TRIANGULAR SYSTEMS IN SURFACE REPRESENTATION

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Before the advent of computers the representation of single valued surfaces such as contour or sub-surface structure maps was a matter of hand drafting based on information provided at certain control points. These were either bore-hole data in the case of sub-surface structures or terrain information gathered either in the field or from air-photographs. In many cases there was an underlying triangular framework provided by the surveyor which was later used as a guide for the hand contouring that followed.

Automatic Triangulation

Algorithms did not at first deal with triangular systems for representation of surfaces, as a data structure relating triangle to triangle is fairly complex and initial programming efforts concentrated on the interpolation of regular rectangular grids based on random data point information. More recently, particularly in the case of topography, considerable work has been undertaken on the best representation methods for surfaces and there has been a resurgence of interest in the triangular approach (Peuker, et al. 1977). In these cases, however, it was usually assumed that the triangulation would be decided by the form of the topography or by factors exterior to the data distribution such as the method of geodetic surveying employed. Mathematical methods of triangulating a random data set were not well developed and the implementations were generally extremely slow. Most of the automatic triangulation algorithms involved optimisation procedures that reduced error defined in terms of producing the "most equilateral set of triangles". Rhynsburger in 1973 put forward a faster method for the computation of a set of thiessen polygons and Delaunay triangles covering an area. Recently Green & Sibson (1977) and later Brassel (1979) published faster algorithms. Further elaboration of the system yields a computation time approximately proportional to the number of data points.

The advantages of the Delaunay triangulation are that it is unique, produces a triangulation equivalent to that which would be manually expected, and is considerably faster than the gridding process used by many surface interpolation programs. The main advantage of the Delaunay triangulation's uniqueness is that maps created based on the triangulation will exactly fit each other where they meet. This means that two map sheets can be joined by a third without seaming problems. Figure 1 shows the triangulation of a part of a marine seismic system. The pattern of triangles that will later be used as a basis for interpolation of surface heights can be seen to agree with those that would be chosen for manual interpretation.

Contouring a Triangular Net

Contour lines could be drawn using the triangulation in Figure 1 as a data structure covering the area. If the values at the data locations are used, it is possible to form a linear interpolation across the triangular facets, giving a rather jagged appearance to the final map. In order to create smooth contours through the area some surface patch including derivative information must be used that will exactly reproduce the heights at the corners of the triangles and will also maintain continuity along the edges. This is a particularly difficult problem, especially for random irregular shaped triangles from the real world. Birkhoff and Mansfield (1974) have proposed a local triangle patch that guarantees first degree continuity within and between triangles based on the values at the corners of the triangles and the first derivatives at those points in terms of X and Y. Their triangular patch is quintic reduced by subtracting a number of rational interpolants to minimise calculation and

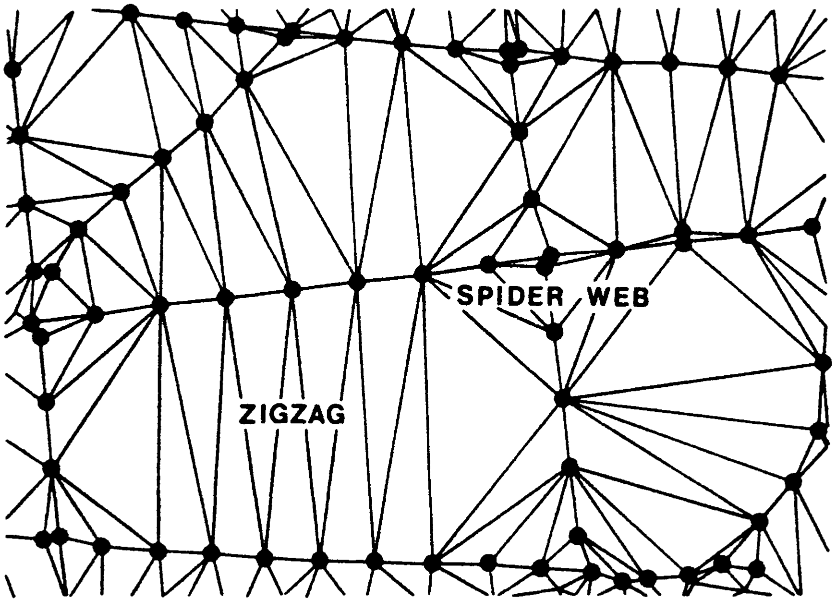

FIGURE 1: Delaunay's Triangulation of Seismic Data

ensure globally continuous first partial derivatives.

The problem is in estimation of these at the control points. A first approximation can be achieved by using the triangulation that has been calculated to provide information for each point in turn as to which surrounding points should be considered nearest relevant neighbours. By passing a least-squares fitted distance weighted plane through the control point in question it is possible to estimate the values of the first derivatives at each control point. Once all the first derivatives have been estimated any value on the surface can be interpolated using the surface patch function covering the triangle surrounding the interpolated location.

Contours can then be laced through the triangles using a linear piece-wise approximation to the continuous surface. The result of such a lacing procedure is shown in Figure 2 which is part of the contouring of the Arbuckle in Kansas based on a data set of about 300 points.

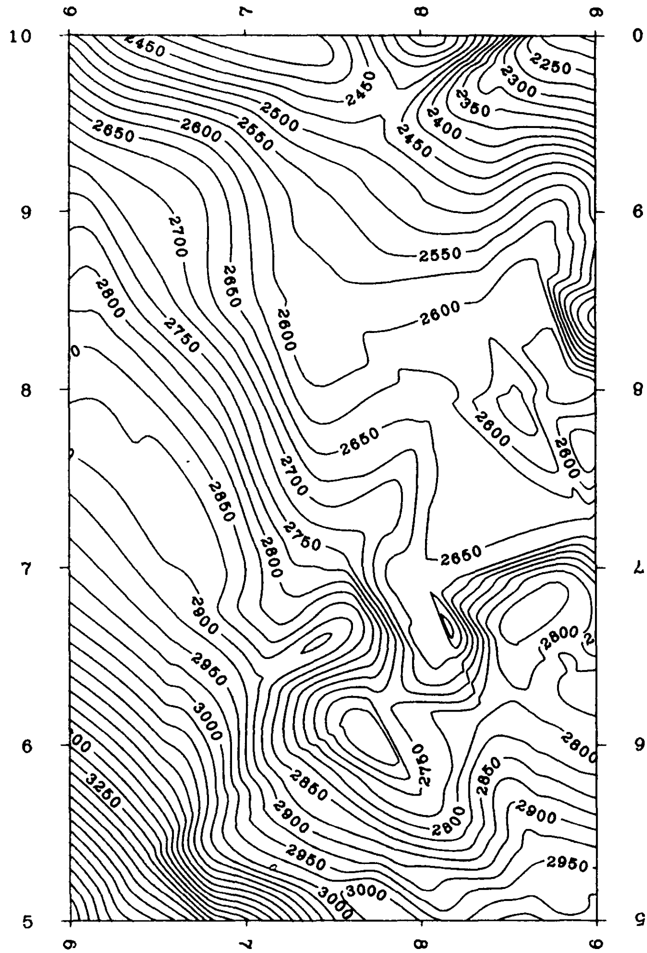

FIGURE 2: Triangular Structure Contour Map

The advantages of a triangular contouring system using continuous local surface patches are as follows. Firstly all data points exist on the interpolated surface. Secondly it is possible to maintain any derivative information which may already be available in the form of dip - meter or other measurements. Lastly in many cases data densities will vary from one place to another on the map. This indicates that information is more available in certain areas and hence structural interpretation should be more detailed in those areas. As more triangles are found in high density areas the effective resolution of the surface is increased and considerably more detail achieved than in an interpolated surface based on a regular grid.

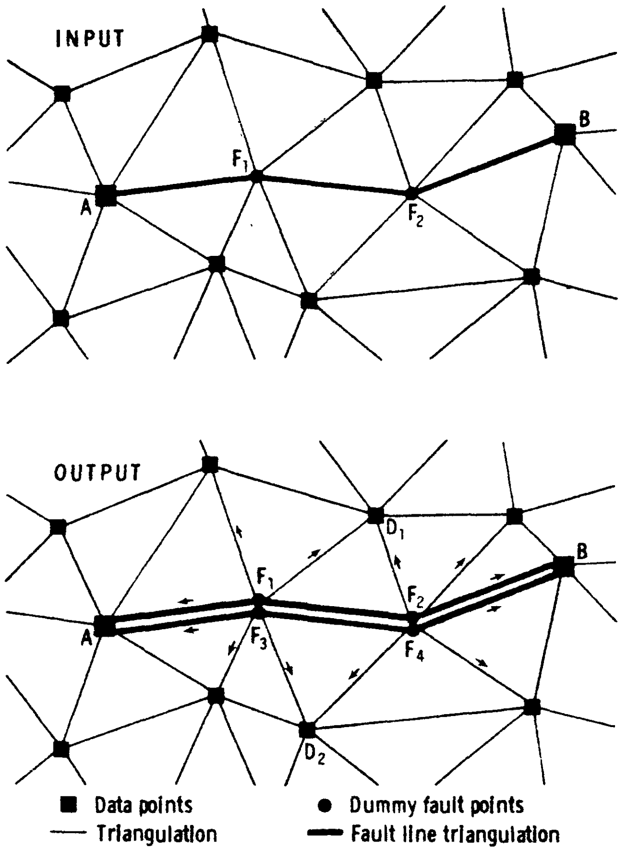

FIGURE 3: Unzipping a Fault in the Triangulation

Faulting

One of the major advantages of using a triangulation system for geologic work is that it is possible to represent accurately the line of a fault in the contouring system, enabling completely automatic contouring of faulted areas. Inspection of Figure 2 will show the location of a possible faulted structure at the coordinates X = 8 and Y = 6.5. When the geologist has decided the location of possible faults, he can insert the locational information into the data file together with the control points and re-triangulate the area producing a new Delaunay triangulation that includes not only the data points but the fault points as well. Figure 3 shows the resulting triangulation for a small fault made up of points F1 and F2. The problem of inserting a fault in a contour map is effectively that of introducing a barrier for interpolation between points on opposite sides of the fault, and allowing interpolation around the ends of the fault. By using special techniques the triangulation can be unzipped along the fault line creating automatically a barrier in the triangulation where the fault line exists. When the program estimates the first derivatives of the surface at the data points the neighbours contributing to the calculation will only be those that are on the correct side of the fault. This is shown in the bottom half of Figure 3. Values along the fault line itself are interpolated at both sides of the fault based on the information present on their respective sides. Thus the surface representation in the final map can be made to have an absolute discontinuity on the fault with interpolation and extrapolation up to the line of the fault itself. At the ends, as at points A and B in Figure 3, the fault will automatically close, producing a continuous surface.

Figure 4 shows the faulting operation in practice where a complex fault pattern has been superimposed on a geologic structure. Situations where the faults extend out of the area and stop inside the area are both handled automatically.

Conclusions

Exploration work is intimately involved with the creation of structural maps and maps of distributions over geographic space. In many cases a triangular

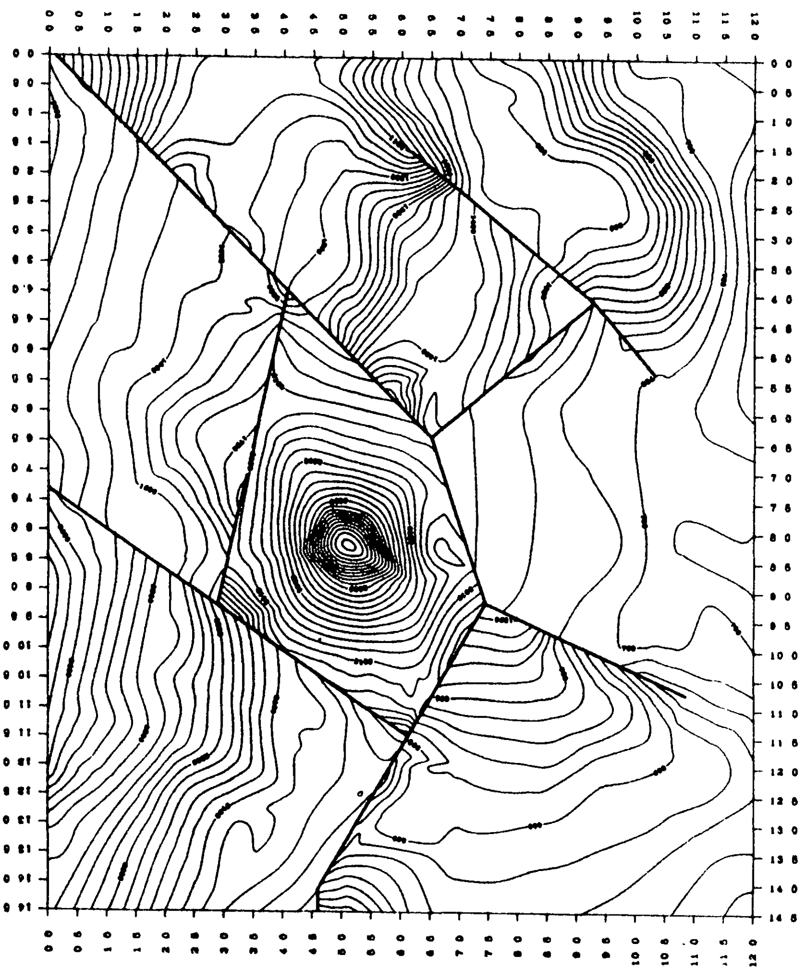

FIGURE 4: Fault Map

system can be a more compact and accurate representation of a reality provided by the data points. In certain cases such as faulting it allows significantly greater flexibility than a conventional grid representation of the surface.

It should also be realised that the increase in precision achieved by the triangular representation is not counter-balanced by a drastic increase in computer time. In general terms the process of creating a grid and the then contouring it by using conventional techniques will be longer for a given product than that required using a triangular system approach.

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