OPTIMUM POINT DENSITY AND COMPACTION RATES FOR THE REPRESENTATION OF GEOGRAPHIC LINES

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ABSTRACT

Previous work on automated line generalization has concentrated on the issues of computer efficiency for line thinning and the choice of a minimum number of critical points which must be retained in the generalized line. This study takes a different view of the problem of line generalization, emphasizing densification, rather than reduction, of the number of points describing a line for the purpose of optimizing representational accuracy. This new perspective raises the two following questions: 1) For a given map scale, what is the maximum number of describing points which can be retained without producing redundant information and 2) What is the relationship between line compaction rates and map scale reductions.

Given a plotter pen size, one may identify the smallest geographic artefact which may be visually recognizable. This, in turn, determines the minimum spacing of the digitized points describing the line. Finally, the concept of fractal dimension may be used to predict the maximum number of describing points for a given map scale, assuming statistical self-similarity for the geographic line.

The function governing the relationship between coordinate compaction rate and map scale reduction is particularly useful for the development of scale independent data bases, assuming that the points selected for small scale representations are always a subset of those used in larger scale representations. A linear relationship has been proposed but preliminary results show that the coordinate compaction rates depend on the generalization algorithm being used, the fractal dimension of the line as well as the map scale reduction.

INTRODUCTION

Not long ago, a consulting firm was commissioned by the US Army Engineer Topographic Laboratories to perform an extensive review of automated methods for line generalization (Zycor, 1984). The results of the study were inconclusive, and gave only broad recommendations as to which generalization algorithms appear potentially more promising than others. The study, among others published in the last few years, reflects the urgency of formalizing the process of cartographic generalization so that it can be adequately automated (Jenks, 1979; McMaster, 1983; White, 1985).

Previous work on the evaluation of automated line generalization has concentrated on the issues of computer efficiency and the choice of a minimum number of critical points which must be retained while preserving the geometric and visual characteristics of a geographic line. This paper proposes a different perspective on the problem of line generalization by emphasizing the idea of densification, rather than reduction, of the number of points describing the line with the purpose of optimizing geometric accuracy. Because storage costs and computing time for line storage and line thinning will become less and less of an issue, I take the view that the original line information should be preserved as much as possible, given the physical and conceptual limitations of map scale and map purposes. If we equate maximizing geometric accuracy to maximizing information --- in this case the number of points describing the curve -- then maximizing information implies minimizing data losses resulting from selection, displacement, simplification and deletion. This new perspective raises the two following questions: 1) For a given line and map scale, what is the maximum number of describing points which the map is able to bear, and 2) what is the relationship between line compaction rates and map scale reductions. The second question is raised in the context of a cartographic data base which is used to produce maps at different scales. It would be useful to know the relationship between scale and information compression since this relation could be implemented to determine automatically the number of describing points that should be kept to draw a line at a given scale. A further issue, of course, is the choice of the describing points which ought to be included. Although the focus of this paper is not the evaluation of generalization methods which most characteristically reproduce the geometry and the geography of a coast line or river, the method adopted for line generalization is a necessary consideration in this research. Specifically, the relationship between data compaction rates and map scales may be seriously affected by the method adopted for compressing the data.

PREDICTING THE OPTIMUM NUMBER OF DESCRIBING POINTS

The digital representation of a cartographic line usually takes the form of a discrete set of points identified by their positions with respect to an arbitrary coordinate system. Successive points are joined by linear elements to make up the line. Thus, this discrete set of points constitutes the core information about the line. One important practical question for the cartographer is how large must this set be in order to ensure a proper description of the line. If one wants an accurate representation, then the number of describing points must be as large as possible for the particular scale at which the line is to be plotted. This, in turn, implies that the points made available in the cartographic data base have been digitized with a resolution equal or higher than the corresponding resolution of the plot.

Let the line width of the plotter pen be 0.2 mm. By definition, open meanders whose width or wavelength is smaller than or equal to 0.2 mm cannot be drawn open (Figure 1). This would also be the case for a raster plotter whose resolution is less than or equal to 127 dots per inch. Assume further that the separation between two line strokes may not be smaller than 0.2 mm. This is a conservative estimate to ensure visual separability, although I am aware that visual acuity or the corresponding angle of visual discrimination measured in minutes of arc may allow much finer separations, depending on the reading conditions. Thus, only meanders whose wavelength is equal or larger than 0.4 mm can be drawn clearly and distinguishably (Figure 1). As a result, the describing points of the line may not be closer than Δ = 0.4 mm from each other. This means that digitizing a map in a stream mode with a distance between digitized points less than 0.4 mm would generate superfluous points. It also means that any further reduction

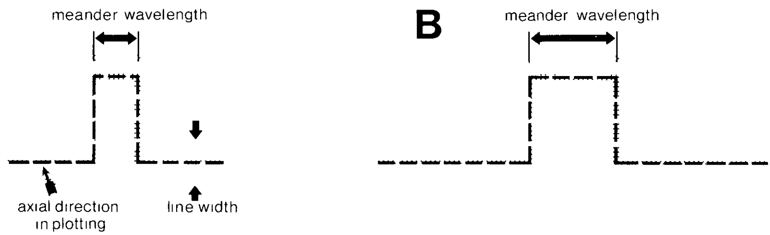

Figure 1. In A, the meander is closed in because its wavelength is no greater than the line width. In B, the meander is shown distinguishably. Its wavelength is no smaller than the line width plus the minimum separation allowed between line strokes.

of the original map to reproduce the line on a smaller map format implies a generalization such that no two consecutive points retained on the generalized curve are closer to each other than a distance $\Delta = 0.4$ mm. For instance, a three-fold reduction would imply the systematic elimination of one of the points which is member of a pair of consecutive points whose distance is smaller than 0.4 x 3 = 1.2 mm on the original copy. The nth point or distance-traversed generalization algorithm could be used to enforce such a condition when the selected points are to be a subset of the original points. Otherwise the walking generalization algorithm is a good candidate for the application of the minimum separation rule (Muller, 1987). It produces a new sequence of points which are equally distant from each other. In either case, collinear points may be subsequently removed in order to reduce the storage space taken by the results.

The above rule provides a guideline for maximum point density. It would be useful to predict the total number of describing points resulting from its application. The concept of fractal dimension may be used to calculate this number.

Assume the geographic line is a fractal, that is, each piece of its shape is geometrically similar to the whole. This property is called self-similarity (Mandelbrot, 1982). From Richardson, we have the equation (Richardson, 1961):

$$L(\varepsilon) = \varepsilon^{(1-D)} \qquad (1)$$

where ε is the step length to measure the length of the line $L(\varepsilon)$, and D is a constant.

Let N be the number of steps ε used to measure the line length. Then $L(\varepsilon) = N \times \varepsilon$. According to (1):

$$N \times \varepsilon = \varepsilon^{(1-D)}$$

$$\ln N + \ln\varepsilon = (1-D)\ln\varepsilon$$

$$\ln N / \ln\varepsilon = -D$$

or

$$D = \ln N / \ln(1/\varepsilon) \qquad (2)$$

D is called the fractal dimension of the line. For all practical purposes, the value $1/\varepsilon$ may be thought of as the number of steps of length ε partitioning the base line (a straight line joining the first and last point of the curve's basic fractal generator, which, in the case of a geographic line, is the whole line itself).

Note that equation (1) can also be rewritten:

$$D = 1 - \ln L(\varepsilon) / \ln(\varepsilon) \tag{3}$$

A geographic line is said to be statistically self-similar when the relationship between $\ln L(\varepsilon)$ and $\ln(\varepsilon)$ is linear. In this case, the limit $(\ln L(\varepsilon + \Delta\varepsilon) - \ln L(\varepsilon))/\Delta\varepsilon$ where $\Delta\varepsilon \to 0$, is estimated through regression analysis and is used to determine the fractal dimension in equation (3).

Furthermore, given the fractal dimension of a geographic line, one can determine the value of N:

$$\ln N = D \times \ln(1/\varepsilon)$$

or

$$N = e^{**}\ln(1/\varepsilon) \times D \tag{4}$$

The steps of length ε are the strokes which are used to draw the curve and, according to the minimum separation rule, may not be smaller than Δ, the minimum distance between the describing points of the curve. Assume again $\Delta = 0.4$ mm. One can calculate the value of N to predict the maximum number of points which may be used to describe the line, depending on its fractal dimension and the size of the plot (Table 1).

TABLE 1. MAXIMUM NUMBER OF DESCRIBING POINTS (N) DEPENDING ON FRACTAL DIMENSION (D) AND PLOT SIZE

PLOT SIZE = 40 mm;	$\Delta = 0.4$ mm;	$\varepsilon = 0.4/40$;	$1/\varepsilon = 100$
D		N	
1.0		100	
1.1		158	
1.2		251	
1.6		1,585	
2.0		10,000	
PLOT SIZE = 160 mm;	$\Delta = 0.4$ mm;	$\varepsilon = 0.4/160$;	$1/\varepsilon = 400$
D		N	
1.0		400	
1.1		728	
1.2		1,326	
1.6		14,564	
2.0		160,000	

Note: Plot size is calculated according to $[(Xn - X1)^{**}2 + (Yn-Y1)^{**}2]^{**}\frac{1}{2}$ where (X1, Y1) and (Xn, Yn) designate the positions of the first and last points of the curve.

For illustration, the coast line of Banks in British Columbia was plotted on a pen plotter using the above approach, with pen size = 0.2 mm, Δ = 0.4 mm, and N = 1226 points for a fractal dimension of 1.2145 and a plot size of fourteen centimeters (Figure 2).

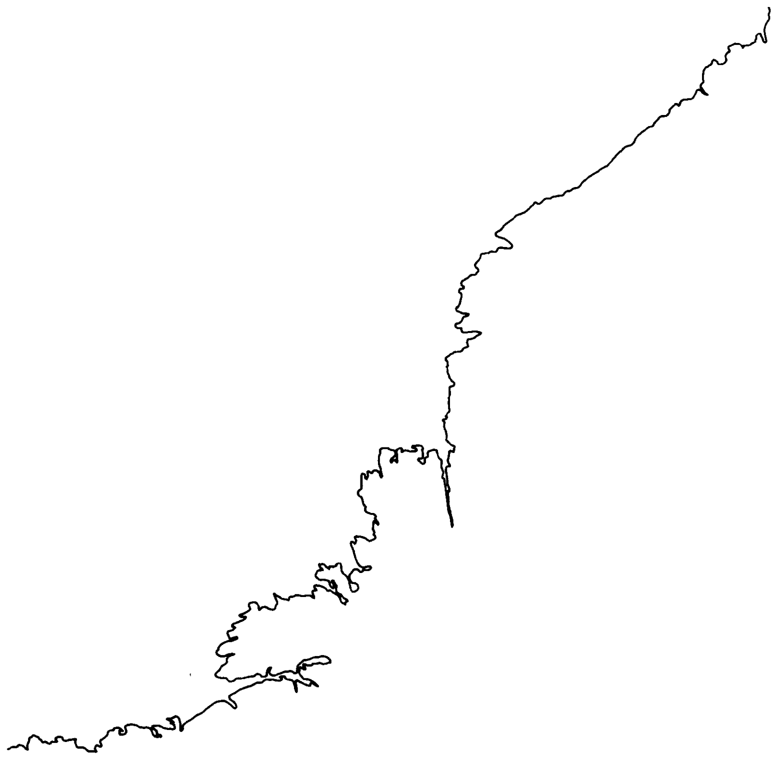

Figure 2. The Banks coastline with 1226 points.

The original Banks coastline was digitized on a 1/50000 topographic map from the NTS series 103 G/8. The number of digitized points was originally 2759 and was reduced to 1266 using the walking algorithm and a walking step Δ = 0.4 mm. Since the Banks coastline is self-similar (Muller, 1986), the value of N could be predicted for any plot size. A plot size of seven centimeters, for instance, would require only 528 describing points. The visual appearance of the plot in Figure 2 may still be unsatisfactory to the reader, as many spikes and meanders appear closed in. This phenomenon is not directly related to the size of the walking steps, however, but to the morphology of the original line which shows many narrow spikes and inlets which are almost circular (Figure 3). The spike problem has already been mentioned elsewhere (Deveau, 1985). Complex lines with narrow spikes and wide but circular meanders have a tendency to collide with themselves through the process of generalization. This is particularly true when a recursive tolerance band algorithm, such as the one of Douglas and Peucker (1973), is applied. A possible solution to this problem would be to identify all line segments which are crossing over, colliding or potentially colliding (within a particular tolerance window) and displace the corresponding points. Research is currently in progress in this area.

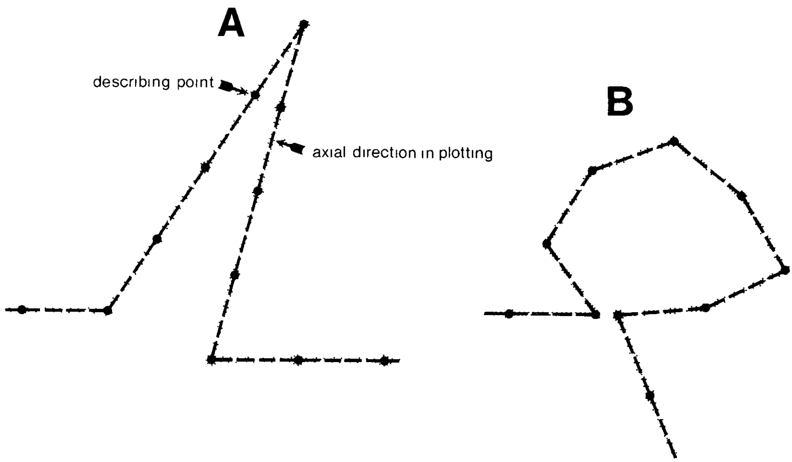

Figure 3. The spike (A) and the inlet (B) are partly or completely closed in.

Note that equation (4) provides the lower and upper limits of the number of points necessary to describe a line corresponding the lower (D=1) and upper (D=2) limits of fractal dimension (Table 1). When D tends toward a value of 2, the line tends to fill the space and thus requires a large number of points to be described (160,000 points for a point sampling resolution of 0.4 mm and a 16 cm plot size). The lower limit corresponds to a fractal dimension of D=1 which characterizes continuous, differentiable curves such as a circle or a straight line. In the later case, a blind application of the proposed approach would be absurd, since a straight line only requires two points and any other point included according to the minimum separation rule would be redundant. Again, a check on collinearity for any straight segment of the line would remove this problem. Another limitation of the proposed approach is the fact that N can be predicted for self-similar lines only. Previous studies have shown that geographic lines are not always self-similar (Hakanson, 1978; Goodchild, 1980).

RELATING DATA COMPRESSION RATES TO MAP SCALE

The relationship between scale and the quantity of information displayed on a map has been studied for quite some time. Several models to formalize this relationship have been proposed, among those the Radical Law or Principle of Selection by Topfer and Pillewizer (1966). For linear information, such as the data describing a geographic line, the Radical Law takes the simplest form:

$$N \times M = \text{constant} \tag{5}$$

when N would be the number of points describing the line and M the denominator of the map scale. Accordingly, a two-fold reduction of the original map translates into a two-fold reduction of the number of describing points. Renewed interest was recently expressed for this type of empirical rule, as it "introduces the possibility of a hierarchical method of line storage, whereby the number of points retrieved is directly related to the scale of the required map". (Jones and Abraham, 1986). This hierarchical structure, however, implies that the points selected for small scale representations are always a subset of those used in larger scale representations, which is not always the case.

The issue here is whether the Radical Law, which proposes a linear relationship between map scale reduction and data compression for line data, has practical value. An empirical test was conducted on two coastlines -- one complex line (Banks, already mentioned) and one simpler line (Isidro, digitized from the 1/50,000, G12-B11 Gulf of California map). They were tested at four different scales, each scale being successively a two-fold reduction of the previous one. Three different generalization algorithms, including the moving average, the Douglas-Peucker and the walking algorithm, were used to represent the lines at the different scales, with generalization rates corresponding to the scale reduction rates (Figure 4 and 5). For the sake of

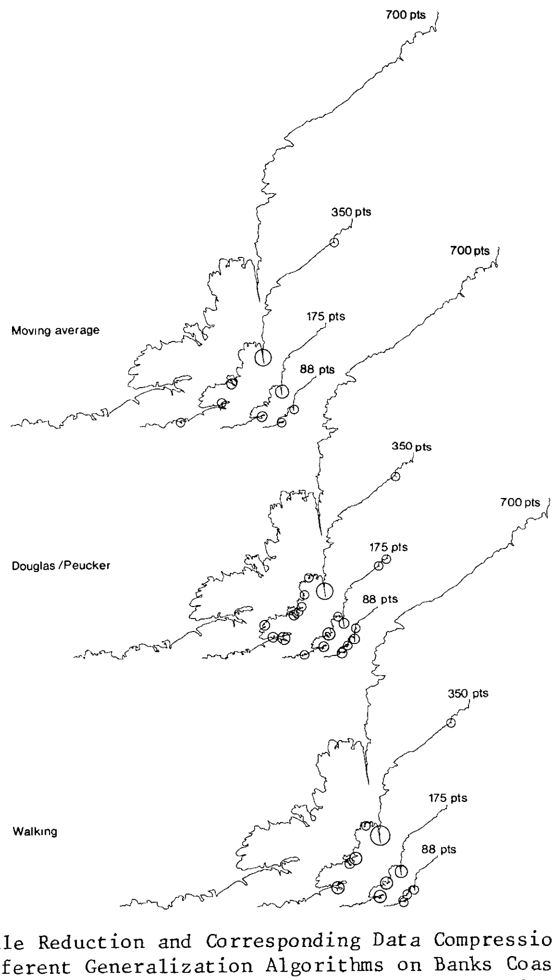

Figure 4. Scale Reduction and Corresponding Data Compression Using Different Generalization Algorithms on Banks Coastline. Problem areas in the smaller scale representations are highlighted by circles.

clarity, the number of describing points on the largest representations was purposely reduced in order to minimize the risk of line collision and afford a better comparison with the smaller scale representations. For all the Banks tests, the smaller scale representations show new problem areas (closing spikes and closed loops) or a worsening of the ones already present on the larger maps (Figure 4). Note, however, that the test using the Douglas algorithm gives the worst result. This suggests that the Radical Law is less suited for this generalization algorithm than for the others. The Isidro tests, on the other hand, were all successful, demonstrating that the Radical Law is applicable for simpler lines (Figure 5). This small experiment shows that the form of the relationship between data compression and scale reduction of linear elements is more complex than the one suggested by Topfer and Pillewizer and is a function dependent on line complexity and method of generalization as well. In the case of statistically self-similar geographic lines, one could incorporate the effect of

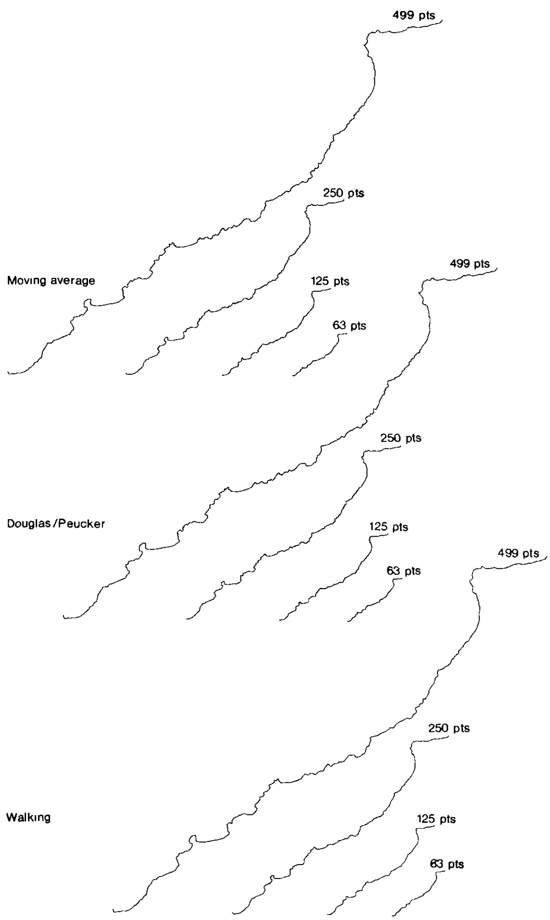

Figure 5. Scale Reduction and Corresponding Data Compression Using Different Generalization Algorithms for the Isidro Coastline.

complexity by suggesting the following relation:

$$N1 = N0\ [(M0/M1)**D] \tag{6}$$

where D is the fractal dimension of the line, N0 and N1 are the number of describing points on the larger and the smaller scale maps, M0 and M1 are the corresponding scale denominators. In the case of a space filling curve, the reduction in the number of describing points would correspond to the reduction in map area:

$$N1 = N0\ [(M0/M1)**2] \tag{7}$$

Although this relationship may be more suited for complex curves, its successful application depends upon the assumption of an appropriate point density on the original source map.

Furthermore, one could incorporate the minimum separation rule in equation (5):

$$\Delta 1 = \Delta 0\ [(M1/M0)**D \tag{8}$$

when $\Delta 0$ and $\Delta 1$ are the minimum spacing between the describing points on the original map and the new derived map after reduction. This would provide a rule for generalization as well as optimize point density for any particular scale. It could be easily applied in a hierarchical data base where the original describing points on the source document were captured through stream mode digitizing with a constant $\Delta 0$ value. The points selected in a smaller scale representation would be a subset of the original describing points according to the new minimum separation value $\Delta 1$.

CONCLUSION

A few guidelines for consideration prior to the process of line generalization have been proposed. There is the view that one ought to maximize the number of points describing the line for any particular scale. A minimum separation rule between describing points may be set as a function of plotting resolution and visual discrimination. For statistically self-similar geographic lines, the total number of points required to describe the curve according to the minimum separation rule may be predicted. The walking algorithm was applied to illustrate this rule. Furthermore, it was found that the Topfer and Pillewizer's Radical Law which suggests a linear relationship between data compaction and map scale reduction was not suited for complex lines. In the case of the self-similar lines, a relationship including the fractal dimension was proposed instead. A problem which deserves further investigation is the tendency of a complex curve to collide with itself through the process of generalization. A purely algorithmic solution to the problem of line generalization does not appear satisfactory. Cartographic generalization is not only a reduction of the amount of information for the sake of preserving map readibility (Salichtchev, 1977). Generalization also involves an understanding of the meaning of the information which is being generalized. Thus, there is a need to add some "intelligence" to the computer generalization process to insure that line segments are not colliding (topological integrity) and that significant geographic features are preserved (geographical integrity,) as would be case if the line was generalized by a cartographer using his geographic knowledge.

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