GRIDS OF ELEVATIONS AND TOPOGRAPHIC MAPS

by

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Abstract

Computer assisted topographic mapping is usually based on a DTM in the form of a regular grid. Grids of elevations can be formed in different ways, which may lead to variations in the map being produced. It is therefore important to select an approach to the grid arrangement which provides an adequate terrain representation.

Selecting the desired method of the grid formation is related to several problems: orientation of the grid, density of the grid, combining the grid with separately measured break-lines and modes of acquiring the topographic data.

The problems mentioned above are considered in the paper. Examples produced on the basis of various grids illustrate the discussion.

General Remarks

The topographic data base, known as DTM or DEM, is an issue under discussion for nearly 20 years. Interpolation techniques, arrangement modes and data acquisition procedures have been discussed in journals and symposia. The reasons for the ongoing occupation with the topic are firstly its practical value, secondly its nature - everything stated on the subject lies in the category of an opinion, and whenever opinions contradict one another one cannot logically prove which is right or wrong.

The authors believe that adding their own opinion on the matter may elucidate some aspects which may have escaped the attention of others.

Everybody may agree that a topographic data base aims to represent the terrain numerically. This may sound plausible, nevertheless a clarification of the statement is in order, for it is not self evident what a numerical terrain representation means.

Obviously, everyone would like to have a topographic data base which provides the best representation of the terrain. But that is only an idea, because such a data base should consist of an infinite number of points located with an absolute accuracy. From a practical point of view, when terrain is described by a finite number of points, it has been claimed that the best terrain representation is achieved if all the salient or characteristic points are located during the terrain survey. However, such a statement is not satisfactory because of the vague definition of the term "characteristic point". It is usually said, that wherever a change in the slope of terrain takes place there is a characteristic point. Two objections may be raised here; it is not known what change in slope should occur in order to regard the respective point as being "characteristic", changes in slopes usually do not occur along distinctly and uniquely defined lines (except for man made features), but rather within certain regions. Selecting characteristic points is therefore a result of a judgement, and since there is no basis to expect that every person will have the same point of view regarding positions of characteristic points, each one may locate them differently.

Considering what has been said above, it is seen that topographic data sets, all comprising an equal number of picked up points and including all which is considered as being characteristic points, inevitably differ from each other on the one hand, and must be regarded topographically equivalent on the other. Referring to what has been maintained previously, each such data set ought to be called a best terrain description, which is unsound, for the best must be unique. Hence, even from a practical point of view there is no such thing as a best terrain representation, and we have to conclude that we can speak only about an adequate terrain description. We may say that a topographic data base provides an adequate terrain representation if it meets the accuracy requirements imposed by the tasks for which the data have been collected.

If that statement is accepted, it becomes irrelevant which techniques and procedures are employed as long as the resulting DTM or DEM describes the terrain adequately. The preference of one method over another now becomes a matter of convenience concerning collecting, processing, storing, updating, retrieving data and so on. In such a case it is indisputable that a DTM in the form of a rectangular grid is the one to be preferred.

Topographic data differ in several respects from other geodetic data.

A geodetic point (triangulation point, traverse point, bench mark) is defined only quantitatively by coordinates and elevation. A topographic point frequently carries also qualitative information (e.g. when the point is one of a string describing a topographic feature), and if qualitative information is considered, it is apparent that no mathematical manipulation can provide it adequately.

A geodetic point represents only itself, besides, it is always a subject of direct application (for example, a line of levels is tied to bench marks, or a trig point is the starting point of a traverse, so we are interested directly in the coordinates and elevations of those points). A topographic point, on the contrary, represents not only its location, but also a terrain portion in its immediate surroundings, besides, the potential user of a DTM is rarely interested in a grid corner as such, he is more concerned with points or lines (contours, profiles) derived from the grid points and located in between them.

An additional question concerning the regular grid of the DTM is the density of the data and the method of its acquisition.

Topographic data are usually collected from photogrammetric models in two ways - by scanning the model in a grid mode, or in a profile mode.

Scanning in a grid mode has its advantages since it saves computational effort. It has also been stated that it provides more accurate results. However, that method has its disadvantages too, which follow from the enforcement to carry out height measurements at predefined locations (grid corners). On many occasions, such a location may not be appropriate for a height determination, either because of a local irregularity (e.g. a top of a tree) which distorts the representation of the terrain portion around the grid corner, or because of unsuitable measuring conditions (shadow for example). Moreover, it usually leads to to excessive measuring operations by imposing height determinations at points which do not contribute to the terrain description. Scanning the model in profiles eliminates the above disadvantages, it enables to locate points which are most relevant to the shape of the terrain and it usually reduces also the volume of the acquired data.

The numerous variations in the terrain forms do not lend themselves to rigid rules, hence the question of how dense a grid should be remains open. From our experience we may cite that a grid with an interval between the grid lines of an order of magnitude of 4-10 mm. at the scale of the final map is adequate, the smaller intervals recommended for small scale maps and the larger intervals for large scale maps.

The above observations give rise to several conclusions.

It is essential to locate all characteristic terrain features, as interpreted by the operator while scanning the model. Since there is no law according to which a feature line has to assume a certain shape, any attempt to produce reliably the shape of the line from data not related directly to it, is doomed to fail, even when the data are distributed densely. Consequently there is no reason to form very dense grids.

It is advisable to scan the photogrammetric model in a profile mode. When forming a regular grid from the profiles, the feature lines - watercourses, watershed lines, brinks etc. - should be regarded as boundary conditions (the same applies to the stage of contouring, if a map is being prepared).

Only when a DTM is composed of a grid combined with data describing features, can an adequate terrain representation be assured.

The quality of a DTM should not be judged solely by the accuracy of the elevations of the grid corners. An essential question is how good does a grid corner represent the terrain portion in its vicinity, hence judging the DTM should be based rather on examining the validity of the derived information (contours, sections etc.).

Results of Tests

Below are presented samples of tests which have been carried out in accordance with the above considerations. The samples relate to a model formed from small scale photographs 1:50000 approximately, taken with an overlap of 70% over an area with varying topography, from which a map at a scale 1:25000 with a contour interval of 10 meters was produced. The tests were performed in a Traster Analytical Plotter (Matra) by an operator with limited experience. The ability of the operator to measure elevations was derived from determining repeatedly heights at 105 points distributed at various locations in the model. These determinations were reiterated four times. The m.s.e. of a single elevation determination derived from the above measurements was 0.7 meters. Histogram nr. 1 shows the distribution of the residuals v_h.

The tests aimed to compare data derived from grids with data acquired directly in the model. The quality of a map can hardly be superior to the quality of the model itself, hence the comparative nature of the tests serves our purpose.

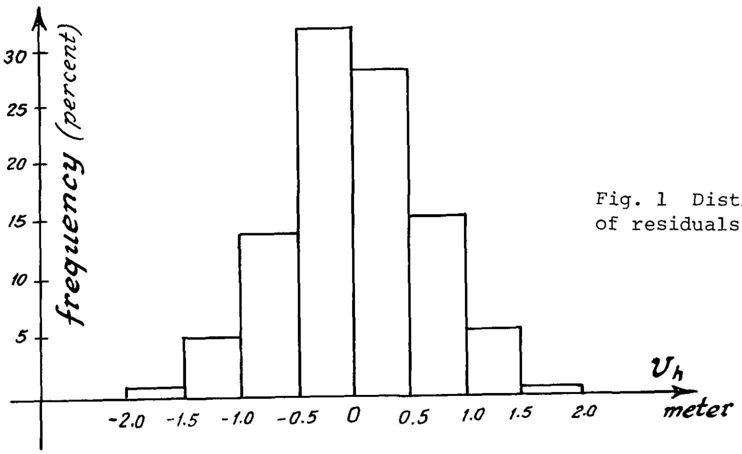


Fig. 1 Distribution of residuals

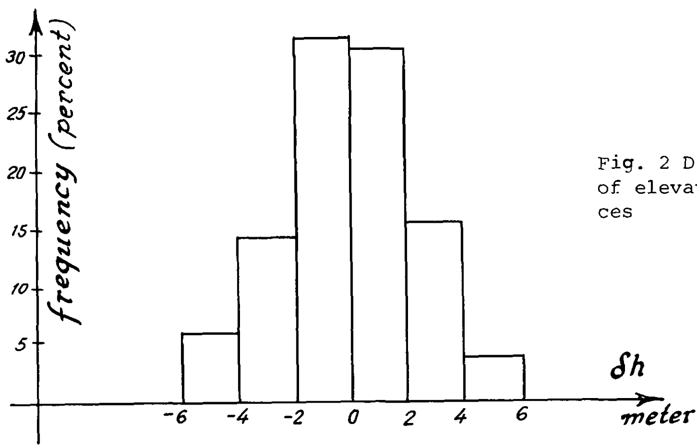


Fig. 2 Distribution of elevation differences

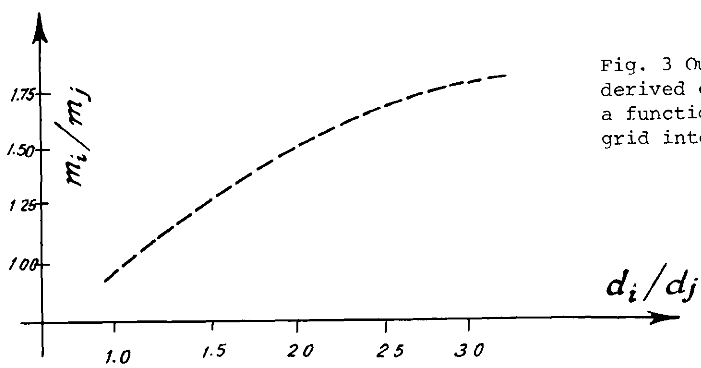


Fig. 3 Quality of derived elevations as a function of the grid interval

In our opinion it is preferable to scan the model in a profile mode. It is usually convenient to scan profiles which are parallel to one of the axes of the instrument (especially in conventional stereoplotters). But a grid of elevations may be oriented arbitrarily with regard to the profiles. It is of interest therefore, to examine whether there is any relation between the orientation of the grid and its quality.

The model was scanned in profiles with an interval of 100 meters between them (4 mm at the scale of the map). Besides, all characteristic features and salient points have been located in the model. Combining the two types of data the following grids have been formed (all with a grid interval of 100 meters):

a: A grid with one family of lines coinciding with the profile lines.

b. A grid parallel to the profiles and shifted with regard to them.

c. A grid rotated by 15° with respect to the direction of the profiles.

d. A grid rotated by 30°.

e. A grid rotated by 45°.

f. A grid rotated by 60°.

From each one of the grids, elevations of about 100 points located variously in the grid squares have been derived by bilinear interpolation. When computing the elevations, all relevant information on the characteristic lines and salient points has been taken into account.

The elevations of the above points have been measured directly in the model. The differences δh between the derived and measured elevations expressed in terms of m.s.e. and extreme values, are summarized in the table below (table 1).

Grid	m.s.e.	min.	max.
	m	m	m
a	2.3	-4.9	4.1
b	2.3	-4.7	5.0
c	2.3	-4.6	4.0
d	2.6	-4.6	4.7
e	2.3	-5.0	4.8
f	2.4	-4.6	4.9

We may also note that in all cases 90% of the differences were smaller than half a contour interval, and none of them exceeded an entire contour interval. It is seen, that the orientation of the grid has no bearing on the quality of the derived information, provided the grids are combined with topographic features.

Histogram nr. 2 represents the distribution of the differences δh.

The model has also been scanned in a grid mode and two grids have been picked up - a grid with an interval of 100 meters, corresponding to the computed grids, and a grid with an interval of 65 meters.

Two cases have been considered here. Elevations of the above mentioned group of points were derived from the "normal" grid (100 meters interval) while combining the heights of the grid corners with the topographic features. Secondly, elevations of those points were computed from the dense grid without taking into account the features. Comparing the results of the above determinations with the elevations measured directly in the model yields:

Normal grid with features -

m.s.e. = 2.2 m , min. = -4.9 m, max. = 4.3 m

Dense grid without features -

m.s.e. = 2.3 m , min. = -4.9 m, max. = 4.9 m

It is also worthwhile to compare the volumes of the acquired data. Assuming that the number of picked up points along the profiles equals 1, the following results are obtained:

Ratio between the profile points and the feature points - 1:1.

Ratio between the grid corners (normal grid) and the feature points 1.2 : 1.

Ratio between the grid corners (dense grid) and the sum of profile points and feature points - 1.4 : 1.

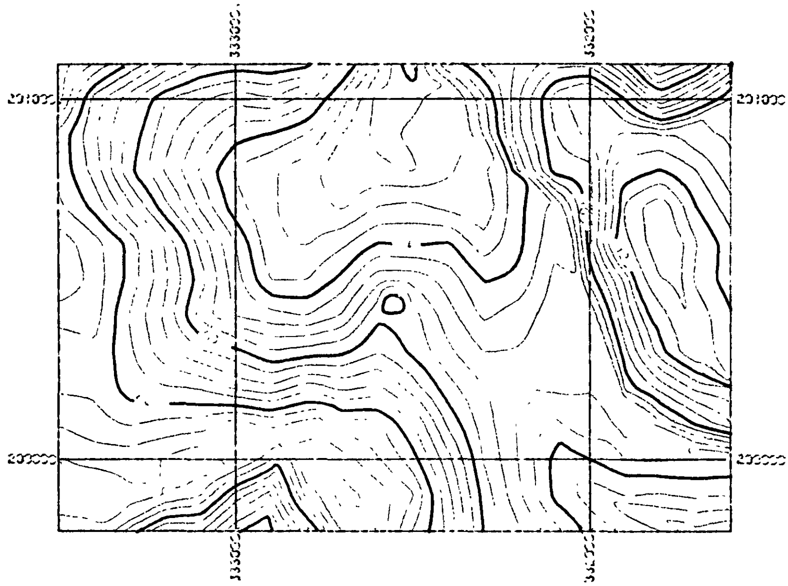
It is seen that there is no significant difference in the quality of the terrain representation as expressed by the various grids, if terrain representation is judged by the quality of elevations derived at discrete points. However, there is a significant difference in the numbers of the measured points. Sparse grids have been formed on the basis of the available data in order to check the effect of extending the grid interval on the quality of the derived elevations. The diagram on figure 3 illustrates the results. On the horizontal axis are plotted the quantities d_i/d_j - the ratios between a grid interval d_i and the intervals d_j. On the vertical axis are given the respective values m_i/m_j - the ratios between the mean square errors of an elevation determination in a grid with an interval d_i , and in a grid with an interval d_j.

The diagram shows a prominent deterioration in the quality of the grid when the grid interval is increasing.

To get exhaustive information on the validity of the terrain models, the tests presented above have been complemented by maps and terrain sections, prepared from a photograph at a scale 1:30000 with a contour interval of 5 meters. Two samples of maps are given in figure 4. The map labelled 1 has been prepared from a grid produced from profiles in combination with topographic features. The map labelled 2 has been generated from a dense grid picked up directly in the model and without taking into account the topographic features.

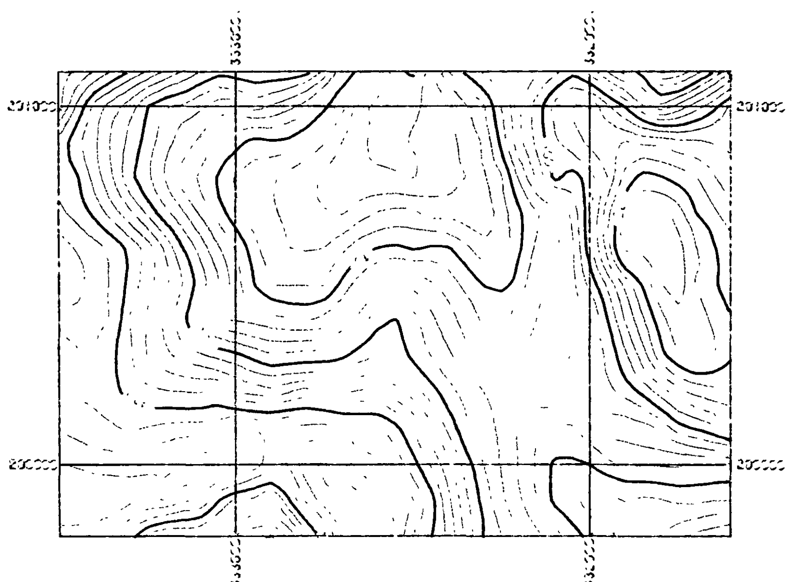
Map nr. 1 shows sharp changes in slope which occur in regions containing topographic features. That effect is eliminated somewhat on map 2,

Figure 4. Samples of maps



Map 1

Map 2



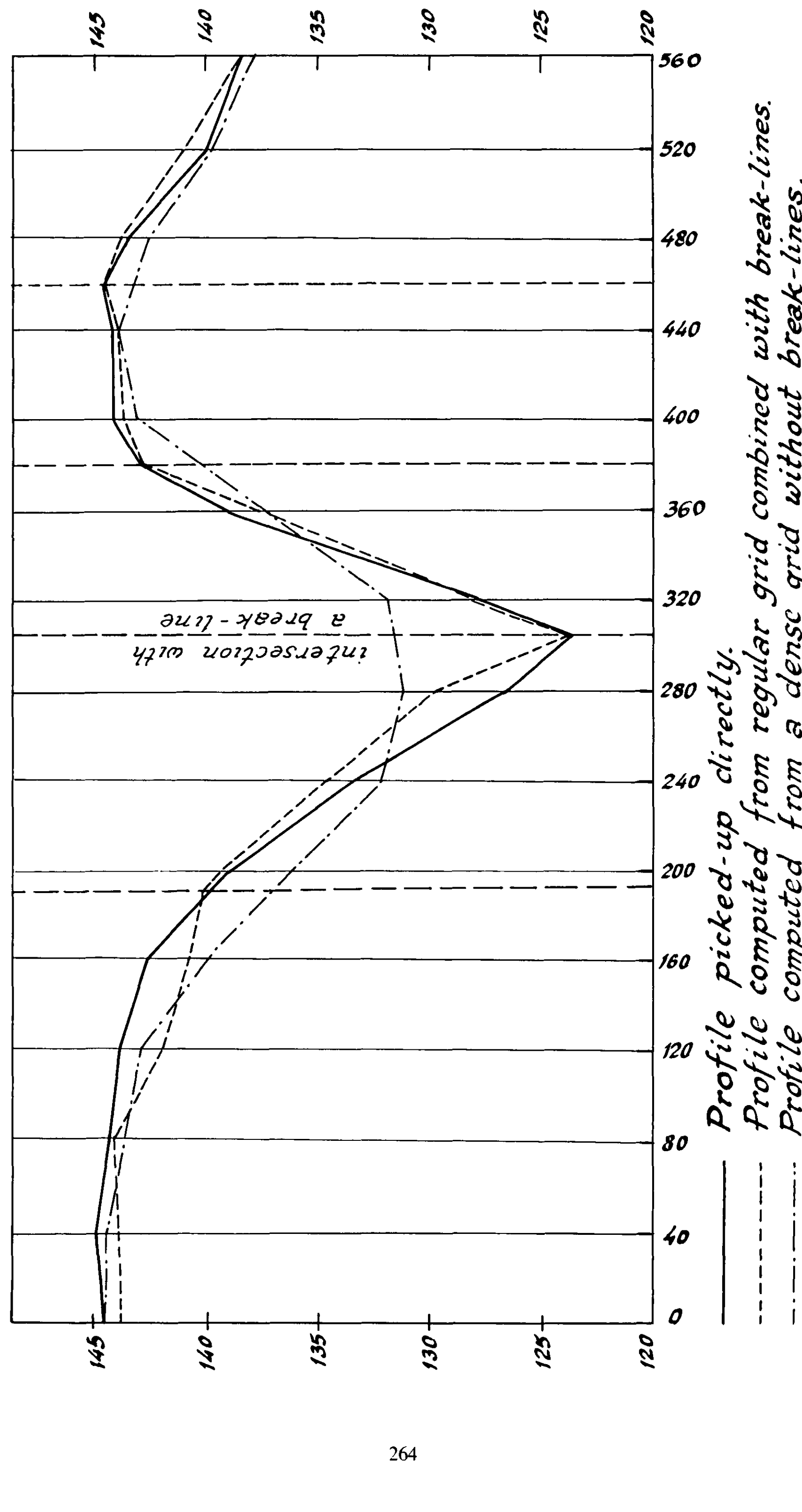

145
140
135
130
125
120
0
40
80
120
160
200
240
280
320
360
400
440
480
520
560
intersection with
a break-line
Profile picked-up directly.
Profile computed from regular grid combined with break-lines.
Profile computed from a dense grid without break-lines.

Figure 5

since there were no boundary conditions imposed on the contour generation. Besides, the contours on map 2 are more noisy, which has to be attributed to the shorter distances between the grid corners.

A number of terrain sections were derived from the data. Each section was formed in three ways: the first was produced from the grid based on profiles and features, the second was determined from a dense grid picked up directly in the model and the third - measured directly. A sample of a section is depicted in figure 5. It shows clearly the agreement between the section derived from the profiles plus the topographic features and that measured directly, and the poor quality of the section derived from the dense grid.

Summary

A topographic data base composed of a regular grid and data describing topographic details represents the terrain adequately.

It is advisable to scan the model in a profile mode. The orientation of the grid produced from the profiles is irrelevant to terrain representation.

It is worth noting that an unskilled operator, after a short training period only, is able to acquire reliably the topographic data necessary for the data base, in contrast with the degree of skill required from an operator who produces maps by conventional procedures.