURACY

It was observed that map contrast was strongly affected by quantization. Quantization created sharp contrast within the maps; and as the number of classes decreased the contrast in blackness between the maps was amplified. As the number of classes decreases, however, the maps became increasingly less accurate (Table III). 8/ This fact was demonstrated by the curvilinear relationship

TABLE III. OVERALL ACCURACY OF THE QUANTIZED MAPS

Number of Classes:	2	3	4	5	6	7	8	9	10
Map Number									
1	0.50	0.67	0.79	0.83	0.86	0.89	0.91	0.94	0.96
2	0.37	0.79	0.85	0.89	0.92	0.93	0.95	0.96	0.96
3	0.33	0.65	0.79	0.88	0.90	0.92	0.94	0.96	0.97
4	0.70	0.73	0.83	0.88	0.91	0.94	0.96	0.97	0.99
5	0.57	0.74	0.80	0.84	0.88	0.92	0.94	0.95	0.96
6	0.51	0.67	0.76	0.82	0.88	0.93	0.95	0.96	0.96
7	0.52	0.70	0.81	0.87	0.90	0.92	0.94	0.96	0.96
8	0.55	0.72	0.80	0.87	0.92	0.94	0.95	0.96	0.97
9	0.28	0.54	0.70	0.80	0.86	0.89	0.92	0.94	0.96
10	0.51	0.67	0.80	0.85	0.89	0.91	0.94	0.96	0.97
11	0.35	0.56	0.67	0.76	0.83	0.89	0.92	0.94	0.95
12	0.54	0.69	0.81	0.85	0.89	0.91	0.92	0.94	0.96
13	0.46	0.59	0.74	0.79	0.84	0.88	0.91	0.94	0.95
14	0.45	0.70	0.79	0.86	0.88	0.90	0.93	0.95	0.96
15	0.49	0.65	0.73	0.79	0.85	0.89	0.93	0.94	0.96
16	0.33	0.63	0.83	0.88	0.91	0.95	0.96	0.98	0.99
17	0.42	0.62	0.73	0.84	0.88	0.92	0.94	0.96	0.97
Mean Accuracy	0.46	0.66	0.78	0.84	0.88	0.91	0.93	0.95	0.96

Note: The overall accuracy varies between 0 and 1.

between the average accuracy of the seventeen maps and the number of classes (Figure 5). The problem, of course, was to find the level of generalization which best conciliated map enhancement and map accuracy. A few criteria for answering this difficult question may be suggested.

Since one purpose of quantization was to increase map contrast, it seems logical to select the generalized maps that most accurately replicate the contrast between the maps without class intervals. Correlation between the overall blackness of maps with and without quantization showed that the four-class maps most closely reproduce the data intensity (Table IV).

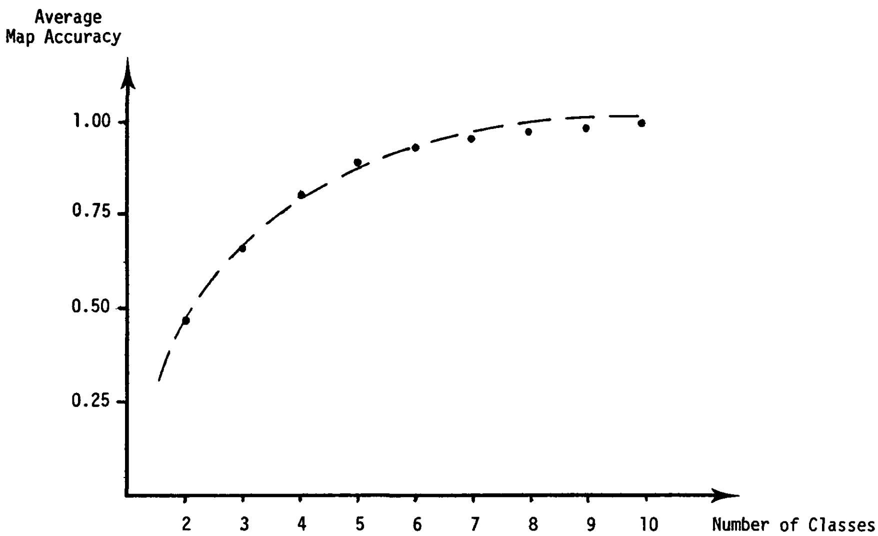

Figure 5. Relationship Between Average Statistical Accuracy and Number of Classes of the Maps.

TABLE IV. CORRELATION BETWEEN THE OVERALL DARKNESS OF THE QUANTIZED AND UNQUANTIZED MAPS

Number of Classes of the Quantized Maps	Correlation (r) with the Unquantized Maps
2	0.34
3	0.67
4	0.69
5	0.57
6	0.68
7	0.63
8	0.58
9	0.51
10	0.37

Another criterion may be provided by the overall accuracy of the maps. One acceptable solution would be to examine the relationship between accuracy and generalization and choose the number of classes which appears to be at a suitable elbow of the resulting curve. For instance, five classes seemed to be the most acceptable solution for map number 3 (Figure 6).

Finally, in the case of multimap comparison, one may be more particularly interested in the representation of statistical relationship between geographic variables. The representation is accurate if the statistical correlation between the maps duplicates the statistical correlation between the variables. This is seldom the case. By compressing the data into classes, some information is lost that usually distorts the relationships between the data sets. The fact that map correlation takes different values on the same mappable set of data would be relatively unimportant, however, if all the correlation coefficients were jointly monotonic. In other words, if all correlation values for pairs of maps on one number of classes were ordered so that they form a monotonic series, the correlation values taken by the corresponding pairs of data sets would also be monotonic. Correlation values were computed between all pairs of variables and all pairs of maps. They showed to be not jointly monotonic, although the relationship between the series became increasingly monotonic as the number of classes increased (Table V).

Two minor elbows were found in the improved relationships of the series, located at four and ten classes which represented the best mapping solutions in this case.

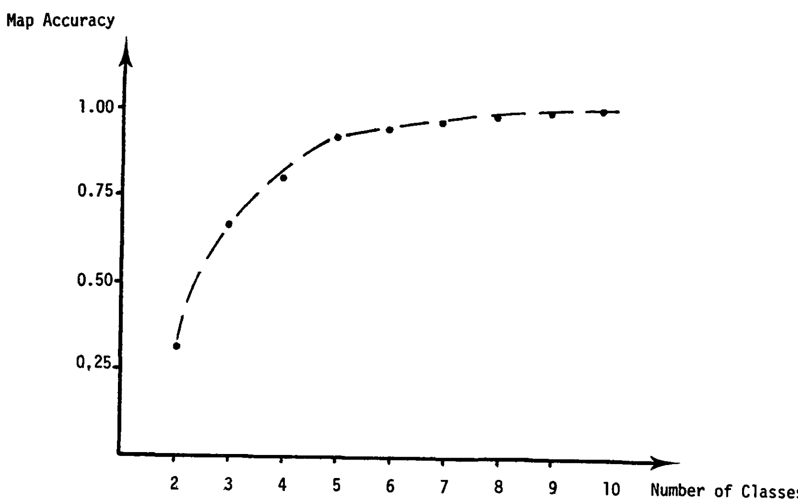

Figure 6. Relationship Between Statistical Accuracy and Number of Classes of Map 3. The optimum mapping solution is indicated by the elbow of the resulting curve which appears at the five-class mark.

TABLE V. CORRELATION (r) BETWEEN DATA SET RELATIONSHIPS AND MAP RELATIONSHIPS

Number of Classes of the Quantized Maps	Correlation (r)
2	0.75
3	0.77
4	0.81
5	0.82
6	0.82
7	0.83
8	0.84
9	0.84
10	0.93

Note: Data-set and map relationships were measured by areally weighted Pearsonian coefficient and areally weighted rank correlation respectively.

CONCLUSION

This study has shown some important aspects of choropleth generalization. Quantization had obvious effects on contrast within map and edge enhancement between regions. Furthermore, experiments have already shown that map readers concentrate most of their attention on the borders between more or less homogeneous regions. 9/ Under these circumstances, data classification becomes a necessity and a key element in the theory of choropleth mapping. It leads to the characterization of well contrasted subsets which can be visually recognized and whose components can be easily assembled into a geographic pattern.

Quantization presents serious statistical drawbacks, however. Map accuracy decreases rapidly as the number of classes decreases. Depending on the distribution of the data, the partition process may lead to a wrong representation of the data intensity. In this experiment, contrast between the generalized maps was not always consistent. Finally the statistical relationship between generalized choropleth maps seldom replicates the statistical relationship of the data sets.

Although a general choroplethic mapping solution, conciliating the contradictory trend of map accuracy and map enhancement does not seem feasible, individual answers to the quantization problem can be found. In this essay, analysis of contrast within and between maps and statistical accuracy suggested that the four or five class maps may represent the optimum mapping solution.

REFERENCES

1. Waldo R. Tobler, "Choropleth Maps Without Class Intervals?" Geographical Analysis, 3 (1973), pp. 262-265.

2. Many cartographers have been concerned with this problem and most of them agree on limiting the number of classes to seven or eight. See G.M. Schultz, "An Experiment in Selecting Value Scales for Statistical Distribution Maps," Surveying and Mapping, No. 3 (Washington, D.C., 1961), pp. 224-230; J. Bertin, Semiologie Geographique (Paris: Gauthiers-Villars, 1967); and A. H. Robinson and R.D. Sale, Elements of Cartography (New York: John Wiley and Sons, Inc., 1969), p. 145.

3. Tobler, op. cit.

4. Michael W. Dobson, "Choroplethic Maps Without Class Intervals?: A Comment," Geographical Analysis, 4 (1973), pp. 358-360.

5. Waldo R. Tobler, "Calcomp Choropleth Mapping Program," Geography Department, University of Michigan (1972).

6. Statistically, it would seem more logical to symbolize the choropleth classes by shading patterns whose visual intensity would be proportional to the data mean of each class.

7. G. Jenks and F. Caspall, "Error on Choroplethic Maps: Definition, Measurement, Reduction," Annals of the Association of American Geographers, Vol. 60, No.2 (1971), pp. 217-244. Jenks and Caspall's solution has the advantage of minimizing the information loss or discrepancy between the numerical values displayed by the map classes and the corresponding numerical values given by the raw data.

8. Map accuracy is measured by the absolute deviation of the map values from the corresponding values of the data set. See G. Jenks and F. Caspall, op. cit.

9. Richard O. Duda and Peter E. Hart, Pattern Classification and Scene Analysis, (New York: John Wiley and Sons, Inc., 1973), p. 268.