

A CARTOGRAPHER'S APPROACH  
TO QUANTITATIVE MAPPING OF SPATIAL VARIABILITY  
FROM GROUND SAMPLING TO REMOTE SENSING OF SOILS

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ABSTRACT

Data collection and the handling of spatial information inherent in the data for natural resource mapping is cumbersome and particularly problematic in large area surveys for which remote sensing provides an excellent tool in resource assessment and process monitoring. In this case ground sampling is used not only to produce maps but also to calibrate remotely sensed data, therefore the statistical and physical relationships between them should be treated quantitatively, as well as the features of the resulting thematic maps. This approach can be extensively used in both ways: in sampling design and in accuracy testing.

From a theoretical point of view the sampling design can be well optimized in terms of sample size, acceptable error and confidence limits to describe the spatial variability of Earth surface features, when measurement errors and characteristics of spatial patterns are taken into consideration. This strategy should be also applied for remotely sensed data.

Once spatial data is constructed either by interpolation or remote sensing, thematic maps are generally derived. A geographic expert system should additionally exploit in this processing the understanding of errors related to the geometric and thematic determination of contours.

INTRODUCTION TO THE IDEAS

The last decade has produced and spread a number of new data sources, consequently processing technology in spatial data handling. Geographers, cartographers, remote sensing scientists and others have been working on the exploitation of these opportunities in quantitative resource mapping. The impact of this development on cartography was well demonstrated, among others, at the previous AutoCarto Conference in London (e.g. Morrison 1986).

A number of authors have proved the potential of sampling theory applied for mapping, in particular in experiment design (McBratney and Webster 1981), calibration of remotely sensed data (Curran and Williamson 1986a) and thematic map accuracy testing (Rosenfield et al. 1982), to provide tools for determining strategies of better sampling to achieve better accuracy for, in general, better (i.e. quantitative) description and understanding of spatial phenomena.

One of the most popular and important keywords for those involved in this type of research is spatial variability. The basic idea of this paper is to raise and outline the issue of incorporating our knowledge of spatial variability in geographic information systems (GIS), thus developing it to a geographic expert system (GES). The approach is thought to be appropriate for a wide variety of applications in both sampling design and in accuracy testing.

Part one is a compact summary of sampling theory with regard to ideal vs. field sampling and remote sensing. It is followed by a section on how maps can be constructed from raw data with regard to spatial variability of the represented variables. Then some aspects of contour definition for thematic maps are outlined and finally an ongoing experimental soil mapping experiment is presented with preliminary results.

## SAMPLING REVISITED

### Sampling theorem for stationary processes

Many properties of several Earth surface features can be treated as continuous signals, however, scientists need to describe their patterns using generally sparse point observations. Field sampling in this respect can be represented as seen in Fig.1(a-c). Having the continuous signal  $g(x)$ , where  $x$  denotes spatial coordinate(s), point sampling can be approximated with a Dirac- $\delta$  series:

$$g(x) \approx \sum_k \delta(x - kd) = \sum_k g(x) \delta(x - kd) \quad /1/$$

where  $d$  denotes the sampling distance. From a number of illustrative experience one can have the impression that the shorter the sampling interval, the better the reconstruction of the signal, although the right hand side of Eq./1/ is equal to zero at every  $x \neq kd$ .

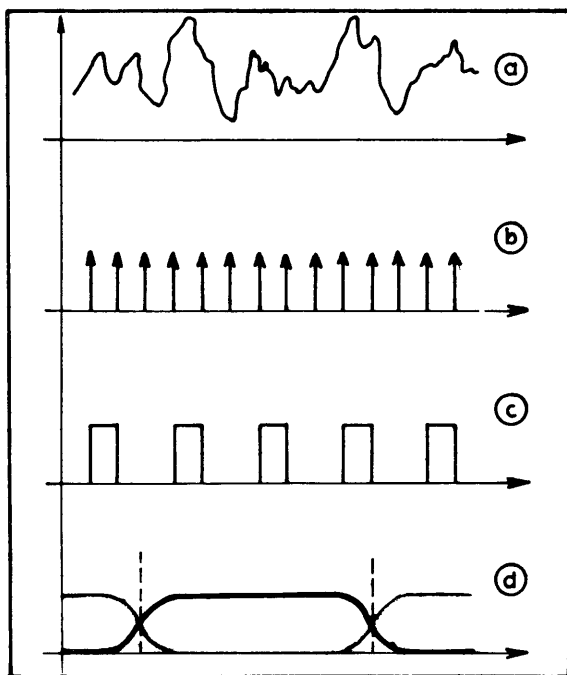


Figure 1.

Schematic representation of a property as a continuous signal (a) and its ideal (b), real in-situ (c) and remote (d) sampling.

The exact relationship between the sampled and original signal is defined by the sampling theorem (see e.g. Meskó 1984). Recalling the Fourier-transform of the Dirac- $\delta$  series, the Fourier-transform of Eq./1/ can be written as

$$G(f) * (1/d) \sum_k \delta[f - (k/d)] = (1/d) \sum_k G[f - (k/d)] \quad /2/$$

where \* denotes convolution,  $f$  stands for (spatial) frequency and the expression gives the sampled spectrum  $G_s(f)$ . The summed terms of the right hand side may be overlapping, however,

$$\begin{aligned} \text{if } |f| > f_{UF} \leq f_{NYQUIST} = 1/2d \\ \text{then } G(f) = 0 \text{ for } |f| > f_{UF} \end{aligned} \quad /3/$$

meaning that the original signal can be reconstructed without any loss of information if the period of the highest spatial frequency is, at least, twice sampled.

Once having the Fourier-transform of  $g(x)$ , the autocovariance function can be derived as follows:

$$\text{COV}(h) = F^{-1}\{|G(f)|^2\} \quad /4/$$

from which spatial patterns (e.g. dominant frequency) can be computed.

#### Quantitative description of variability of not strictly stationary processes

Eq./4/ implies that our process for which the  $g(x)$  signal is recorded should be stationary in both mean and variance. As it happens many properties of the land appear not to be stationary in this sense (Oliver and Webster 1986). This led Matheron (1965) to consider the somewhat weaker assumptions of stationarity:

$$E\{g(x) - g(x+h)\} = 0 \quad /5/$$

and

$$\text{VAR}\{g(x) - g(x+h)\} = E\{[g(x) - g(x+h)]^2\} = 2\mu(h) \quad /6/$$

where  $E$  denotes expectation and the function  $\mu(h)$  is called the semi-variogram.

(If the process is second order stationary then than the semi-variogram is related to the autocovariance function by:  $\mu(h) = \text{COV}(0) - \text{COV}(h)$ , and either  $\mu(h)$  or  $\text{COV}(h)$  can be used to describe the spatial process. If, however, only the so called intrinsic hypothesis holds (c.f. Eqs./5/ and /6/), than the covariance is undefined and  $g(x)$  is called a regionalized variable. The semi-variance can be estimated without bias according to the definition implicit in Eq./6/, or to the formulas for two or more dimensions and irregular sampling (Webster 1985).)

## FROM DATA TO MAPS

### Optimal interpolation and estimation for spatial units

The method of estimation embodied in regionalized variable theory is known in Earth sciences as kriging (Webster 1985). It is essentially a means of weighted local averaging:

$$g'(y) = \sum_k L_k g(x_k) \quad /7/$$

in which the weights ( $L_k$ ) are chosen so as to give unbiased estimates at  $y$  ( $g'(y)$ ).

It is optimal in the sense, that at the same time it minimizes the estimation variance:

$$\begin{aligned} \sigma_{\epsilon}^2(y) &= E\{[g(y) - g'(y)]^2\} = \\ &= 2\sum_k L_k \mu^*(x_k, y) - \sum_k \sum_m L_k L_m \mu(x_k, x_m) - \mu^*(y) \end{aligned} \quad /8/$$

where  $\sigma_{\epsilon}(y)$  is the estimation variance at  $y$  (that can be either a point or a block),  $E$  denotes expectation,  $\mu(x_k, y)$  stands for the semi-variance of the property between  $x_k$  and  $y$ , taking into account of both the distance and angle, while  $\mu^*(x_k, y)$  and  $\mu^*(y)$  denotes the average semi-variance between  $x_k$  and all points within the block, and within the block, respectively. (For formulas to obtain the estimation variance see Webster 1985.)

#### Application to large area surveys, remote sensing and GIS

Large area surveys of natural resources, in general, require an enormous amount of samples when applying systematic sampling. Remote sensing techniques, however, confine ground sampling to training areas only. Regionalized variable theory (or otherwise geostatistics) can reduce the necessary sample size to even less, and provide information on the accuracy of a constructable map as a function of location while raising the efficiency of data processing in the following ways:

(1) To obtain the semi-variogram from ground observations to describe the scale and patterns of spatial variables over several orders of magnitude nested sampling is a very economic way (Oliver and Webster 1986);

(2) Once the semi-variogram is obtained the spatial resolution with acceptable estimation error, or, conversely, the error for a given spatial resolution for a given property can be determined;

(3) This can be then compared to the optimum spatial resolution derived from remote sensing pilot studies of "homogeneity" (Curran and Williamson 1986b);

(4) Finally structural information of the remotely sensed data themselves can be used in digital image processing (Carr and Myers 1984).

From the above listed aspects (2) is in the focus of our interest, because it is not only applicable for calibration purposes with ground and remotely sensed data, but can be introduced as an expert system function in a GIS. Thematic accuracy have not only been neglected untill now in standard GIS products, but "intelligent map edition" may serve as a permanent temptation for constructing meaningless but nice graphic products from a very limited number of samples. Even with sufficient data, accuracy as a function of location can be used as an overlay or auxilliary information in a GIS for data representation and/or for further processing (like scale changes etc.), providing a quantitative cartographic tool for deeper understanding of spatial features.

MAPS AND CONTOURS

Once spatial data is constructed either by interpolation or remote sensing (or by both), thematic maps are generally derived. It is out of the scope of this paper to discuss classification and its accuracy in general, nevertheless, some notes should be made with regard to cartography.

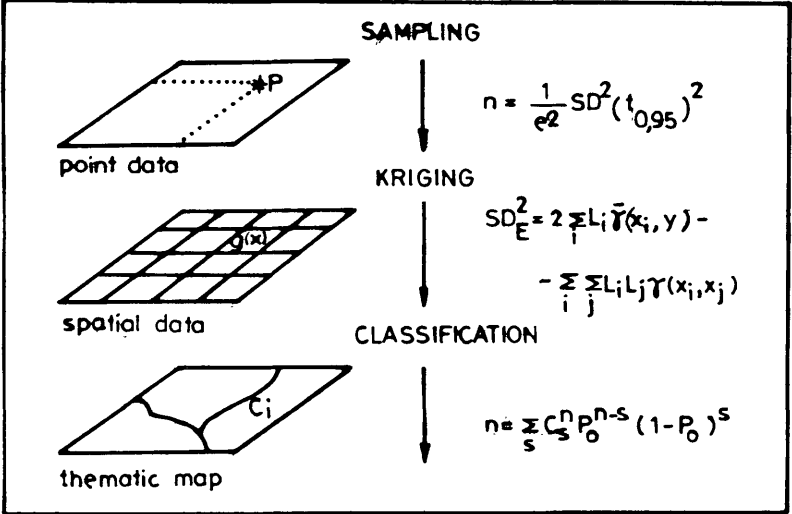


Figure 2.

Error estimation at different stages of data processing (see text for details)

### Thematic and geometric approach to the definition of class boundaries

When some kind of preconditions (e.g. tradition) define the classes of a thematic map, it is only separability of the data set that effects recognition accuracy of such patterns. In case of a purely statistical data set, however, class intervals for a map should be determined with statistical considerations, such as to ensure the significant differences between adjacent classes beside the usual "equal-frequency" coding (Csillag 1986, Stegena and Csillag 1986).

There are some instances when the distribution of contours of a given thematic map is known. Burgess and Webster (1986), for example, have developed an algorithm incorporating a given distribution function to estimate the risk in constructing choropleth maps by point sampling along transects.

### Accuracy of thematic maps

When, as a supposed final product, a thematic map is constructed, its accuracy should be tested, too. Evaluating the accuracy of a thematic map requires sampling statistically the classified polygons to determine if the thematic classes, as mapped, agree with the field-identified categories.

Rosenfield et al. (1982) developed a method to validate the accuracy for each class with specified confidence from the cumulative binomial distribution. This method determines the minimum sample size for each category with a preliminary estimate of the accuracy.

## AN EXPERIMENTAL SOIL MAPPING PROJECT

The above outlined ideas are being tested in an experimental soil mapping project in East-Hungary. Error propagation will be controlled in each step of processing from measurement error through interpolation to classification (see Fig.2). Soil samples have been collected at specific sites for calibration purposes and transect and gridded data have been collected for testing quantitative treatment of mapping errors. The project is envisioned as a pilot study to introduce this methodology in the TIR soil information system (Csillag et al. 1986).

A considerably large number of laboratory and in-situ calibration measurements have been completed to estimate the proportion of variance due to measurement errors.

Spatial patterns have been then described with semi-variograms (see Fig.3). These will serve as the basis for plotting spatial estimation error against aerial block size compatible with different sources of remotely sensed data, and for the interpolation of the training data with regard to spectral reflectance characteristics of soils (Baumgardner et al. 1985). Finally thematic maps are planned to be constructed and tested.

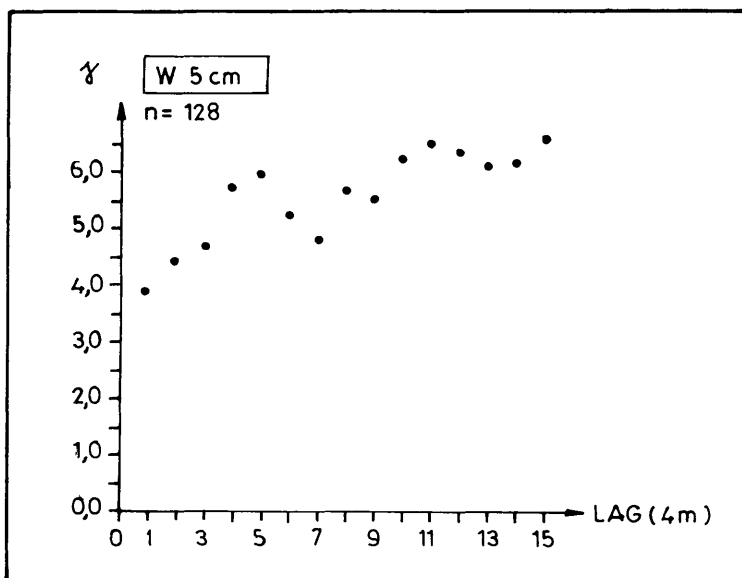


Figure 3.

Sample semi-variogram of soil moisture of the 0-5 cm layer

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## REFERENCES

Baumgardner, M.F., L.F.Silva, L.L.Biehl, E.R.Stoner (1985)  
Reflectance Properties of Soils  
Advances in Agronomy 38:1-44.

Burgess, T.M., R.Webster (1986)  
A Computer Program for Evaluating Risks in Constructing  
Chloropleth Maps by Point Sampling Along Transects  
Computers and Geosciences 12:107-127.

Burrough, P.A. (1986)  
Five Reasons Why GISs Are Not Being Used Efficiently for  
Land Resource Assessment  
in: AutoCarto London (ed:M.Blakemore) Vol.II. 139-148.

Carr, J.R., D.E.Myers (1984)  
Application of the Theory of Regionalized Variables to  
the Spatial Analysis of Landsat Data  
Proc. PECORA IX., IEEE Publ.CH-2079-2:55-61.

Csillag, F. (1986)  
Comparison of Some Classification Methods on a Test-Site  
(Kisköre, Hungary): Separability as a Measure of Accuracy  
International Journal of Remote Sensing (in press)

Csillag, F., S.Kabos, Gy.Várallyay, P.Zilahy, M.Vargha (1986)  
TIR: A Computerized Cartographic Soil Information System  
in: AutoCarto London (ed:M.Blakemore) Vol.II.

Curran, P.J., H.D.Williamson (1986a)  
Sample Size for Ground and Remotely Sensed Data  
Remote Sensing of Environment 20:31-43.

Curran, P.J., H.D.Williamson (1986b)  
Selecting Spatial Resolution for the Estimation of  
Grassland GLAI  
in: Mapping from Modern Imagery, IAPRS 26:407-416.

Matheron, G. (1965)  
Les Variables Regionalisées et Leur Estimation  
Masson, Paris

McBratney, A.B., R.Webster, T.M. Burgess (1981)  
The Design of Optimal Sampling Schemes for Local  
Estimation and Mapping of Regionalized Variables  
Computers and Geosciences 7:331-334.

Meskó, A. (1984)  
Digital Filtering: Applications in Geophysical  
Exploration  
Akadémiai-Pitman-Halsted, Budapest-London-New York

Morrison, J. (1986)  
Cartography: A Milestone and Its Future  
in: AutoCarto London (ed:M.Blakemore) Vol.I. 1-13.

Oliver, M.A., R. Webster (1986)  
Semi-Variograms for Modelling the Spatial Pattern of  
Landform and Soil Properties  
Earth Surface Processes and Landforms 11:491-504.

Rosenfield, G.H., K. Fitzpatrick-Lins, H.S. Ling (1982)  
Sampling for Thematic Map Accuracy Testing  
Photogrammetric Engineering and Remote Sensing 48:131-  
137.

Stegen, L., F. Csillag (1985)  
Statistical Determination of Class Intervals for Maps  
(manuscript)

Webster, R. (1985)  
Quantitative Spatial Analysis of Soil in the Field  
Advances in Soil Science 3:1-70

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