PROPORTIONAL PRISM MAPS: A STATISTICAL MAPPING TECHNIQUE

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ABSTRACT

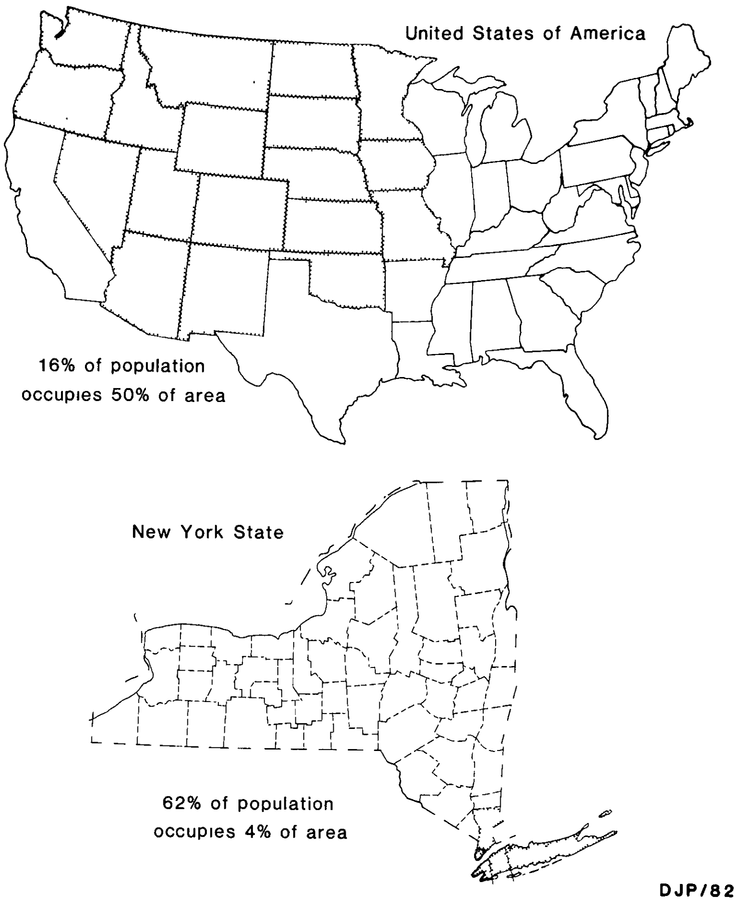
This paper explains a methodology for computer assisted construction of proportional prism maps. Proportional prism maps combine the design of non-contiguous proportional polygons with a three-dimensional prism map. The prism bases are formed by proportional polygons and represent one variable. The polygons are then assigned a height representative of a second variable, thus forming prisms. The map produced as a result of this research provides one solution to three persistent statistical mapping problems: 1) the problem of large, sparsely populated areas dominating the map space; 2) the limitation of displaying one set of data per map, thus restricting the reader's analytical powers; and 3) the problem of displaying two variables monochromatically.

INTRODUCTION

Most statistical maps rely upon standard land division shapes such as states or countries for a base since these shapes are often important and necessary recognition aids. One problem with statistical maps using standard land divisions as a base is that large areas displaying low values carry disproportionately heavier weight on visual perception than smaller, often denser divisions. For example, a choropleth map of the United States showing population by state shows sparsely populated, large, western states which are visually dominant over the smaller, northeastern states which become obscure in the space they occupy. This property of uneven population distribution and varying division sizes is common to statistical maps compiled at all levels (nations, states, counties, metropolitan areas), and helps to demonstrate the clustering phenomenon of geographic information (see Figure 1). This clustering phenomenon pertains to many statistical attributes in such diverse topics as politics, energy, agriculture and manufacturing, since spatial distributions are related to irregular population patterns or idiosyncrasies in the landscape. The cartographer is challenged to diminish any adverse effects that the clustering phenomenon may have on map perception.

As civilizations become more complex, the demand for displaying interrelationships between phenomena increases. The purpose of this research is to rationalize and develop a

Figure 1
An example of contrasting population concentrations.



methodology for computer construction of a three-dimensional prism map with the prism bases formed by noncontiguous proportional polygons and prism heights representing a second related phenomenon. The resultant plotted maps will overcome three important statistical mapping problems: 1) the limitation of displaying one set of data per map thus restricting the analytical powers which the reader can apply; 2) the problem of large, sparsely populated areas dominating the map space and related perception problems; and 3) cartographic presentation of multiple variables in black and white. As with many types of computer maps, the sophistication of the final plotted result is governed by the amount of programming effort expended weighed against the amount of manual effort to complete a satisfactory product. Often the computer produced image should be one compilation stage towards creating a reproducible final map. Computer maps are sometimes impeded, though not stymied, in their time/cost effectiveness by demanding requirements of line widths, legend, customized labeling or shading, and lettering requirements, to mention a few. One specific time/cost consideration that pertains to a proportional prism map program is discussed in a later paragraph.

An often stated adage for novice cartographers is "keep it simple" and many cartographers adhere to this rule by displaying one theme per map using symbols or shading within area outlines. Hopefully, the result is a simple, easy to read map. However, if two phenomena are examined, a statistical relationship between the two variables is calculated and mapped, thus sacrificing the two individual sets of information for one value. Maps can display two sets of related data, utilizing such techniques as shaded proportional symbols or proportional pie shading. The drawback to reading such maps is that symbols must be read in two or more aspects and then associated with a location --- a multistep perception process. Proportional prism maps display two related variables on one map: a total population set (independent) and a subgroup of that population (dependent).

Confronting the second problem of large areas dominating the map space, proportional polygons afford one solution to the dilemma of emphasizing each area division in accordance with a particular statistical population rather than in accordance with its true area (see Figure 2). Each polygon reflects a size in proportion to its own population with respect to the total population. In addition to the primary advantage of balancing divisions by their populations, proportional polygons yield empty space around the polygon which becomes significant in showing density differences among the divisions with respect to the base unit (Olson, 1976). That is, the ratio of new polygon size to original polygon size is a measure of population density with respect to the base unit density. However, it should be noted that one problem with noncontiguous proportional polygons occurs when the base unit possesses such a high density value that many other units become small to the point of non-recognition. Therefore, base area divisions should not be too small or so dense as to force most other areas to unrecognizable smallness, but be dense enough to accommodate any potential boundary overflows.

Accepting that proportional polygons efficiently display one variable, the challenge becomes to effectively map additional parameters. Three-dimensional prism maps, where each prism base is formed by the true geographical outline of the unit area, and the height is proportional to some value, can be impressive in showing statistical information (see Figure 3). However, while eyecatching, there are shortcomings to most three-dimensional maps which must be tilted to a perspective view in order to show height dimensions. One problem is that the tilting of the plane surface to obtain the best perspective view may cause some shorter, remote prisms to be wholly or partially masked by near prisms particularly if the prisms are contiguous. In Figure 3 for example, six countries are hidden. A second problem is that the bottom

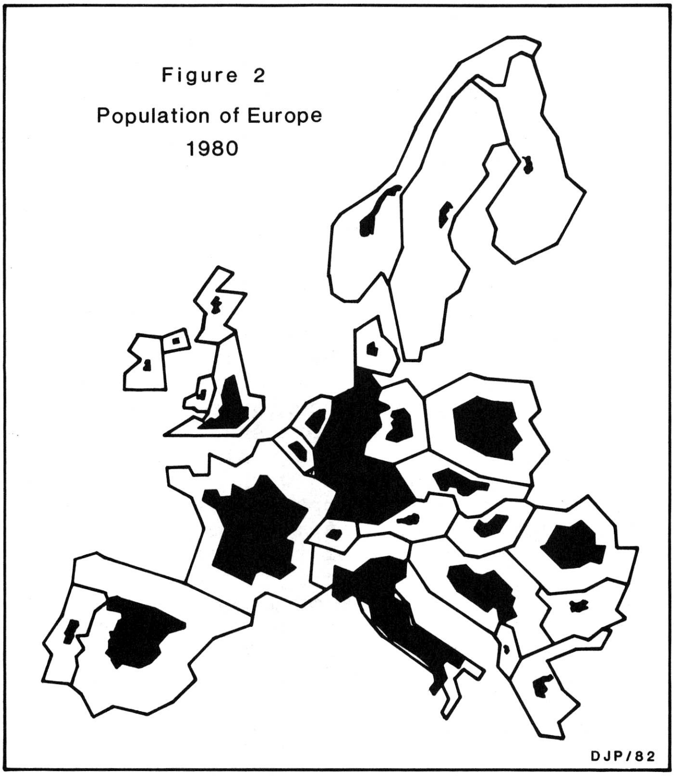


Figure 2
Population of Europe
1980
DJP/82

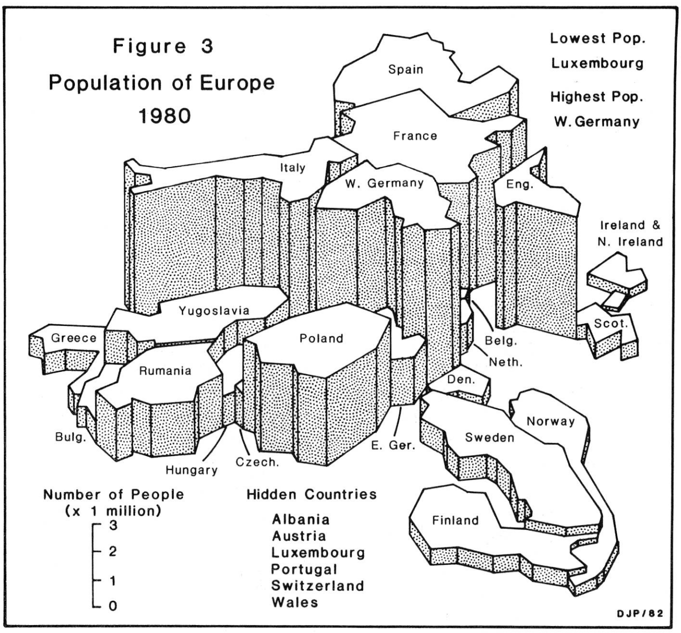


Figure 3
Population of Europe
1980
Lowest Pop.
Luxembourg
Highest Pop.
W. Germany
Spain
France
Italy
W. Germany
Eng.
Ireland &
N. Ireland
Scot.
Yugoslavia
Greece
Poland
Belg.
Neth.
Rumania
Den.
Norway
Sweden
Bulg.
E. Ger.
Hungary
Czech.
Number of People
(x 1 million)
3
2
1
0
Hidden Countries
Albania
Austria
Luxembourg
Portugal
Switzerland
Wales
Finland
DJP/82

of the prism is not always visible, making some prism height measurements more difficult. A noncontiguous proportional polygon prism map which has some open space between the prisms alleviates both of these problems. Therefore, the combination of a noncontiguous proportional polygon base and its prisms efficiently represents a total population by polygon area (x,y) and an attribute of that population by height (z). This exploits the three dimensions of the prism to show two related variables whose product is a relative quantity represented by prism volume. So while many statistical maps sacrifice actual data and display statistical relationships such as percentages, regressions, and the like, proportional prism maps present the reader with a visual relationship beyond the original input.

Limitations on multivariate mapping are also related to reproduction decisions; black and white versus the luxury of color printing. The Census Bureau's Urban Atlas series has attempted to overcome the limitations on black and white maps by employing a four by four color matrix for two variables (Olson, 1975). Unfortunately, the resulting color matrix for these maps requires sixteen color hues from three basic colors and distinction becomes difficult particularly in the middle ranges. One advantage of the proportional prism map is that three values can be readily shown solely utilizing the black and white printing capabilities of the plotter.

METHODOLOGY

To produce a three-dimensional proportional polygon map, the computer logic proceeds as follows: A base map will be defined in terms of x and y coordinates. One high valued areal division within the base map (e.g. a country) will be selected as a basis for area division values, and its prism base (base map area) will be congruent with its true outline. Within the other area divisions a polygon will be placed, factored down in size within its true area according to its population. A rectangular prism will then be formed with this polygon as its base. This prism then will contain two parallel faces which are congruent polygons. Each corresponding pair of line segments of each polygon will be connected with a rectangular face perpendicular to each polygon. The rectangular face will thus be bounded by the pair of line segments and lines connecting each segment and point with its corresponding end point on the other segment. The equal lengths of all perpendicular lines, or heights of the vertical faces, are prism heights. Each prism height is to the base prism height, as its percentage of subgroup population or its attributes value is to that of the areal division selected for the base height. A reference height is selected from the highest subgroup populations or attributes. If the height represents a percentage of total population, the prism volume then represents the total subgroup population. If the height represents a mean (average) attribute, the volume becomes the total attribute. For example, if the total population is census population, the subgroup population percentage of persons over 65 years old; then the base area of the prism is proportional to total population, the height is proportional to percentage of persons over 65, and the volume is proportional to total persons over 65. If the height is proportional to an attribute such as personal income per capita, then the volume is proportional to total personal income.

Preliminarily, the prism will be constructed orthogonally to the base map so that two congruent faces will be superimposed upon each other. The connecting lines will be merely end points. After all graduated polygon prisms are constructed, the map will be tilted by perspective projection. The resultant three-dimensional map must be oriented in such a direction that a minimum of prism surfaces is obscured; the tilt should be slight, but enough to discern heights easily. A base polygon unit will be selected upon which all other area sizes will be proportioned. The base map boundary inclusion is optional.

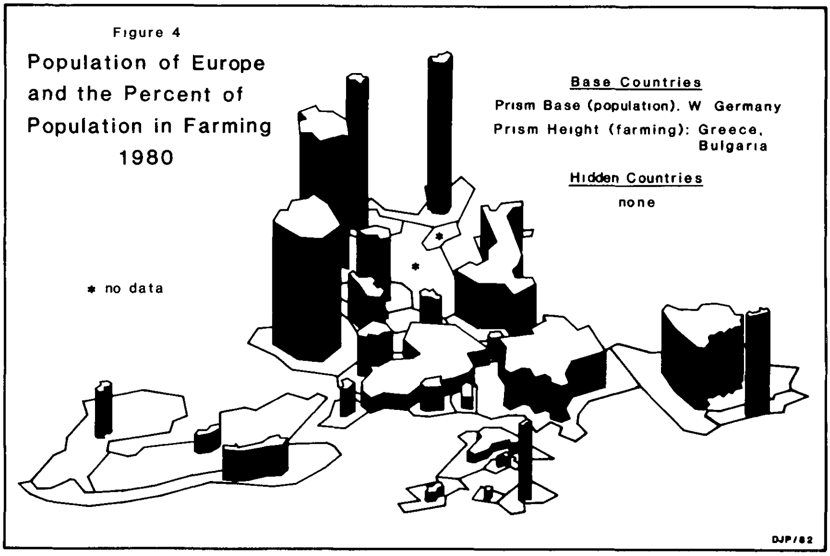
Production of proportional prism maps by computer requires eight major steps, each with a number of sub-tasks. In order to avoid such terms as region, area or polygon, consider a map of Europe showing total population together with number of workers in farming. The steps are:

1. Read a digital representation of Europe. A digitized equal-area projection of Europe is best. The program is capable of accepting data input in various formats and types of organizations.
2. Read data for total population and percentage of population in farming for each country. As a user option, multiple variables may be read as a matrix with rows representing countries, and columns representing different variables. This enables the user to quickly produce multiple plots. Other variables which can be supplied as designated on the control cards include the country's central point stated in x,y and the country's area. Both of these values can also be calculated by the program using the outline points.
3. Construct a proportional polygon figure for each country. Construction of the polygon requires a central base point. A desirable central point is one well within the boundaries of the figure, and situated to maximize the probability of totally containing the proportional polygon within the country boundary. Geographic centers are often satisfactory and are readily available by latitude and longitude from various sources. Ideally, a rigorous examination of the boundary in each case of size reduction should be pursued to determine the alternative possibility of using median centers, but the volume and complexity of the calculations would increase computer time extensively. Population density will be given or calculated to develop a factor for contracting the polygon within the country boundary. From all

densities calculated, a base density must be selected - the densest to guarantee contraction (no overlapping) for all countries. Areas with low values should be examined carefully in this regard. The base country density (West Germany in this case) is then assigned a contraction factor of 1.0 to designate no contraction; all other country densities are then divided by the base country density for a ratio of polygon size to state outline size.

4. Determine a height (z) for each polygon. A base height is selected as the maximum height desired for the prisms. This parameter is user supplied and a maximum percentage country must be selected by the user or by the computer to employ the maximum desired height.
5. Determine a point distribution around the perimeter of each polygon to construct vertical lines connecting the base of prisms with the top surfaces. The selection of vertical line end points is made on the basis of choosing vertical faces which will show the right and left limits of the prism and show major changes of direction in the shape of the polygon. Another criterion for choice of vertical lines can be for the purpose of shading. Equal spacing of points can cause equal shading. In order to indicate the direction of a hypothetical light source, shading can be performed on the basis of face angle from the source. Such an approach can give a realistic, three-dimensional effect.
6. Project the base maps and prisms to a three-dimensional view, given a user-defined perspective. The degree of tilt and direction of view can be user specified.
7. Remove the hidden lines and plot (Hedgley, 1982). This step in the program consumes the greatest amount of computer time. While the plot is essential, it may be plausible not to remove the hidden lines. The result will appear as a glass transparent figure with many vertical lines running through the prism. The decision here depends upon whether or not the intent is to obtain a final product, or a map for the purpose of further compilation.
8. Add appropriate descriptive information. For a finished product, title, legend, source and possibly area labels may be added. This step is also computer time dependent and some may opt for manual enhancement.

This paper presents a computer-assisted cartographic technique that combines two existing statistical mapping techniques (Nimmer, 1981), and an example is shown in Figure 4. By following the eight steps outlined above, the resulting proportional prism map should be pleasing to view if the parameters are carefully selected to minimize prism interference, to enhance interpretation of the areas and heights, and to facilitate observation unit identifications.



SUMMARY

There is a great need to derive cartographic techniques to display information which consist of two components. Often, the mapped variables are largely dependent on the aggregate population of which they are a part. For instance, at the global scale, gross national product is portrayed with a missing dimension. Either gross national product per capita or total gross national product by nation is shown, not both. The proposed mapping technique could show both and add population information by employing population as the cartogram base and gross national product per capita as the prism height. The resulting prism quantity would represent the total gross national product.

In summary, the following are considered to be major advantages and disadvantages of computer-assisted proportional prism mapping:

Graphic Advantages

1. The base plane viewed in perspective is more apparent since the prisms do not fill the entire plane as a traditional three-dimensional map does. This enables the reader to view the study space boundaries as well as the subdivisions.
2. Because the base plane is more apparent, prism heights are easier to see, thereby making "z" values easier to compare. The reader can see most of the tops and bottoms of the prisms since the polygons are noncontinguous, while traditional three-dimensional maps usually mask some subdivisions.
3. Proportioned polygons are easy to recognize since the shapes and locations are maintained unlike contiguous cartograms.

Graphic Disadvantages

1. Some base units may become so small that the polygon or prism generated may not be recognizable. Depend-

ing on the data, some units may overflow their boundaries slightly, while others remain too small.
2. Centering the proportional polygon in the base unit may not always yield the most graphically pleasing result if the unit is a complex, irregular shape.
3. Since the proportional polygon is categorically an area symbol, actual distributions are disrupted in the sense that the symbol is centralized within the base unit. The areal nature of the symbol has disadvantages similar to a choropleth map. That is, a certain amount of localized information may be lost since enumeration unit values are applied.

Computer-Assisted Advantages

1. The proportional prism map program can be interfaced with existing geographic information systems which contain a chain file and values for individual polygons (i.e. the Census Bureau's GBF-DIME).
2. The program does all calculations for polygon areas and prism heights.
3. The program generates a plot that can be incorporated into the cartographic process for refinement (line weights, prism face shading, lettering). Such manual enhancement may make the map more pleasing to view, and more reproducible.

Computer-Assisted Disadvantages

1. The routine for removing hidden lines from the polygons consumes an excessive amount of computer time since it requires a sorting procedure and external storage.

With any new cartographic technique, the question becomes whether or not the information displayed is effectively transmitted and discerned by the map reader. To date, the proportional prism map has not undergone any type of formalized perception evaluation. The maps have been scrutinized however by several geographers and computer programmers. The communication aspects of the proportional prism map have to be examined in several regards. Can the proportioned base units still be recognized by the reader as a geographic entity? How well does the map reader discern the values being represented? Can spatial relationships be realized using this mapping technique?

There is an increasing demand in statistical oriented research for effective geo-processing techniques which organize data into meaningful entities, both in tabular and graphic form. This computer-assisted proportional prism mapping program presents one approach to the challenge of visually communicating multi-variable information.

REFERENCES

Dutton, G. 1978, ed. Geographic Information Systems: Survey, Overviews and Criteria: Harvard Papers on Geographic Information Systems, Volume II.

Hedgley, Jr., D.R. 1982, A General Solution to the Hidden-Line Problem: NASA Reference Publication 1085, Ames Research Center, Dryden Flight Research Facility, Edwards, California, 14 pages. This is the most efficient solution to date in the opinion of Russell A. Brown of Evans and Sutherland Corporation, Salt Lake City, Utah.

Nimmer, J. 1981, Proportion Prism Program: unpublished, Department of Geography, The University of Akron, Akron, Ohio, 10 pages.

Olson, J. 1975, The Organization of Color on Two-Variable Maps: Proceedings of the International Symposium on Computer Assisted Cartography - Auto-Carto II, American Congress on Surveying and Mapping, Reston, Virginia, pp. 289-294.

Olson, J. 1976, Noncontiguous Area Cartograms: The Professional Geographer, Volume XXVIII, No. 4, November, pp. 371-380.