THE INWARD SPIRAL METHOD:
An Improved TIN Generation Technique and Data Structure for Land Planning Applications

David G. McKenna
Foundation Land Company
4176 Burns Road
Palm Beach Gardens, Florida 33410

ABSTRACT

This paper introduces the concept of the Triangulated Irregular Network (TIN) for computer representation of topographic surfaces. Discussion focuses on the TIN's benefits for interactive topographic modeling and site design applications. The paper then presents an alternative method of TIN generation termed the Inward Spiral Method. This method of TIN generation represents an improvement over previous methods by maintaining the integrity of site boundary edges and by automatically augmenting sparse or widely disparate data sets. The paper concludes with a discussion of the method's data structure and a sampling of its potential applications.

INTRODUCTION

The triangulated irregular network (TIN) is a topological data structure used to represent three-dimensional topographic surfaces. The TIN was developed out of the desire of geographers and cartographers for a more accurate and efficient means of collecting and storing topographic data in a digital format. A TIN may be visualized as a set of triangles which connect surface data points in a continuous coverage of irregularly-shaped triangular facets. (Figure 1)

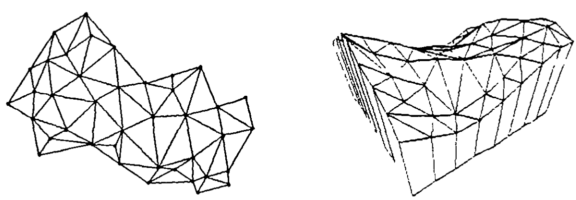


Figure 1 TIN representation of a topographic surface

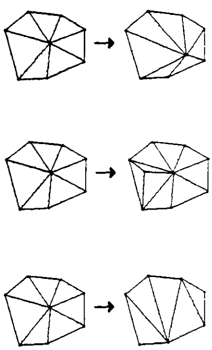
TIN representations of topographic surfaces have several advantages over more commonly used grid representations. Mark (1975) and Peucker, et al (1976) have convincingly demonstrated that TIN systems result in a more accurate surface representation with far less storage, and McCullagh and Ross (1980) have shown that the generation of TIN surfaces can be accomplished much faster than the generation of gridded surfaces.

Grid systems do not permit vertical surfaces, and irregular boundaries or interior holes in a surface area are difficult to define with a grid. TIN systems, on the other hand, can describe nearly any surface, including those with holes, irregular boundaries, or vertical surfaces. In addition, the resolution of a gridded surface representation is limited to the resolution of the superimposed grid, while a TIN representation is limited only to the resolution of the original data.

THE TIN AS AN INTERACTIVE SITE DEVELOPMENT TOOL

These benefits have been exploited for a variety of applications. The TIN is now commonly utilized by automated survey systems, contour map generation software, earthwork calculation software and geographic information systems. Most of these applications employ the TIN purely as an internal data structure - that is, they utilize the TIN as a structure for storing and retrieving topographic data. But perhaps the most exciting advantage of the TIN, and until now the one most underutilized, is the TIN's tremendous potential as an interactive site design and development tool.

The irregular structure of the TIN is well suited to interactive design applications because it allows a surface to be freely manipulated and edited. Surface points, for example, can be moved in any direction without affecting the data structure of the original surface. Points can be added to or deleted from a TIN and the change accommodated by a simple, local triangulation of the altered triangles. (Figure 2)



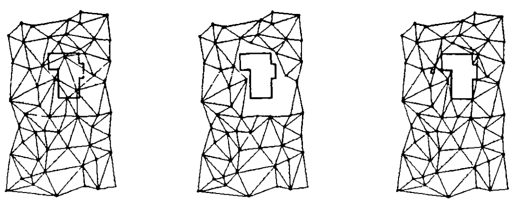


Figure 2

Points moved, added, and deleted. These basic operations can be combined into powerful interactive site design capabilities - for example, fitting a building footprint onto the topographic surface.

These advantages are being recognized by developers of three-dimensional site modeling systems. Several turnkey CAD vendors, for example, now offer the TIN as their topographic data structure. The use of the TIN in microcomputer-based CAD and engineering systems is also growing. The Inward Spiral Method of TIN generation described here is a component of one such microcomputer-based system.

METHODS OF TIN GENERATION

There are several methods of generating a triangulated irregular network from a set of data points. The process of each method is essentially a problem of "connect the dots;" that is, determining the connections between data points that will yield the best overall triangulation. The "best" triangulation is that which most accurately describes the surface being represented. In practice, this has proven to be the triangulation in which the triangles are most equilateral in shape.

Triangulation methods fall into two general categories. Methods of the first category actually involve two steps. The first step generates an initial, arbitrary network of triangles from the surface data points, while the second step refines the network by optimizing triangle shapes. Examples of this category can be found in the work of Gold, et al (1977) and Mirante, et al (1982).

The second general category of triangulation methods consists of methods which seek to generate the optimal triangulated network in a single step. These methods exploit the geometric principles underlying the organization of a TIN and produce what has come to be called the Delaunay tesselation (or triangulation) of a set of data points. Examples of these methods can be found in Brassel and Rief (1979), McLain (1976), McCullagh and Ross (1980) and Tarvydas (1983).

All these methods of TIN generation produce suitable triangulated networks but have several disadvantages. Some require that data be input manually, thus diminishing some of the economic benefits of automation. Some methods do not order data points or resultant triangles in an efficient and flexible manner, making interactive edit operations difficult. Some methods employ localized search procedures which, in extremely sparse or disparate data sets, can produce overlapping triangles. Most notably, all the reviewed methods typically encounter problems at concave boundary edges. Triangles may be generated outside the boundary, or triangle edges may intersect boundary lines. These problems can be eliminated only with substantial difficulty and loss of overall efficiency.

THE INWARD SPIRAL METHOD FOR TIN GENERATION

The Inward Spiral Method for generating triangulated irregular networks utilizes many concepts of previous methods. It introduces several enhancements to existing methods and is a superior method for certain applications. The method was developed to fit the requirements of SCHEMA, a three-dimensional modeling system being developed at the Harvard Laboratory for Computer Graphics and Spatial Analysis. One of the system's primary applications is the modeling of urban areas, and the special requirements involved with this application dictated that a different triangulation method be devised.

SCHEMA structures an urban model around the street pattern of the city. Blocks enclosed by streets define the surface areas to be triangulated, and the streets themselves constitute the boundaries of the triangulated areas. It was of paramount importance, therefore, to maintain the integrity of the street edges. No triangle could be generated outside the street boundary, and no triangle edge could intersect a street edge. This constraint led to the development of the Inward Spiral Method.

The Inward Spiral Method generates an optimal triangulated network in a single iteration, it maintains the integrity of boundary edges and it minimizes the possibility of overlapping triangles by automatically augmenting sparse or widely disparate data sets.

The heart of any TIN generation method is the algorithm which determines the points to connect to form an optimal triangle. The Inward Spiral Method uses the algorithm devised by McLain (1976). The McLain algorithm operates by assigning two points as endpoints of a triangle edge, examining neighboring data points and applying Euclidean geometry to determine the point which, when connected to the assigned edge, defines the optimal triangle.

The efficiency of this method depends on the efficiency with which it can determine the best point for the creation of a new triangle. Obviously, if every point on the surface area were examined for each new triangle, the efficiency would be considerably diminished. The Inward Spiral Method addresses this problem by superimposing a rectangular grid over the data set (Tarvydas, 1983). Data points are sorted into rows and columns within the grid. The search for the point defining the optimal triangle, then, is limited to those data points in adjacent grid cells.

After a complete data set has been input, the boundary edges of the surface are tested against the cell size of the superimposed grid. If edges are longer than the grid cell dimension, additional data points are inserted along the boundaries such that the distance between all boundary points is less than the grid cell size. This process eliminates the long, narrow triangles which typically arise at boundary edges and greatly diminshes the possibility of an error occurring at a boundary edge.

The data is then sorted into rows and columns within the superimposed grid. Each cell of this grid is then examined in turn. When a grid cell within the boundary of the surface area is found to be empty, a new data point is generated within that cell and its elevation determined by a distance-weighted averaging of nearby data points. This procedure ensures that every grid cell within the boundaries of the surface area will contain at least one point, thus minimizing the possibility of the generation of overlapping triangles and enhancing the aesthetic appearance of the surface.

The data are now ready for triangulation. The first boundary edge is chosen as a starting baseline, and the McLain algorithm examines data points to the inside of the boundary edge. The point selected to be the best point for a triangulation from that edge is tested to determine whether a triangle drawn to that point will intersect any boundary edge. If the test shows that an intersection will occur, the point is flagged and the next best point tested. If no intersection occurs, the triangle is drawn and added to a list of triangles in the data structure. The process is repeated for each consecutive boundary edge.

When the boundary triangulation is completed, the method looks to the triangles thus formed and establishes new triangles on the interior edges of previous ones. This procedure is repeated for each triangle, working from the lowest numbered triangle to the highest numbered triangle, until no new triangles can be created. By this sequence of triangulation, the method traverses the data points in a spiral pattern moving inward from the boundary edges. (Figure 3)

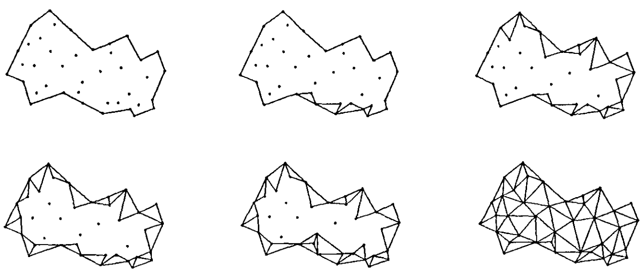


Figure 3 The Inward Spiral Method

The inward spiral pattern and the tests associated with the boundary triangulation ensure the integrity of the boundary with minimum effort, and thus the method represents an improvement over other methods in applications where maintenance of the boundary is essential. The automatic insertion of data points significantly improves the method's reliability and enhances the aesthetic quality of the resulting surface.

DATA STRUCTURES

There are two data structures that have been used in the generation of triangulated irregular networks. The first data structure regards the triangles themselves as the primary entities. Each triangle is an element in the data structure and is defined by pointers to the triangle's three vertices. The data structure also maintains pointers to each triangle's three adjacent triangles. (Figure 4) This is the more widely used of the two data structures and can be found in the work of McLain (1976), Gold, et al (1977), McCullagh and Ross (1980), Mirante, et al (1982), Tarvydas (1983) and others.

The second data structure regards the vertices of triangles as the primary entities. The network is defined by allowing each vertex to point to a list of the vertices connected to it by triangle edges. The list is sorted in clockwise or counter-clockwise order around the center vertex, starting at "north" in the local Cartesean coordinate system. (Figure 5) This data structure requires about one-half the storage of the first method. The structure is attributable to Peucker and Chrisman (1975) and can be found in the work of Fowler (1976).

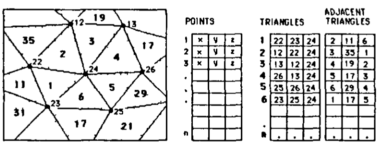


Figure 4 TRIANGLES data structure

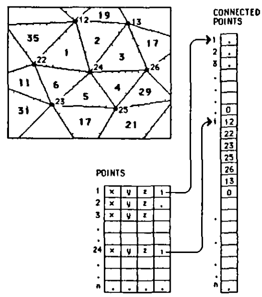


Figure 5 CONNECTED POINTS data structure

Each data structure has its own advantages and limitations, and each is suitable for different applications. A list of triangles, for example, is essential for efficient display functions such as hidden surface removal, surface shading, or fractual surface texturing. A list of connected points, on the other hand, is useful for a variety of editing operations such as "rubber-banding" of a modified surface, data point insertion and deletion or analytic functions, such as volume calculations or contour cutting. The Inward Spiral Method combines elements of each data structure into a dynamic, two-tiered data structure that is useful for all these applications. (Figure 6)

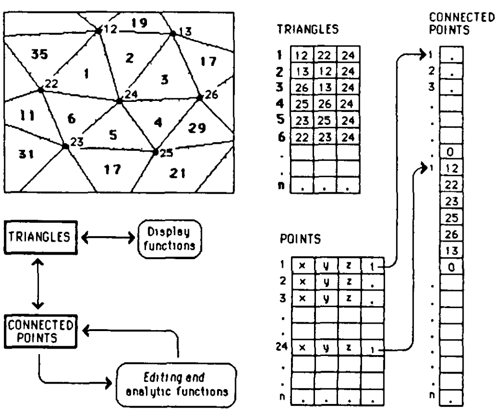


Figure 6 Inward Spiral Method data structure

An array (POINTS) contains the x, y and z coordinates of each data point. As triangles are generated in the triangulation process, the indices of the points defining each triangle are placed in the TRIANGLES list. This is similar to the first data structure described, but it is different in that the records of adjacent triangles are not kept. Instead, the list of triangles is used to generate the data structure of connected points (CONNECTED POINTS). A fourth element is added to the POINTS array which indicates the position in the CONNECTED POINTS list to which each data point refers. Editing and analytic operations are performed within the CONNECTED POINTS structure. If editing operations result in triangle changes, CONNECTED POINTS is transformed to a new, updated TRIANGLES list which is then used for display operations.

This combined data structure takes advantage of each of its component structures' best potential. It results in a system that is capable of both efficient interactive editing and sophisticated display. Although the data structure contains redundant information, it is of comparable size to the most commonly used data structure of triangles and adjacent triangles.

APPLICATIONS

One of the applications for which a triangulated irregular network is especially useful is interactive three-dimensional modeling. The irregular nature of a TIN allows it to be freely edited and manipulated, and the data structure of the Inward Spiral Method improves the efficiency of such operations.

One of the basic editing operations is the ability to move or "drag" a point along the topographic surface, with the edges connected to the point "rubber-banding" to the new point location. To accomplish this, a point is indicated with a cursor device. The search for the desired point involves only those points contained in the grid cell in which the cursor was activated and is, therefore, extremely fast. As the point is moved, the data structure of connected points instantly indicates the lines to be redrawn to create the rubber-banding effect.

Another basic operation involves adding new data points to an existing surface. This is done by determining the triangle which circumscribes the new point and connecting the point to the vertices of the circumscribing triangle. Mirante, et al (1982) describes a method for determining the triangle containing a point which employs extensive use of matrix algebra. The data structure of connected points, however, allows this function to be reduced to a few simple arithmetic operations (McKenna, 1985).

Data point deletion can also be performed very efficiently within the connected points data structure. The selected point is simply removed from the POINTS array and its associated points removed from CONNECTED POINTS.

The resulting polygon "hole" is then treated as the boundary of a surface with no interior data points. A boundary triangulation is performed and CONNECTED POINTS is updated to reflect the change.

The connected points structure is also useful for calculating the volume under the surface area. The volume under any single triangle is found by multiplying the surface area of the triangle by the average height of its three vertices. Computing the volume under an entire TIN in this fashion would require the computation of the area of each triangle and the average height of each triangle's three vertices. This would involve many separate area calculations, and since any vertex is shared by several triangles, the height of each vertex would be considered several times.

An alternative method of volume calculation takes advantage of the connected points structure. The number of points connected to a given point (i.e., the number of triangles which share that vertex) is easily determined. The volume under the entire surface, therefore, can be reduced to averaging the height of each data point by the number of points connected to it, summing the average height of all data points and multiplying the sum by the total area of the surface. This method represents a considerable savings, as each data point is considered only once, and no additional area calculations need be conducted.

A common application of the TIN is the automatic generation of contour maps. A contouring method which uses the connected points data structure is described by Peucker and Chrisman (1975). Other methods and discussions of this application can be found in most of the literature related to TINs.

TIN DISPLAY

Any computer-aided design system must be capable of fast, sophisticated three-dimensional display operations in order to be a truly useful design tool. The Inward Spiral Method employs the data structure of triangles for display operations. Triangles can be thought of as separate surface facets which combine to form the overall topographic surface. In this way, the TIN can be processed by many of the display operations commonly used in three-dimensional modeling systems such as hidden surface removal, surface shading or fracted surface texturing. Moreover, because a TIN is a continuous, connected surface, many of these operations can be simplified for TIN displays.

Hidden surface removal on raster display devices, for example, can be accomplished by a simple "back-to-front" display of the triangular facets. This process can be made more efficient by maintaining the values of the vectors normal to each triangle. Triangles whose normal vectors point away from the direction of view (i.e., those triangles which face away from the viewer) would not be seen from that viewpoint, and these triangles are never considered. The normal vectors can also be used for applying cosine shading to the surface (Figure 7) or for calculating slope or solar aspect of the surface.

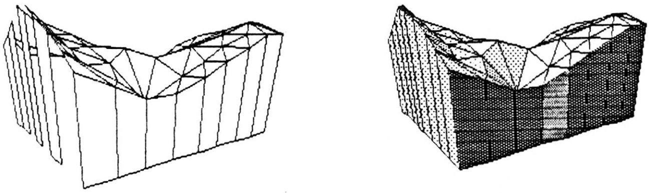


Figure 7 A TIN surface in perspective, with and without surface shading

CONCLUSION

The triangulated irregular network has been shown to be a superior system for topographic surface modeling. TIN systems execute faster than grid systems and produce more accurate surface representations with far less storage. In addition, TIN representations can be freely edited and manipulated and thus provide significantly greater potential for interactive surface modeling and site design.

The Inward Spiral Method for the generation of triangulated irregular networks is an improvement over other generation methods for certain applications. The method produces the most optimal triangulation in a single iteration. It maintains the integrity of boundary edges with minimum effort. And, by automatically augmenting sparse or widely disparate data sets, the Inward Spiral Method minimizes program error and enhances the aesthetic quality of the resulting triangulated network.

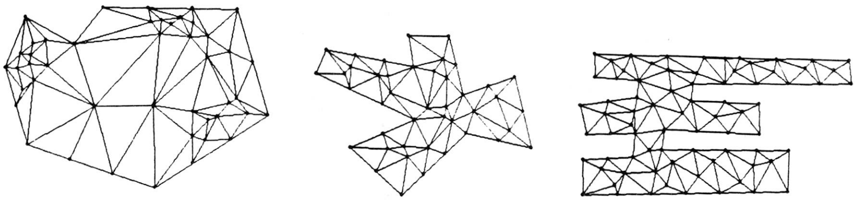


Figure 8 Complex surfaces successfully triangulated with the Inward Spiral Method

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