MAPS OF SHADOWS FOR SOLAR ACCESS CONSIDERATIONS

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BIOGRAPHICAL SKETCH

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ABSTRACT

A computer program for shadow mapping has been developed at the University of Wisconsin. The program is intended as a tool for site-selection and planning purposes when access to sunlight is a design criterion. The program operates on two files which have a common three-dimensional reference system: 1) a DTM file, and 2) an "improvements" file. The "improvements" file contains X,Y,Z coordinates of structures and other physical features. Given the altitude and azimuth of the sun, the program will plot the shadows which the improvements cast upon the DTM. The program is also capable of mapping shadows which the terrain casts upon itself. In order to assess the accuracy of the mapping algorithm, two tests have been made. In the first test, computer-generated shadows of structures were compared to shadows mapped from aerial photography on a stereoplotter. In the second test, positions of computer-generated shadows of the terrain itself were compared to those obtained by ground survey.

INTRODUCTION

With recent advances in technology and inevitably declining supplies of fossil fuels, the practical use of solar energy has become more and more attractive. Associated with this trend is the recognition of the value of sunlight and the need for protecting individual rights to the sun. Solar access may be protected by granting easements (Franta,1980), (McLaughton,1980), by zoning, or by including restrictive covenants in the owners' certificates of new subdivisions (Hayes,1979).

Specialized surveying instruments and photogrammetric techniques (Elgarf,1981), (Colcord,1982) have been developed for positioning solar collectors in shade-free areas and for locating individual solar easements. Planning and design of solar subdivisions have been done using templates of shadows placed upon maps (Erley and Jaffe,1979) and physical models of structures and terrain illuminated by a light source simulating the sun (Knowles,1980). The process of planning solar access was partially automated by Arumi and Dodge (1977). They developed software for generating building height limits for solar access in an urban redevelopment project.

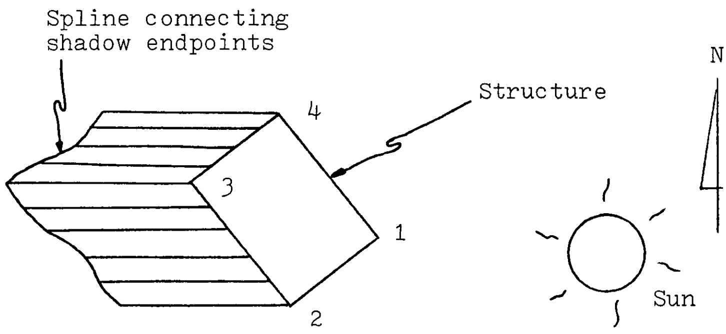
The computer program described herein is intended as a general tool in collector site-selection and in the large-scale design of subdivisions. Given three-dimensional models of terrain and structures, maps of shadows for any solar azimuth and altitude may be produced. The program should be particularly useful when a large scale topographic map, usually required for subdivision design, is already available. Such a map can supply the needed digital terrain model.

THE MAPPING ALGORITHM

Shadows of Structures

In order to map the shadows of structures, the algorithm requires a DTM and an "improvements" file containing X,Y,Z coordinates of building corners. The DTM must be on a uniform grid. The improvements file may contain 1) real data, in which case the application is solar collector site-selection, or 2) fictitious data, in which case the application is planning and design. The azimuth and altitude of the sun are required as additional input.

The corners of each structure must be numbered consecutively clockwise. As illustrated in Figure 1, any wall of a structure casts a shadow if the sun lies to the right of the plane containing the wall as the building is traversed clockwise. No shadow is cast if the sun lies to the left of that plane. Shaded areas are located by scanning along profiles in vertical planes which are separated by even increments along walls which cast shadows.



Structure with Corners Numbered Clockwise

Figure 1.

Figure 2 illustrates the manner in which the endpoint of a shadow in a given profile is found. The elevations of points A and B, on the perimeter of a DTM grid cell, are found by assuming a linear slope between the adjacent grid corners. The vertical angles, α, from A and B to the point casting shadow are computed. Let

$$\alpha_S = \text{solar altitude.}$$

If

$$\alpha_A > \alpha_S \quad \text{and} \quad \alpha_B > \alpha_S ,$$

both A and B are in shadow and scanning along the profile continues. If

$$\alpha_A > \alpha_S \quad \text{and} \quad \alpha_B < \alpha_S ,$$

the shadow ends within the grid cell and the endpoint is found by iterative interpolation. The first approximation (L) for the endpoint is found on the line AB where

$$\alpha_L = \text{solar altitude.}$$

The elevation of the DTM at that point (A') is computed by inverse weighted distance between the four surrounding grid cell corners. The second approximation (L') lies on the line A'B. The process continues until the difference in elevation between the latest approximation and the DTM becomes negligible at C. Once the shadow endpoints for a given wall are computed, they are connected by a spline curve as illustrated in Figure 1.

Figure 3 shows computer-generated shadows cast by a building upon fictitious terrain for three positions of the sun. The three solar azimuths are 135°, 180°, and 225° with respective solar altitudes of 12°, 30°, 12°. These are approximate values for 9:00 AM, 12:00 PM, and 3:00 PM on the winter solstice at a latitude of 36½° N. The winter solstice is commonly used for design because that is the time at which shadows are longest. In Figure 3 the lengthening of shadows in the terraced depression and the shortening of shadows on the terraced hillside can clearly be seen.

Shadows of Terrain

Shadows cast by the terrain itself are found in a manner similar to those cast by structures except that profiles covering the entire DTM are scanned. Profiles for morning shadows are begun along lines connecting the centers of the easternmost tier of grid cells and the centers of the southernmost tier of grid cells. Profiles for afternoon shadows are begun along lines connecting the centers of the westernmost tier of grid cells and the centers of the southernmost tier of grid cells. With reference to Figure 2, as the perimeter of a grid cell is intersected, if the vertical angle from point B to point A is greater than the altitude of the sun, then A casts a shadow on B. The endpoint of the shadow cast by A lies in another cell along the profile being scanned and is found in a manner similar to that of the endpoints of shadows cast by structures.

Figure 4 illustrates shadows cast by terrain upon itself for

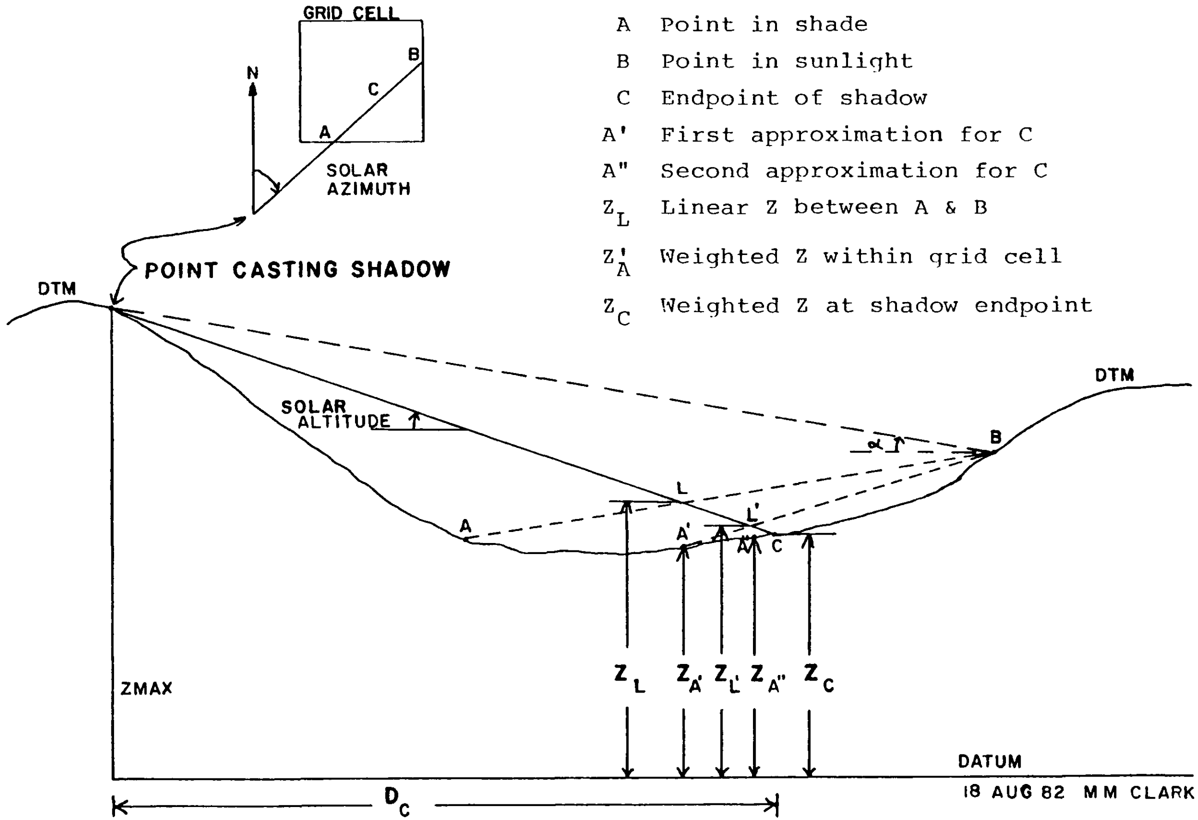
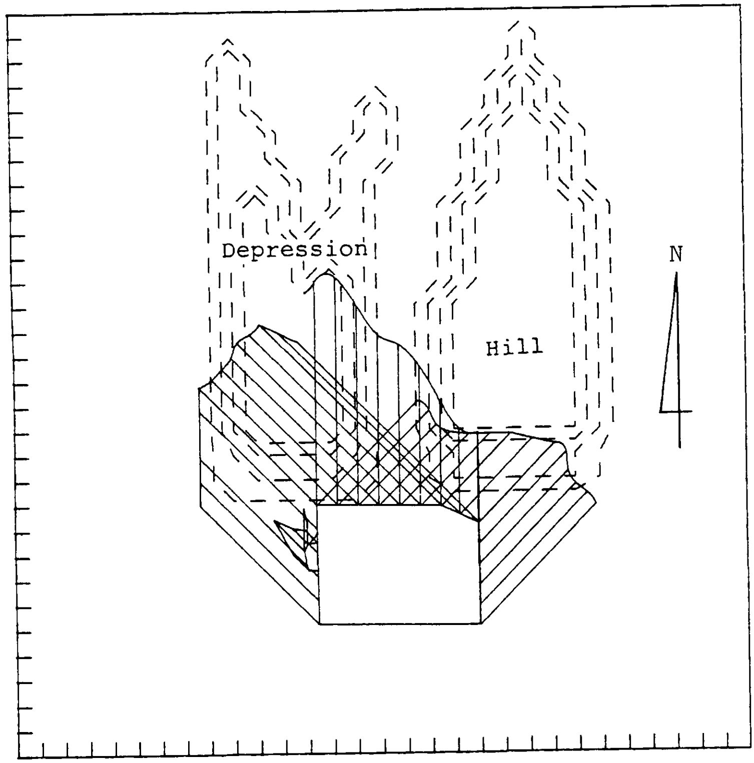


Figure 2. Interpolation for Shadow Endpoint.



Three Positions of the Sun on Building with Porch and the Effect on Varied Terrain. Contour Interval 10 ft.

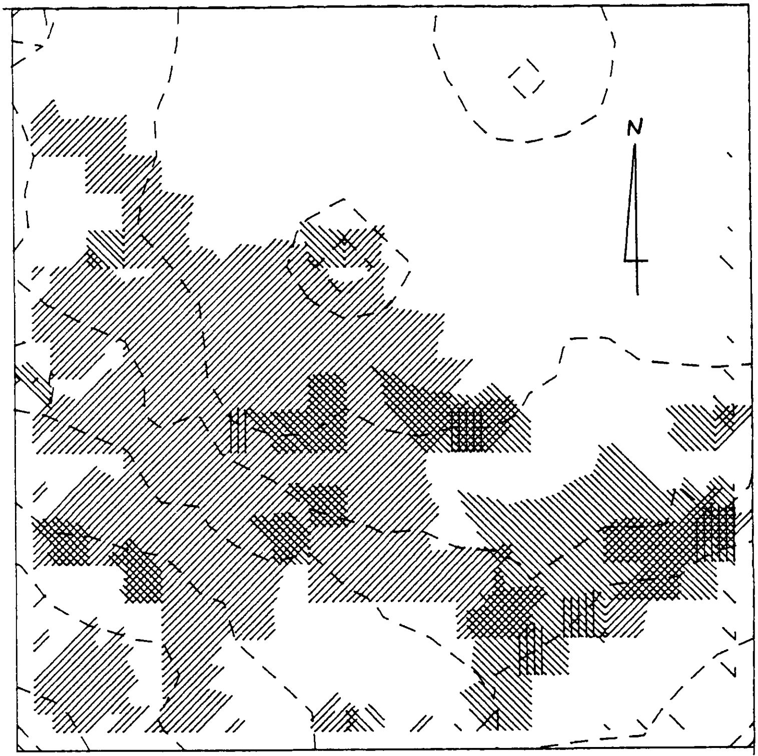
Figure 3.

the same three positions of the sun used in Figure 3. There are hills in the north-central, southeastern, and southwestern parts of the map. There is a shallow valley which runs northwest-southeast. Areas which are shaded for various parts of the day can be seen. A few small areas are in shadow throughout the day. The peak in the north-central part of the map is in sunlight all day long, but is surrounded by shadows, some of which are cast by itself, in the morning and afternoon.

COMPARISON TO MEASUREMENTS

Shadows of Structures

In order to provide a basis for comparison of computed shadows to measurement data, a map of shadows cast by structures was compiled on a Kern PG2 stereoplotter. Existing aerial photography, taken at approximately local noon on November 6, 1974, was used to form a single stereomodel at a scale of 1:720. This photography was ideal in that ground control was already available and the shadows of structures were sharply defined. A portion of the map from the stereo-



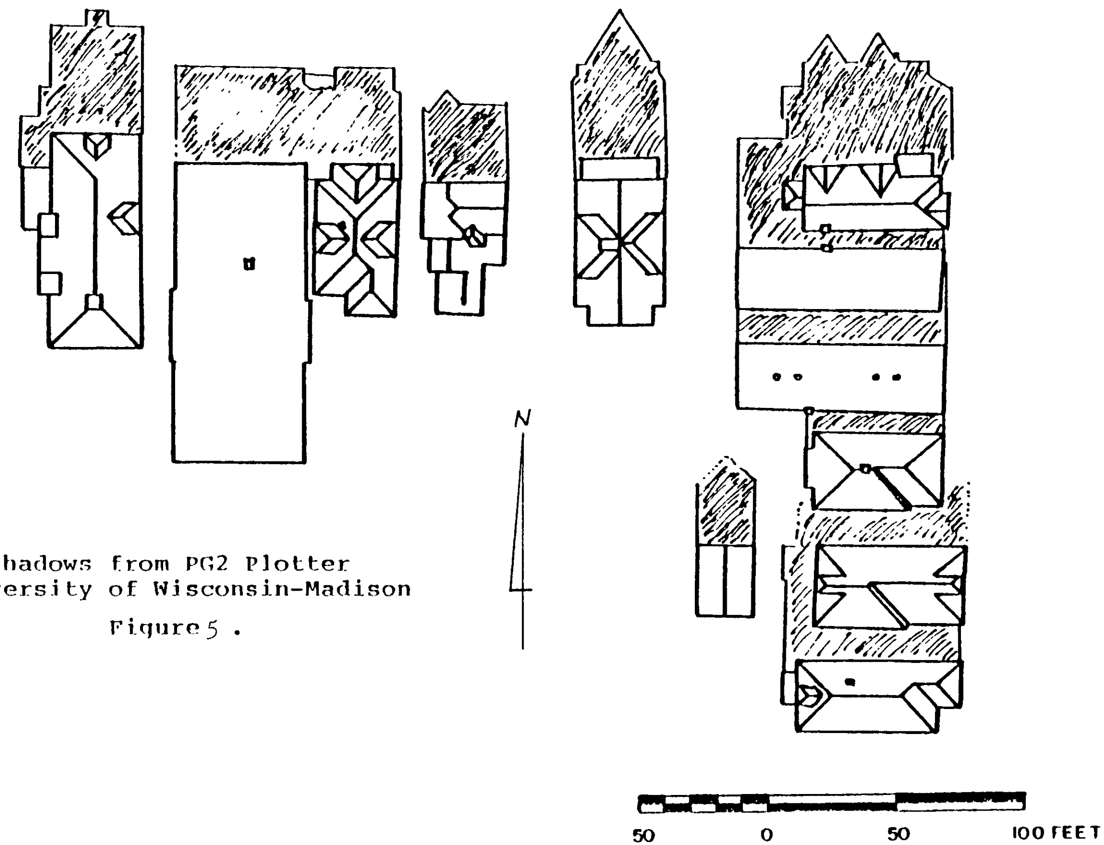
Three Positions of the Sun with the Sample Terrain
Casting Shadows on Itself
Contour Interval 10 ft.

Figure 4.

plotter appears in Figure 5.

The stereoplotter is equipped with a digitizer. As the map was being compiled, model coordinates of control points, building corners, shadow corners, and random spot elevations were recorded. The parameters of a three-dimensional, conformal coordinate transformation were computed by a least squares fit to the ground control. The measured model coordinates were then transformed into a ground-scale system.

A digital terrain model, on a uniform grid, was then produced by inverse weighted distance interpolation to the four data points nearest each grid cell corner. The shadow mapping algorithm was then employed to generate the map which appears in Figure 6. Since the exact time of photography was not known, the azimuth of the sun (0°55'53") was computed as the average direction of the shadows of a number of vertical lines in the transformed stereomodel. The altitude of the sun (31° 00'24") was then computed from the azimuth using ephemeris data.



Shadows from PG2 Plotter
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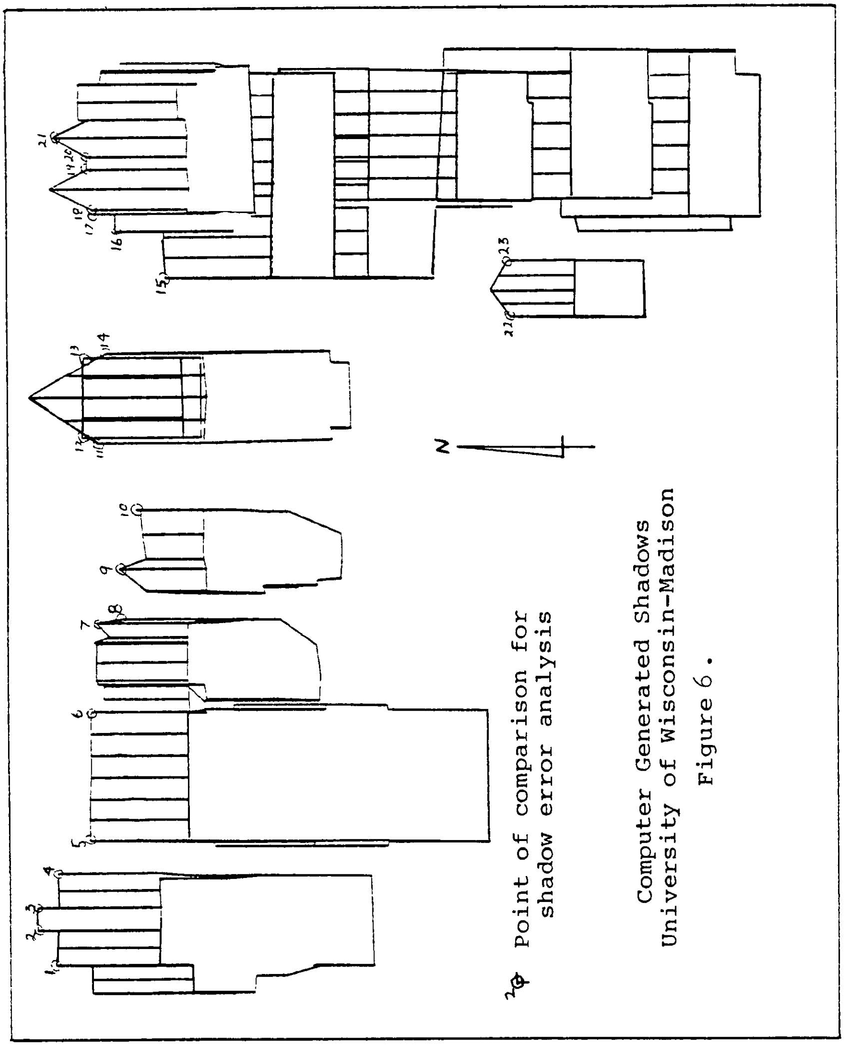
Figure 5.



Computer Generated Shadows
University of Wisconsin-Madison
Figure 6.

Figure 6 indicates twenty-three sharply defined points which were used to obtain the data in Table 1. That table contains the discrepancies, at ground scale in feet, between coordinates of shadow endpoints as measured on the stereoplotter and as computed by the mapping algorithm. Since the sun was nearly on the local meridian, it is the discrepancies in the Y coordinates which most nearly reflect the errors in the lenghts of shadow lines. These discrepancies also are indicative of the errors in the planimetric positions of the shadow endpoints on the computer-generated map. The mean discrepancy in Y indicates bias introduced by systematic errors in the DTM and error in the computed altitude of the sun. The corresponding standard deviation reflects random error in the DTM.

Table 1.

Discrepancies Between Stereoplotter Shadow Coordinates and Computer-Generated Coordinates (Units are Feet)

Point	Delta X	Delta Y	Delta Z
1	0.1	2.6	-0.2
2	0.2	1.7	0.0
3	0.2	1.4	0.1
4	-0.1	1.7	-0.5
5	0.5	-0.2	0.8
6	-0.6	-0.8	0.3
7	0.2	1.1	0.1
8	0.7	3.1	0.6
9	0.2	2.2	0.1
10	0.2	2.0	-0.3
11	0.0	-2.7	-0.2
12	-0.1	-2.1	0.0
13	0.0	-1.4	-0.1
14	0.6	0.0	0.2
15	0.0	1.0	-0.7
16	-0.4	2.3	-0.3
17	0.2	2.1	0.2
18	0.4	2.1	0.2
19	-0.4	0.4	0.0
20	0.1	0.3	0.0
21	-0.3	-0.2	0.0
22	-0.3	0.2	0.0
23	0.0	0.0	-0.2
Mean	0.0	0.7	0.0
Std. Dev.	0.3	1.5	0.3

Shadows of Terrain

Soldiers Grove, Wisconsin is a small community in the southwestern part of the State. Its residents are unfortunate enough to live and do business in the floodplain of the Kickapoo River. Just recently it was decided to escape the river by moving the entire community approximately one kilometer. The village board showed foresight by requiring that the new businesses and residences be equipped for solar energy. The new town-site is just north of the base of a very steep and high bluff. Layout of the town-site required knowledge of the position of the shadow cast by the bluff. Design was based upon the edge of the shadow cast at 2:00 PM on the day of the winter solstice in 1981 (solar azimuth = 208°22!6, solar altitude = 17°52!3). Soldiers Grove provided a fine opportunity for the testing of shadows cast by terrain.

A topographic map, at a scale of 1:600, was prepared from a stadia survey of the site. The orientation of the map was controlled by a solar azimuth which was checked by running traverse to a triangulation station approximately three kilometers distant. The contours on this map (contour interval = 10 feet) were digitized at a resolution of 0.001 inches. A digital terrain model, on a uniform grid, was generated by inverse weighted distance from the digitized contours.

A ground survey of the design shadow line was performed in July of this year. The theoretical shadow line was located by running, in the direction of the design azimuth of the sun, along offsets from a baseline. The endpoint of each offset was taken to be the place where the vertical angle to the tops of trees atop the bluff equaled the design altitude of the sun. An estimate of the heights of the mature hardwood trees (60 feet) was obtained from the Crawford County Forester. The computer-generated terrain shadow line, the ground surveyed tree shadow line, and a terrain shadow line based upon tree heights could then be plotted (see Figure 7). Due to errors in the survey, it is estimated that the uncertainty in the tree shadow line is $\pm$ 4 feet in the design azimuth of the sun. If the estimate of the tree heights is uncertain by $\pm$ 5 feet, the corresponding uncertainty in the ground surveyed terrain shadow is $\pm$ 16 feet in the azimuth of the sun.

CONCLUSIONS

A computer program for shadow mapping has been developed at the University of Wisconsin. The program is intended for use in solar collector site-selection and in subdivision design.Tests indicate that the shadows of structures can be mapped to within 2.5 feet. A comparison of shadows of terrain as surveyed and as mapped by the program has also been made.

The results reported herein should be regarded as preliminary. The stereoplotter map and the computer map of structure shadows are not totally independent because the solar altitude used on the computer map was computed from an azimuth taken from the stereoplotter. The authors hope to repeat the described test using aerial photography for which the time of exposure is known precisely. The authors also intend to return to Soldiers Grove in winter when the shadow cast by the large bluff can be mapped directly.

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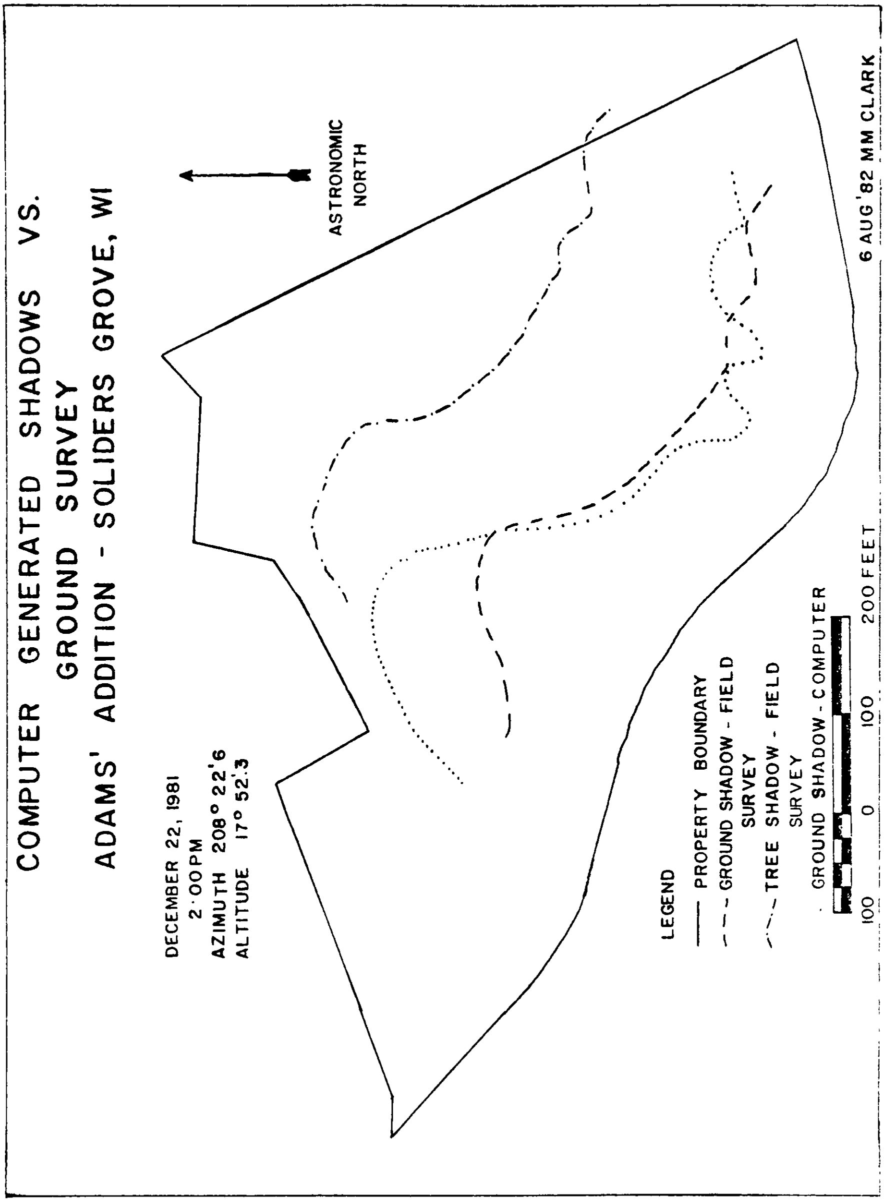
COMPUTER GENERATED SHADOWS VS.
GROUND SURVEY
ADAMS' ADDITION - SOLIDERS GROVE, WI
ASTRONOMIC
NORTH
DECEMBER 22, 1981
2:00PM
AZIMUTH 208° 22'6
ALTITUDE 17° 52'3
LEGEND
PROPERTY BOUNDARY
GROUND SHADOW - FIELD
SURVEY
TREE SHADOW - FIELD
SURVEY
GROUND SHADOW - COMPUTER
100
0
100
200 FEET
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Figure 7.

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