COORDINATE TRANSFORMATIONS IN MAP DIGITIZING

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ABSTRACT

One important task in map digitizing is the conversion of coordinates given in some local device coordinate system to map coordinates. Manual and automatic digitizing devices (digitizing tables, line followers, scanners) send coordinates in inches or metric units to the host. Map data are stored and processed either in rectangular (meters, feet) or geographical units (longitude, latitude). Device coordinates are converted to map coordinates with the help of user defined control points. This conversion may be done with n-parametric polynomial transformations or inverse functions of cartographic map projections. In this paper we investigate both approaches and give some comparative statistics. In the case of polynomial transformations parameter estimation is done with least squares as well as with robust statistical methods.

INTRODUCTION

Manual and automatic digitizing devices produce coordinates that have to be converted to map coordinates. In order to derive results from map data processing the coordinates must be stored in a coordinate system that is suitable for all required tasks. Usually geographical coordinates in degrees of longitude and latitude are chosen, because any cartographic projection may be applied to the data without causing troubles in overlapping zones as it is the case with some projections.

The general problem is to define a transformation between two coordinate systems. In the case of map coordinate data there are two ways of coordinate conversion, using inverse projections and polynomial approximations. Both have already been treated by various authors for digitizing (Fischer 1979) and converting from one projection to another (Brandenberger 1983).

In the case of manual digitizing of paper maps the coordinate conversion is done with the help of control points that are used to compute transformation parameters. The given values of the control point coordinates are used together with the measured values from the digitizing device. It is obvious that these control points have to be measured with utmost care in order to derive useful results. Only one wrong measurement will render unacceptable parameters. In the sequel we shall also investigate methods to decrease the effects of wrongly measured control points.

In this paper we see two procedures applicable to coordinate transformations:

1. Device coordinates --polynomial--> geographical coordinates

2. Device coordinates --polynomial--> intermediate rectangular coordinates (e.g. UTM) --inverse-projection--> geographical coordinates

The first procedure converts device coordinates to longitude, latitude without any intermediate step. In the second case device coordinates are first converted to meters or feet in the projection of the map sheet. Then the inverse projection is used to compute geographical coordinates.

POLYNOMIAL TRANSFORMATIONS

The relationship between digitizing device coordinates (x,y) and geographical map coordinates (long,lat) is expressed by the following formula

$$\begin{aligned} long &= F1(x,y) \\ lat &= F2(x,y) \end{aligned} \qquad (1)$$

If we define F1 and F2 to be power series we can write (1) as

$$\begin{aligned} long &= \sum_{i=0}^{n} \sum_{j=0}^{i} a_{j,i-j} x^j y^{i-j} \\ lat &= \sum_{i=0}^{n} \sum_{j=0}^{i} b_{j,i-j} x^j y^{i-j} \end{aligned} \qquad (2)$$

The vectors **a** and **b** of the coefficients are estimated as

$$\begin{aligned} \mathbf{a} &= (C^T C)^{-1} C^T \mathbf{u} \\ \mathbf{b} &= (C^T C)^{-1} C^T \mathbf{v} \end{aligned} \qquad (3)$$

with **C** being the design matrix composed of the measured values of m control points

$$C = \begin{bmatrix} 1 & x_1 & y_1 & x_1 y_1 & \cdots & x_1^n & y_1^n \\ 1 & x_2 & y_2 & x_2 y_2 & \cdots & x_2^n & y_2^n \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & x_m & y_m & x_m y_m & \cdots & x_m^n & y_m^n \end{bmatrix}$$

and **u** and **v** the vectors of the given control point coordinates for longitude and latitude

$$\mathbf{u} = (u_1, u_2, \ldots., u_i)^T$$

$$\mathbf{v} = (v_1, v_2, \ldots., v_i)^T$$

The goodness of fit is determined by inspection of the residuals of the given values versus the estimated values of the control point coordinates.

INVERSE PROJECTIONS

When the map projection and all necessary projection parameters are known the inverse projection function can be used to derive longtitude/latitude from given meters or feet. In order to get easting and northing values in meters or feet from digitizing device coordinates a polynomial transformation as in (1), (2) and (3) with control points in meters or feet is used:

$$\begin{aligned} \text{easting} &= F1(x,y) \\ \text{northing} &= F2(x,y) \end{aligned} \qquad (4)$$

These values are then used with the inverse of projection P to compute geographical coordinates.

$$\begin{aligned} \text{long} &= P^{-1}(\text{easting}) \\ \text{lat} &= P^{-1}(\text{northing}) \end{aligned} \qquad (5)$$

ROBUST PARAMETER ESTIMATION

Least squares parameter estimation ~~fails~~ fails when only one control point is wrong. We have investigated methods of robust parameter estimation using robust and bounded influence regression (Huber 1981, Dutter 1983). The basic principle of robust regression is that we do not minimize the sum of squares of the residuals as with least squares. Instead of the square function robust regression works with functions that give less weight to large residuals, i.e. their first derivative must be bounded.

The program BLINWDR (Dutter 1983) offers linear least squares and three robust options known as "psi bends at c", psi bends at a, b and d" and "psi has the form of sine" which all have bounded first derivatives.

Applying robust parameter estimation in map set-up procedures decreases the effect of inaccurately measured control points (cf. test results below).

TEST RESULTS

For testing the above stated procedures we took one sheet of the Austrian map series 1:50000. The projection of this series is Gauss-Krüger, a variant of the Transverse Mercator projection. The sheet has a Transverse Mercator grid of 2 kilometers. We selected 20 points at grid line intersections. Geographical coordinates were computed using the U.S.G.S. General Cartographic Transformation Package (U.S.G.S. 1982).

Table 1 lists the control points and their coordinates.

Table 1: control point coordinates

	easting meters	northing meters	longitude dd	mm	ss.s	latitude dd	mm	ss.s	digitizer x-mm	y-mm
1	100,000	5,154,000	14	38	12.0	46	31	05.9	294.525	135.625
2	104,000	5,154,000	14	41	19.6	46	31	03.7	374.250	136.375
3	108,000	5,154,000	14	44	27.2	46	31	01.4	454.175	136.850
4	112,000	5,154,000	14	47	34.8	46	30	59.1	534.425	137.575
5	100,000	5,160,000	14	38	16.6	46	34	20.2	293.450	255.575
6	104,000	5,160,000	14	41	24.4	46	34	18.0	373.325	256.075
7	108,000	5,160,000	14	44	32.2	46	34	15.7	453.100	256.800
8	112,000	5,160,000	14	47	40.0	46	34	13.4	533.375	257.425
9	100,000	5,166,000	14	38	21.3	46	37	34.5	292.425	375.575
10	104,000	5,166,000	14	41	29.3	46	37	32.3	372.275	376.250
11	108,000	5,166,000	14	44	37.3	46	37	30.0	452.100	376.900
12	112,000	5,166,000	14	47	45.2	46	37	27.6	532.350	377.500
13	100,000	5,172,000	14	38	26.0	46	40	48.7	291.325	495.425
14	104,000	5,172,000	14	41	34.1	46	40	46.5	371.225	496.075
15	108,000	5,172,000	14	44	42.3	46	40	44.3	451.150	496.750
16	112,000	5,172,000	14	47	50.5	46	40	41.9	531.400	497.325
17	100,000	5,178,000	14	38	30.6	46	44	03.0	290.000	614.725
18	104,000	5,178,000	14	41	39.0	46	44	00.8	369.825	615.300
19	108,000	5,178,000	14	44	47.4	46	43	58.5	449.700	615.975
20	112,000	5,178,000	14	47	55.7	46	43	56.2	530.050	616.600

Performing parameter estimation for polynomial transformations of degrees 1 and 2 gives 6 and 12 parameters for for both longitude and latitude respectively. Tables 2 shows the results for both procedures described above.

Table 2: residual root mean square in meters

	6 parameters easting	northing	12 parameters easting	northing
procedure 1	10.42	11.54	4.65	7.21
procedure 2	8.69	11.49	4.65	7.21

For testing the effect of wrong measurements we set the values of point 14 equal to those of point 9, i.e. two different control points are measured at the same location. The test was carried out for procedure 1, table 3 lists the results.

Table 3: root mean square in meters (procedure 1) robust estmation

	6 parameters		12 parameters	
	easting	northing	easting	northing
least squares	903.95	1364.37	950.35	1425.00
robust estimation	10.86	13.77	5.36	9.04

We have carried out extensive tests with other data sets all leading to the same results as stated above.

CONCLUSION

One can see that with at least 12 parameters (i.e. polynomial degree 2) we can achieve the same result for both procedures. This can be expected as long as the map covers a relatively small area and the projection in use is not too "strange". For very small scale maps procedure 2 seems to be more suitable. It turns out that propably the best way for map digitizing is first to gather coordinates in some local device coordinate system and then apply transformations and/or projections to compute coordinates for a desired coordinate system.

Robust parameter estimation gives little weight to erroneous measurements thus yielding good and acceptable results even if some measurements are wrong.

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