SPECIFIC FEATURES OF USING LARGE-SCALE MAPPING DATA IN PLANNING CONSTRUCTION AND LAND FARMING

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SUMMARY

New principles to estimate the accuracy of the functions of point coordinates measured on a topographic map are discussed. Particular attention is given to the consideration of the coordinate errors' autocorrelations allowing to obtain original results for the estimation of the accuracy in the determination of farming land areas, distances, slopes, and other data widely used in planning construction and land farming.

USING LARGE-SCALE MAPPING DATA

Topographic maps are used in town designs, land development and rational land use, management of land evaluation cadastre, etc. The reliability of the pertinent decisions depends on the accuracy of positioning the points on the map. It is necessary also to estimate the accuracy of the functions of the points' coordinates (i.e. slopes, elevations, plot areas, etc.) since that is relevant for the reliability of the end results and their possible use in projecting construction and land farming.

In the general case, the problem is to estimate the accuracy of the function

$$u = f\,(l_1,\ l_2,\ \ldots,\ l_n), \tag{1}$$

where l_i $(i = 1,2,\ldots,n)$ are the results of some measurements with the mean square deviations σ_{l_1}.

To estimate the accuracy of the function (1), it is possible to make use of the approximate formula

$$\sigma_u^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial l_i} \sigma_{l_i} \right)^2 + \sum_{j>i} \frac{\partial f}{\partial l_i} \frac{\partial f}{\partial l_j} r_{ij} \sigma_{l_i} \sigma_{l_j} \tag{2}$$

where $\frac{\partial f}{\partial l_i}$ are partial derivatives of the function (1) calculated by the approximate (measured) values of the arguments; $r_{i,j}$ are correlation coefficients of the values l_i, l_j specified in the correlation matrix.

In many publications dealing with the accuracy of the points' coordinates, the second summand of the formula (2) is neglected supposing the correlation (autocorrelation) coefficients are too small to be taken into consideration. More often than not, this can be accounted for by unawareness of the reasons causing the correlation. In consequence, the result is underrating the effect of this factor on the calculation of the accuracy of the sought values.

We have worked out general formulas to estimate the accuracy of topographic points' coordinates. For instance, at a priori accuracy assessment of the elevation calculated on the basis of horizontals, the mean square deviation formula has the form

$$\sigma_h = \sigma_H \sqrt{2(1 - r_H)}\,, \tag{3}$$

where σ_H is mean square deviation of the elevations

r_H is correlation coefficient of the elevations equal to

$$r_H = \exp\left(- \frac{0.18}{\sqrt{S_o}} S\right), \tag{4}$$

where S_o is the maximal distance in m between the points of the digital model used for plotting the map;
S is the distance in m between the points in the map.

Calculations [1] show that when $S \leq 1.5\ S_o$ the correlations should not be neglected. The correlations of the coordinate errors should be taken into account when evaluating the accuracy of plot area calculation and when measuring distances by the mapping materials.

The known formula to calculate a plot area by the coordinates of its vertices is

$$2\ P = \sum_{i=1}^{n} x_i\ (y_{i+1} - y_{i-1}), \tag{5}$$

where x_i and y_i are abscissas and ordinates of the plot vertices.

On the basis of (2) we get

$$4\sigma_p^2 = \frac{\sigma_t^2}{2}\sum_{i=1}^{n} C_i^2 + \sigma_t^2 \sum_{j>i} C_i C_{i+j} \operatorname{Sin} d_i \rightarrow$$

$$\operatorname{Sin} d_{i+j} \times r_{x_i, x_{i+j}} + \sigma_t^2 \sum_{j>i} C_i C_{i+j} \operatorname{Cos} d_i \rightarrow$$

$$\operatorname{Cos} d_{i+j}\, r_{y_i, y_{i+j}}, \qquad (6)$$

where C_i is the diagonal connecting the points (i-1) and (i+1);

d_i is the directional angle of this diagonal in the direction from the point (i-1) to the point i+1);

σ_t is the mean square deviation of the turning point of the contour on the map ($\sigma_{t_1} = \sigma_{t_2} = \ldots = \sigma_{t_n} = \sigma_t$);

r are autocorrelation coefficients.

The summation is taken first for j = 1, then for j = 2 and so on until j = q at which

$$C_{i,i=q} > T,$$

where T is the correlation interval.

For the model of a rectangular land plot with equal sides S and N turning points, assuming

$$r_{x_i, x_{i+j}} = r_{y_i, y_{i+j}} = r_o$$

we get from (6)

$$4\sigma_p^2 = \frac{1}{2}\sigma_t^2\left[(N-4)4S^2 + 8S^2\right] + 4\sigma_t^2 S^2 N r_o. \qquad (7)$$

On the basis of [1] it is possible to take for a priori estimations $r_o = 1/2$. Taking this into consideration and bearing (7) in mind we can get

$$\sigma_p = \sigma_t S \sqrt{N-1} \qquad (8)$$

Without due regard of the correlation, the formula for the mean square deviation of the plot area in the example has the form

$$\sigma_p' = \frac{\sigma_t S}{\sqrt{2}}\sqrt{N-2}\,. \qquad (9)$$

It follows that neglecting the correlation of the contour points' coordinates results in underestimation of the plot area accuracy approximately by [2] times.

Using the formula (2) it is possible to carry out analysis of contours and land plots and draw conclusions on the accuracy of area calculations on the basis of mapping materials.

The problems of distance estimation accuracy measured by region models are of great interest. A detailed discussion of this problem is presented in [2] . We shall only note that neglecting the correlation of the errors in the coordinates of the segments' ends increases the distance estimation error approximately by $\sqrt{2}$ times.

Since analytical solution of the problems to estimate the accuracy of the functions of the points considering the dispersion (correlation) matrix is cumbersome, the method of statistical trials may be used (the Monte-Carlo method).

Computers are applied at that to model the process to produce topographic maps, formation of the contour points' coordinate errors, appearance of the correlations, etc.

For instance, in estimating the accuracy of farming land areas on the maps, a probability model of the plot is produced using the mathematical statistics apparatus. The following factors are taken into consideration:

-- the farming land area (P)
-- the coefficient of the plot elongation (K)
-- the number of the contour turning points (N).

Then the formula (5) is applied to calculate the area of the plot by the specified coordinates x_i^o, y_i^o of the turning points assumed as true. At each modelling step the coordinates x_i and y_i of the contour points are taken from the expressions

$$x_i = x^o_i + \delta x_i ,$$

$$y_i = y^o_i \pm \delta y_i , \tag{10}$$

where δx_i and δy_i are pseudorandom errors with a specified distribution.

Thus we obtain a series of random area error realizations

$$\delta p_i = P_i - P^o_i . \tag{11}$$

The number of the realizations δp_i may be as large as desired and depends on the required accuracy of the end result.

A special algorithm for modelling random errors of the contour points' coordinates has been elaborated at the Chair of Geodesy of the Moscow Institute of Land Use Planning Engineers. It is based on generating random errors in the form of a random vector. The components of the vector are presented as sums of the products of some non-random coefficients and independent random errors taking into consideration the correlation moments

$$K_{x_i y_j} = r_{x_i x_u} \sigma x_i \, \sigma x_j \,,$$

$$K_{y_i y_j} = r_{y_i y_j} \sigma y_i \, \sigma y_j \,. \qquad (12)$$

The number of realizations was taken 1000.

The analysis of the results has shown that the mean deviation σ_p of the area of the probability model of the plot, considering (12), may be presented as a curve

$$\sigma_p = \alpha P^{\beta} \,. \qquad (13)$$

The formula (13) with $r_{x_i x_j}$ and $r_{y_i y_j}$ obtained in [2] has the form

$$\sigma_p \doteq 1.4 \, \sigma_t \sqrt{P} \,, \qquad (14)$$

where σ_t corresponds to that of the formula (6).

In conclusion it should be noted that in order to come to the right decisions in construction or land farming, the customers should have a clear concept about the accuracy of the various obtained data while using the materials of large-scale mapping.

The results of the studies presented in the paper allow for the solution of this problem with a sufficient degree of reliability since they take into account the correlation of the coordinate errors due to the very technological processes of compiling large-scale mapping materials.

REFERENCES

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2. Neumyvakin, Yu.K., Estimation of error correlation of photogrammetric coordinates. - Geodeziya i kartografiya, N 4, 1980.