ABSTRACT

Mathematical optimization techniques have long been used to develop schedules for timber harvesting activity. These techniques provide a rational basis for decisions concerning what to cut and when to cut it. They seldom deal, however, with the issue of where. This paper describes an attempt to combine mathematical optimization techniques with the spatial allocation capabilities of a geographic information system in order to develop an on-the-ground harvesting plan. Methods are described in cartographic rather than mathematical terms and examples are drawn from a case study involving forest land located in central Maine.

INTRODUCTION

Shown in Figure 1 is a topographic map of an area in central Maine. The area includes approximately 22,000 acres of essentially undeveloped forest, lakes, and mountainous New England topography. In Figure 2 is a cartographic overlay of this area generated as part of a timber harvesting plan. The overlay identifies acreage to be harvested in the third decade of a 10-decade harvesting schedule. It also shows the pattern of logging roads to be constructed by that point in time.

The harvesting plan (Tomlin 1981) from which Figure 2 is taken is one which attempts to maximize selected economic returns associated with harvesting over time. To do so, it must account for changes in the value of timber over long periods due to economic as well as biological growth. It must also account for the costs associated with access to the timber involved.

The first of these problems is one of temporal dimensions. It is essentially a matter of optimal scheduling. Here, this is done by expressing changes in the nature and value of the forest resource in the form of an IRAM (Gould 1977) model. Linear programming (Dantzig 1963) is then used to derive optimal harvesting schedules. Results for the first three decades of the proposed plan call for the harvesting of 1284 acres of mixed-age hardwoods, 366 acres of older hardwoods, 165 acres of mixed-age softwoods, 131 acres of...
Figure 1. Portion of the USGS topographic map of an area in central Maine used to demonstrate a computer-assisted technique for allocating timber harvesting activity. Cartographic overlays of the area are shown in Figures 2 through 6.

Younger softwoods, and 635 acres of younger mixed growth in decade one; 1556 acres of mixed-age hardwoods, 95 acres of older hardwoods, 296 acres of mixed-age softwoods, and 635 acres of mixed-age mixed growth in decade two; and 1561 acres of mixed-age hardwoods, 90 acres of younger hardwoods, 296 acres of mixed-age softwoods, and 635 acres of younger mixed growth in decade three.

The problem of efficiently accessing timber is one of spatial as well as temporal dimensions. It is a matter of determining when to cut where and how to get there. In the present context, this must be done in a manner which accommodates the scheduling requirements outlined above while minimizing transportation expenses. This paper describes an approach to the problem which utilizes digital cartographic modeling techniques (Tomlin in preparation).
Figure 2. Cartographic overlay showing areas to be harvested and logging roads to be constructed as part of a timber harvesting plan. Lighter tones indicate roads. Darker tones indicate areas to be harvested in one particular decade.

METHODOLOGY

The data processing capabilities used for this purpose are those of the Map Analysis Package (Tomlin 1980), a geographic information system descending from IMGRID (Sinton 1976), GRID (Sinton and Steinitz 1969), and SYMAP (Fisher 1963). Like its predecessors, the Map Analysis Package is an overlay mapping system employing a grid cell data structure. Most significantly, it features an ability to flexibly transform and combine map overlays through sequences of algebra-like operations (Tomlin and Tomlin 1981).

To spatially allocate timber harvesting activity, these operations are used first to estimate the cost of providing access to each grid cell in the study area, then to prioritize harvestable areas on the basis of those access costs, and finally to locate the routes of minimum-cost access to those areas.
Figure 3. Cartographic overlay showing areas of varying suitability for the construction of logging roads. Lighter tones indicate areas where characteristics such as steepness or wetness result in higher incremental transport costs.

Estimating Access Cost
To estimate the cost of providing logging road access throughout the study area, each grid cell must first be characterized in terms of the incremental cost of constructing a segment of roadway at that particular location. Given a point of access to the study area, these incremental costs are then accumulated as distance from that point increases. Finally, the cumulative transport costs which result are distributed over the entire area ultimately served.

Incremental transport cost. One estimate of logging road construction costs in the central Maine study area ranges from $20,000 to $70,000 per mile depending on site conditions. In general, costs of road construction are greater in areas that are steep, wet, developed, not easily acquirable, and/or subject to legal restriction. An overlay summarizing these conditions in the study area is presented in Figure 3.
To create the overlay shown in Figure 3, overlays respectively describing topographic slopes, water bodies, roads and trails, structures, ownerships, and regulation districts are first transformed into new overlays by assigning cost coefficients to geographic characteristics. In transforming the water bodies overlay, for example, coefficients are assigned to indicate that open water is more costly than wetlands or streams which, in turn, are more costly than dry land. The resulting overlays are then mathematically superimposed to characterize each cell in terms of a single value - Cumulative transport cost. Next, one or more points of access to the study area must be identified. For this exercise, it is assumed that the study area is to be accessed from a single point located at its upper left corner. The cost of accessing any other point within the study area can then be expressed in terms of proximity to this upper left corner. Here, however, proximity is measured not in terms of miles as the crow flies but in terms of dollars as the logging road extends. The dollars involved are those of the incremental transport cost overlay presented in Figure 3.

One way to measure proximity in terms of this travel-cost metric (Warntz 1965) involves treating distance as an expression of the motion of waves that are subject to refraction and diffraction as they pass through media of varying density. In this case, the media involved correspond to areas defined by incremental transport costs. The effect can be approximated as outlined below.

**STEP 1.** ASSIGN A "PROXIMITY VALUE" OF 0 TO THE GRID CELL ASSOCIATED WITH THE POINT OF ACCESS AND A VALUE OF 99999... TO ALL OTHER CELLS.

**STEP 2.** IDENTIFY THE GRID CELL ASSOCIATED WITH THE POINT OF ACCESS AS "ACTIVE."

**STEP 3.** COMPUTE, FOR EACH CELL ADJACENT TO THE ACTIVE CELL, A VALUE EQUAL TO THE SUM OF THE INCREMENTAL TRANSPORT COST OF THAT NEIGHBOR (TIMES 1,4142 IF THE NEIGHBOR IS DIAGONALLY ADJACENT) AND THE PROXIMITY VALUE OF THE ACTIVE CELL. IF THE RESULTING VALUE IS LESS THAN THE CURRENT PROXIMITY VALUE OF THE NEIGHBORING CELL, REPLACE THE HIGHER VALUE WITH THE LOWER.

**STEP 4.** ONCE EVERY CELL HAS BEEN ACTIVE, STOP.

**STEP 5.** OF THOSE CELLS THAT HAVE NOT YET BEEN ACTIVE, IDENTIFY THE ONE (OR ONE OF THOSE) WITH THE LOWEST CURRENT PROXIMITY VALUE AS THE NEXT CELL TO BE ACTIVE.

**STEP 6.** RETURN TO STEP 3.

This technique (Tomlin 1977) is comparable to those associated with the generation of minimum spanning trees in graphs. Its effect can be seen in the overlay of cumulative transport costs presented in Figure 4.
Figure 4. Cartographic overlay showing areas of varying travel-cost proximity to the upper left corner of the study area. Lighter tones indicate areas of higher cumulative transport costs.

Cumulative transport cost per area served. The cumulative transport cost associated with each grid cell of the overlay shown in Figure 4 indicates the lowest possible cost of constructing a logging road from the upper left corner of the study area to that particular cell. Note that if two or more cells are to be harvested, however, the total cost of providing access to those cells as a group may be considerably less than the sum of their individual costs. This will be true whenever two or more cells can be accessed using a common segment of roadway. In this case, the cost of constructing the segment involved can, in effect be shared by all cells ultimately served.

By applying this reasoning to all harvestable cells within the study area, each cell can be characterized in terms of its access cost as part of the overall road construction package. To do so, each cell must first be characterized
according to the total acreage that would be served by a segment of logging road through that cell. The incremental transport cost associated with each cell is then divided by this total to yield an incremental cost-to-benefit ratio. Finally, the resulting cost-per-benefit values are accumulated in the manner of the travel-cost proximity measuring technique outlined above.

To estimate the acreage that would be served by a segment of roadway through each cell, a technique is used which, in effect, determines the minimum-cost path from every cell of harvestable timber to the upper left corner of the study area and then computes the number of paths ultimately passing through each intervening cell. This can be done as outlined below.

**STEP 1.** ASSIGN A "TRAFFIC COUNT" OF 0 TO ALL GRID CELLS.

**STEP 2.** ASSIGN A "TIMBER VOLUME" OF 1 TO ALL HARVESTABLE CELLS AND 0 TO ALL OTHER CELLS.

**STEP 3.** DETERMINE, FOR EACH CELL, THE ADJACENT NEIGHBOR(S) WHOSE CUMULATIVE TRANSPORT COST B MAXIMIZES THE QUANTITY \((A-B)/C\) WHERE \(A\) IS THE CUMULATIVE TRANSPORT COST OF THE CELL ITSELF AND \(C\) IS A VALUE OF EITHER 1 OR 1.4142 DEPENDING ON WHETHER OR NOT THE NEIGHBOR INVOLVED IS DIAGONALLY ADJACENT. IF THE CUMULATIVE TRANSPORT COST OVERLAY IS VIEWED AS A THREE-DIMENSIONAL SURFACE ON WHICH HIGHER VALUES ARE ASSOCIATED WITH HIGHER ELEVATIONS, THESE WILL CORRESPOND TO "STEEPEST DOWNHILL" NEIGHBORS.

**STEP 4.** ADD THE TIMBER VOLUME OF EACH CELL TO ITS TRAFFIC COUNT AND THEN TRANSFER THAT TIMBER VOLUME TO THE CELL'S STEEPEST DOWNHILL NEIGHBOR(S), DIVIDING IT AMONG THEM IF THERE ARE MORE THAN ONE.

**STEP 5.** WHEN THE TRAFFIC COUNT ASSOCIATED WITH POINT OF ACCESS TO THE STUDY AREA IS EQUAL TO THE TOTAL NUMBER OF HARVESTABLE CELLS, STOP.

**STEP 6.** RETURN TO STEP 4.

The process, in effect, drains timber over the three-dimensional surface of cumulative transport cost values. This is done in recognition of the fact that each cell's steepest downhill neighbor on that surface is also next on the path of minimum-cost from that cell to the upper left corner.

By dividing the incremental transport costs of Figure 3 by the acreage values of Figure 5 and accumulating the resulting cost-to-benefit ratios (in the same way Figure 3 values were accumulated earlier to create the overlay in Figure 4), travel-cost proximity can now be measured in terms of cumulative transport costs per area served. This effect can be seen in Figure 6.
Prioritizing Harvestable Areas

Given the overlay of harvesting costs presented in Figure 6, the task of prioritizing harvestable areas reduces to one of sorting the stands of each forest type by this measure of travel-cost proximity. As a result, the areas to be harvested first tend to be those which are not only nearby but which also pave the way (literally) for the greatest amount of harvesting opportunity in subsequent decades. This is what accounts for the pattern of harvesting illustrated in Figure 2.

Locating Access Routes

Once the areas to be harvested in a particular decade are located, access routes to those areas can be established by following the veins of higher traffic on the overlay shown in Figure 5. The resulting pattern of logging roads is one which attempts to minimize hauling distances as well as the ultimate cost of the harvesting transportation network.
Figure 6. Cartographic overlay showing areas of varying travel-cost proximity to the upper left corner of the study area. Lighter tones indicate areas of higher cumulative transport costs distributed over areas served.

CONCLUSION

By applying this allocation technique over several decades, a reasonable pattern of timber harvesting units and access routes can be identified. In doing so, however, it becomes apparent that the spatial distribution of forest types in a given study area may well preclude strict adherence to a harvesting schedule established through non-spatial optimization. If 99 of a prescribed 100 acres to be harvested, for example, are located within a half-mile radius while the 100th acre of the forest type called for is five miles away, it may be wise to reconsider. On the other hand, if access costs can be estimated, they can often be incorporated into the scheduling process as linear programming constraints.

This paper has described a digital cartographic modeling technique for the spatial allocation of timber harvesting activity. The technique described, albeit simplistic,
suggests that automated procedures can effectively allocate over space as well as time. It also suggests that, by emphasizing graphics, this can be done in a way which anticipates the heuristic nature of harvest planning in general.

ACKNOWLEDGMENTS

Much of the work described in this paper was carried out under the direction of Dr. Ernest M. Gould, Jr. at the Harvard Forest. Presentation slides were prepared by Glenn Hansen and Doug Johnston at the Harvard Graduate School of Design using IMAGO MINDI, a color relief-shading program developed by Denis White at the Harvard Laboratory for Computer Graphics and Spatial Analysis.

REFERENCES

Dantzig, G.B. 1963, Linear Programming and Extensions, Princeton University Press, Princeton, New Jersey


Sinton, D.F. 1976, IMGRID, Department of Landscape Architecture, Harvard Graduate School of Design

Tomlin, C.D. 1977, Hierarchical Paths of Minimum Resistance, Yale School of Forestry and Environmental Studies (unpublished manuscript)

Tomlin, C.D. 1980, The Map Analysis Package, Yale School of Forestry and Environmental Studies


Tomlin, C.D. in preparation, Digital Cartographic Modeling Techniques in Environmental Planning, Yale School of Forestry and Environmental Studies (doctoral dissertation)

Tomlin, S.M. 1981, Timber Harvest Scheduling and Spatial Allocation, Harvard Graduate School of Arts and Sciences, (unpublished master's thesis)

Warntz, W. 1965, A Note on Surfaces and Paths and Applications to Geographical Problems: MICMOG Discussion Paper No. 6, University of Michigan