PERFECTING AUTOMATIC LINE DRAWING

Kurt K. Kubik
Joseph C. Loon
Department of Geodetic Science and Surveying
The Ohio State University
Columbus, Ohio 43210-1247

ABSTRACT

New interpolation methods are presented, yielding results which are closer to manually interpolated lines than classical interpolation methods. By better modelling the line structure, these methods do not need the definition of breakpoints, the definition of which is necessary due to shortcomings in the classical methods.

CLASSICAL LINE DRAWING

In automated cartography it is often required to interpolate lines or surfaces in between given points. In one dimension this operation can readily be visualized: Given a set of points (x,z), find the interpolated value z at location x. Various methods are in use for this purpose. One may use local linear interpolation in between neighbouring control points, local polynomial interpolation, spline interpolation, prediction and moving average rules or other methods. These methods can usually be related to each other, as was shown by Frederiksen et al, 1984. Usually these methods use smooth interpolation, minimizing the average curvature of the line.

Take for example the cubic spline. It minimizes the function

$$I = \int_{\Omega} \left(\frac{\partial z}{\partial x}\right)^2 d\Omega \; ; \quad \Omega \text{ domain} \tag{1}$$

In order to understand the behavior of this mathematical spline, we may think in its physical analogue, the draftsman's spline. In drawing a smooth curve through a number of points, the draftsman uses a thin elastic rod forced to pass through the locations of the points. The shape of this mechanical spline is such that it has a minimum bending or curvature. Consider now points (x,z) in figure 1.1, representing a step function. Both the physical (draftsman) spline and the mathematical spline cannot properly describe the abrupt changes in height values and tend to oscillate heavily. In order to get reasonable results, one must introduce two "break points" where the tangent (slope) of the curve is discontinuous, and interpolate independently within the resulting three sections, figure 1.2.

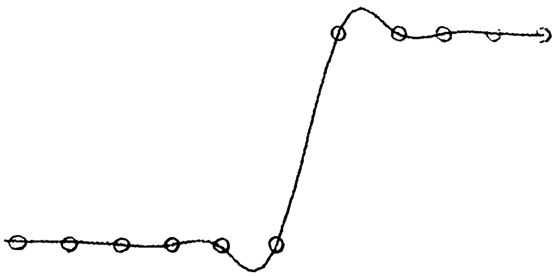

Figure 1.1: Spline interpolation of a step function

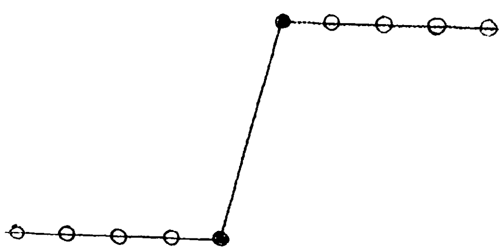

Figure 1.2: Spline interpolation with break points

Similar considerations are valid for other interpolation methods (finite elements, Prediction method, local interpolation).

The necessity of defining break points (or break lines in two-dimensional interpolation) may thus be seen as due to the deficiency of the interpolation methods to properly model the terrain. The economic consequences of this deficiency are considerable: in modern DEM (Digital Elevation Model) applications one spends on the average most of the time digitizing break lines in order to get an acceptable result (see figures 1.3 to 1.5, ref: Kubik, 1985). It thus pays off to investigate other interpolation methods which reduce the need of break point and break line definition.

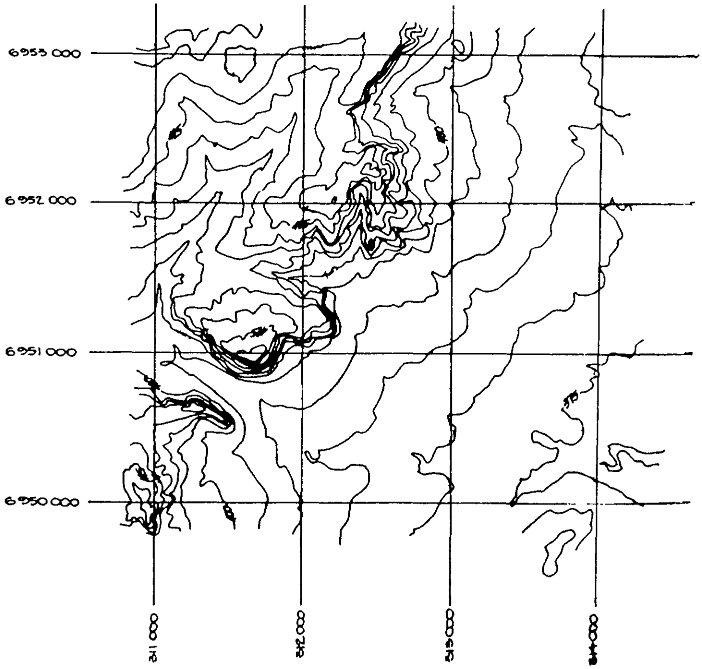

Figure 1.3: Contour lines of test area (5 meter contours) 7,964 points digitized, 90 minutes measuring time.

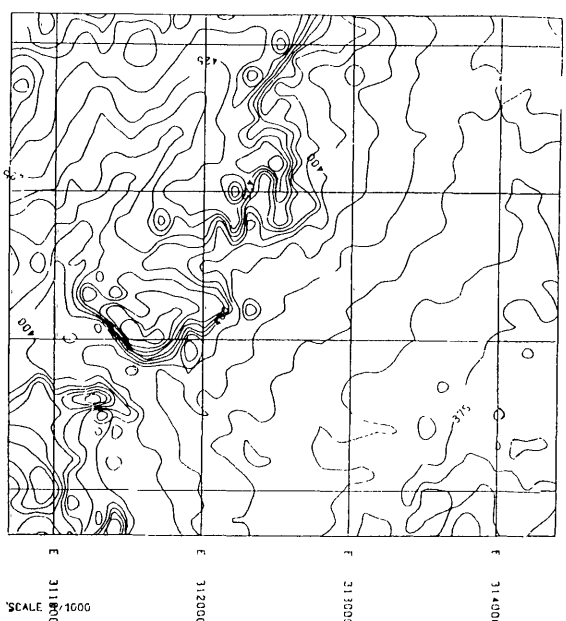

Figure 1.4: Results of HIFI profiles of 100m spacings; characteristic profile points selected by operator; no breaklines (1,162 points digitized, 43 minutes measuring time)

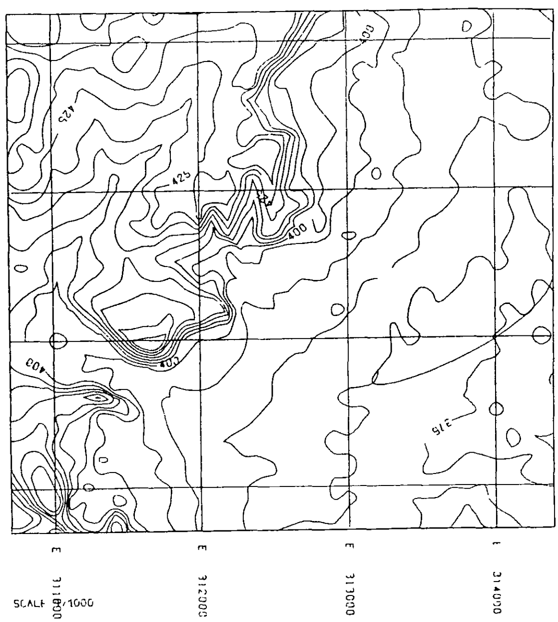

Figure 1.5: Results of HIFI profiles of 100m spacing operator selected points; breaklines included
(1,121 profile points and 2,542 breaklines points digitized, 96 minutes measuring time)

NEW INTERPOLATION METHODS – MINIMIZING OTHER DERIVATIVES

As concluded in the earlier chapter, the classical interpolation programmes use relatively inflexible interpolation functions. By minimizing the average second derivative of the interpolation function, these methods (approximately) describe the bendings of a mechanical spline or plate, but not the undulations of a cartographic line. As an alternative, we may try to model the behavior of this line by the differential equation

$$z^{(n)} = \varepsilon \tag{2}$$

where (n) denotes the n-th derivative of the values z, and ε is an independent random variable (white noise). Here n may also be non-integer. In that latter case this fractional n-th derivative is defined by continuous interpolation into the integer differences (Frederiksen et al, 1984).

This differential equation (2) may now be chosen as a model for interpolation, for instance in L-spline or finite element interpolation. The proper functional to be minimized is then

$$J = \int_{\Omega} \left\{ \frac{\partial^{(n)} z}{\partial x^{(n)}} \right\}^2 dx \rightarrow \min. \tag{3}$$

The results of the L-spline interpolation are indentical to the results of the Wiener prediction method using a proper variogram or covariance function, as it was shown already in 1971 by Kimeldorf and Wabha (see also Dolph and Woodbury, 1952 and Kubik, 1973).

Figure 2.1 shows examples of interpolation according to these new principles. The digitized points represent the profile of a well known cartographer. For n = 1 we obtain piecewise linear interpolation (linear spline), for n = 2, piecewise 3rd degree interpolation (cubic splines) and for n between 1 and 2 we obtain interpolation forms which properly model break points in the terrain profile while preserving relative smoothness in the other profile sections. From extensive analysis of various terrain forms, the authors found n values in between 1.2 and 1.4 as most appropriate for use in DEM applications. Manmade cartographic lines are modelled on the average with a slightly larger n value. Algorithms for online determination of the proper n value for individual lines were developed by the authors to enable proper interpolation according to the line structure inherent in the digitized points (Kubik and Loon, 1985).

[1] The coefficient n can be derived from analysis of the cartographic line, using the variogram or spectrum concepts, see (Frederiksen et al, 1984).

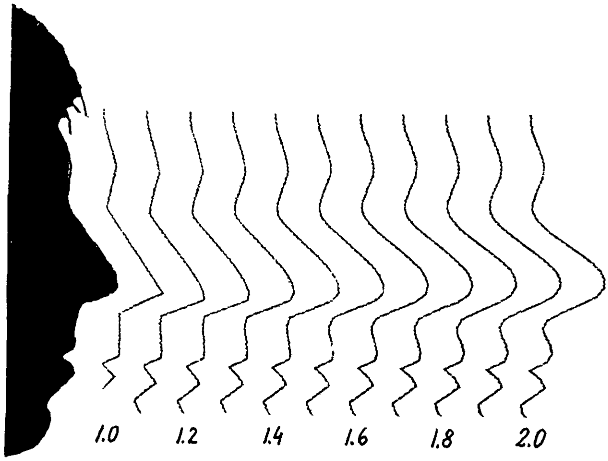

Figure 2.1: Interpolation with fractionals derivatives; use of different values of n for interpolation

NEW INTERPOLATION METHODS – MINIMIZING OTHER FUNCTIONALS

The above idea can be further generalized by minimizing well chosen functions of the derivatives, instead of their square sum:

$$J = \int_{\Omega} f\left(\frac{\partial^{(n)} z}{\partial x^{(n)}}\right) d\Omega \rightarrow \min.$$

In order to illustrate this principle, we consider the simple functionals

$$J_p = \int_{\Omega} \left|\frac{\partial^2 z}{\partial x^2}\right|^p d\Omega \quad ; \quad 0 \leqslant p \leqslant 2$$

minimizing the integral of non-integer power of the second derivative of the line.

The numerical interpolation methods for solving (5) are analogous to the methods described in Chapter 2. In order to demonstrate the effect of this class of interpolation principles, we choose again the profile of a well known cartographer (Figure 3.1). Classical cubic spline interpolation (using $p = 2$) yields unsatisfactory results and would need the definition of numerous break points to yield an acceptable result. 3.4 shows the interpolation results for decreasing values of p. Notice that the profile becomes more recognizable for decreasing values of p, with an optimal choice of p equal to 1.2.[2] Lower p values yield an increasingly rough profile, with piecewise linear interpolation obtained for $p = 1$.

[2] This optimal value of p can also be derived from covariance or spectral analysis of the sample points.

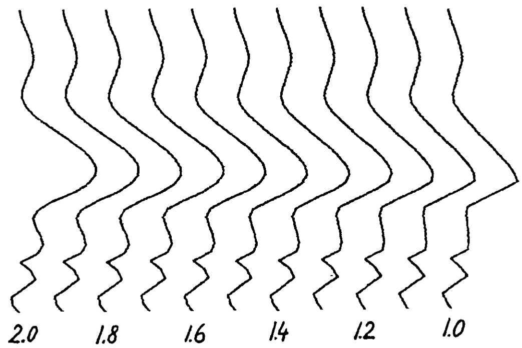

Figure 3.1: Interpolation with fractional powers; use of different values of P for interpolation.

Thus, with the principle (5), we have obtained a new transition of interpolation forms from a cubic spline to a linear spline, different from Chapter 2. In both cases, no break points were needed to yield realistic interpolations, which are close to the lines drawn by draftsmen.

Obviously, other functionals (4) may be chosen, which may give both worse and better interpolation results. However, proper use of these new principles allows a very effective interpolation of cartographic data, and considerable savings in data capture, as compared to classical methods.

FINAL REMARKS

Algorithms for rapid interpolation according to these new principles have been developed by the authors, and the methods were fine tuned for various classes of line interpolation (and approximation). As proposed in this paper, adaptive interpolation strategies are possible, taking into account the internal structure of the data set. However, much work still is necessary in order to fully understand the potentials of these new classes of interpolation methods. In particular, online data capture intertwined with interpolation appears desirable in order to help the operator in the digitization (or measuring) process and in understanding the nature of the interpolation function. This approach will allow a very considerable reduction of data capture time as compared to today's process.

REFERENCES

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