THE PSI-S PLOT: A USEFUL REPRESENTATION FOR DIGITAL CARTOGRAPHIC LINES

Michael P. O'Neill*
Department of Geography
Virginia Polytechnical Institute and State University
Blacksburg, Virginia

and

David M. Mark*
Department of Geography
State University of New York at Buffalo
Buffalo, New York 14260

ABSTRACT

The Psi-s plot represents the geometry of a line by determining Psi, the orientation angle or heading, at points along the line, and plotting that value against s, the cumulative curvilinear distance along the line. No matter how convoluted the line is, the Psi-s plot will be a single-valued function of s; it is much easier to parameterize such a single-valued function. Straight lines on the Psi-s plot represent either straight lines or arcs of circles in x-y space. The Psi-s representation can be used to characterize the shapes of irregular polygons. It also has been used in analyses of river meander planform, and in the automated detection of contour crenulations. The Psi-s plot should be valuable in the generalization of digital cartographic lines; it should have an advantage over standard methods for representing geographic lines which include substantial proportions of circular arcs or straight lines. Circular arcs and straight lines are common components of rivers, roads, and railroads. The technique would appear to have great potential for feature recognition and shape characterization of digital cartographic lines.

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* Electronic mail (BITNET) addresses for authors: ONEILLMP@VTVM1, GEODMM@UBVMS

INTRODUCTION

In any application of computers, the choice of an appropriate **representation** for the phenomenon being studied is crucial to algorithm design and in fact to the definition of problems themselves. Winston (1984, p. 21) defined two key concepts in artificial intelligence:

> "A **representation** is a set of conventions about how to describe a class of things. A **description** makes use of the conventions of a representation to describe some particular thing."

Winston then went on to emphasize the role of an appropriate representation in problem-solving. There has, however, been little attention in computer cartography to representations of geographic and/or cartographic lines. With few exceptions, cartographers have adopted the representation of a digital line as an ordered set of two or more coordinate pairs (a **chain**, or **polyline** representation).

One apparent exception to this lack of explicit attention to representation is Poiker's **Theory of the Cartographic Line** (Peucker, 1975), which attaches an explicit "width" parameter (W) to a line, and then represents the line as a set of "bands" or rectangular boxes, with widths no greater than W, which enclose the line. In fact, this "theory" was developed **a posteriori** to describe the model underlying the so-called "Douglas-Peucker" line reduction algorithm (Ramer, 1972; Douglas and Peucker, 1973) and related line-handling algorithms.

Recently, Buttonfield (1985) reviewed representations of cartographic lines and their variability. In addition to an overview of Poiker's model and the fractal model of line variability, Buttonfield briefly discussed a parameterization of a cartographic line, in which the x-and y-coordinates of successive points along the line are plotted against s, the accummulated distance along the line (Buttonfield, 1985, pp. 3-4). Although this approach simplifies the line, it produces two curves. Furthermore, equations fitted to either of these curves have no clear geometric interpretations.

In this paper, we present an alternative parameterization of a cartographic line. This **Psi-s curve** is an effective representation for cartographic lines, because straightforward geometric interpretations of the form of the transformed line are possible. The Psi-s curve appears to have potential both for line generalization and for pattern recognition in cartographic lines.

THE PSI-S PLOT

The Psi-s plot represents the geometry of a line by determining Psi, the orientation angle or heading, at points along the line, and plotting that value against s, the cumulative curvilinear distance along the line (see Figures

1 to 4, below). One advantage of this transformation is that, no matter how convoluted the line is, the Psi-s plot will be a single-valued function of s; it is much easier to parameterize such a single-valued function. Another advantage is that the shape of the Psi-s plot has a straight-forward geometric interpretation. Straight lines on the Psi-s plot represent arcs of circles in x-y space; in fact, if Psi is measured in radians, the radius of curvature of the arc is exactly the inverse of the slope of the Psi-s curve. Straight lines in x-y space can be considered to be arcs of circles of infinite radius, and appear on the Psi-s plot as horizontal straight lines (zero slope).

The Psi-s plot has been used in computational vision as an aid in characterizing the shapes of irregular polygons representing the outlines of machine parts (cf. Ballard and Brown, 1982). However, it has a longer history of use in geomorphology. Speight (1965) applied this transformation to river meanders when conducting a power spectral analysis of meander form. Then, Langbein and Leopold (1966) discussed this representation for meanders, pointing out that straight lines in Psi-s space represent circular arcs in x-y space. Brice (1974a, 1974b) extended this approach, claiming that a 'circles-and-straights' geometry (straight lines on the Psi-s curve) is more closely characteristic of meanders than are trigonometric functions (sine-waves in Psi-s). More recently, Mark (1986) applied the method to the automated detection of contour crenulations, and O'Neill (1987) extended the use of Psi-s plots for characterizing the planform geometry of river meanders.

PSI-S PLOTS OF GEOGRAPHIC LINES: SOME GENERAL EXAMPLES

The concept of the Psi-s plot and its relation to geometry in geographic (x-y) space can be made clear through the presentation of examples. In this section, we present maps and Psi-s plots of segments of three geographical lines: a meandering river (Figure 1), a winding mountain highway (Figure 2), and a contour line (Figure 3).

A Meandering River. The Hay River in northern Alberta provides an good example of a wandering stream with "free" (i.e., relatively unconstrained) meanders. The segment of the Hay River going downstream from X to Y (Figure 1, above) is represented on the Psi-s plot (Figure 1, below). Letters show the correspondence between bends or straight reaches on the river and straight line segments on the Psi-s plot. Note that sharp bend (such as the one at "g" on the Hay River) plot as very steep segments on the Psi-s plot, whereas more open bends (eg., "c") have lower slopes. Bend "c" in fact appears to be a compound curve, with a more open middle component (lower slope segment on the Psi-s plot) between sharper entrances and exits from the bend. There are no straight segments in this reach.

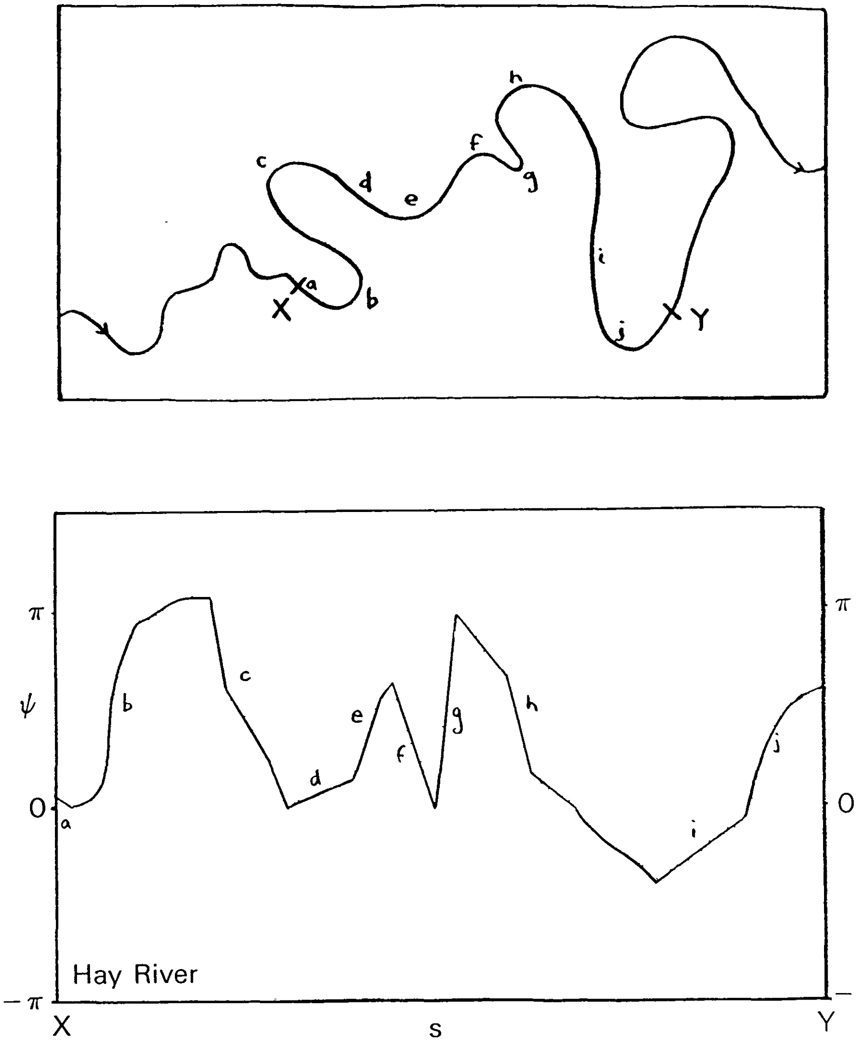
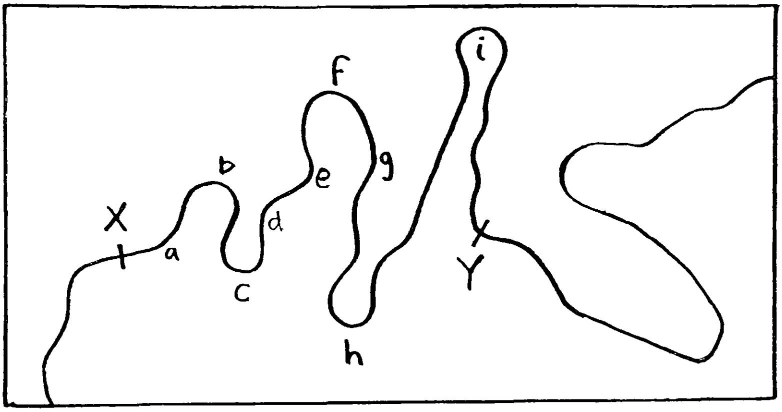


Figure 1: A portion of Hay River, Alberta (above) and its Psi-s plot (below). Equivalent parts of the two plots are marked with lower case letters. Straight lines on the Psi-s plot represent straight lines if horizontal, and circular arcs otherwise.

<u>A Mountain Highway</u>. We noted above that roads were another type of geographic line which should be well-suited to the Psi-s representation. As in the case of rivers, roads commonly are composed of relatively straight segments

joined by approximately-circular bends. Here, there a few straight segments, but most of the highway is composed of curves. Although the two lines are very similar in x-y space (cf. Mark, 1985, Figure 2 E and K, p. 49), there is a fairly distinct difference in the Psi-s plots. Note that the geographic length of a segment is its extent along the s axis; thus a visual examination of the Psi-s plot tends to exaggerate the importance of bends.



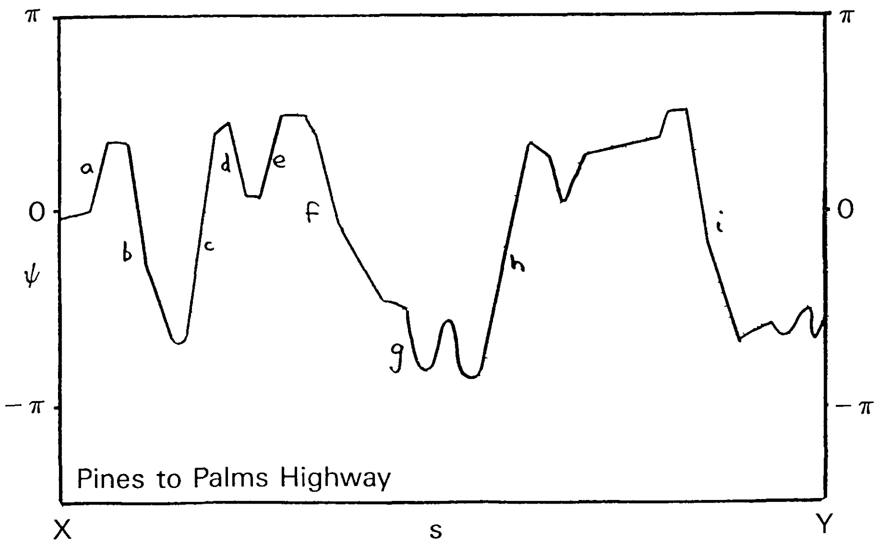


Figure 2: A portion of the Pines to Palms Highway, southwest of Palm Springs, California (above) and its Psi-s plot (below).

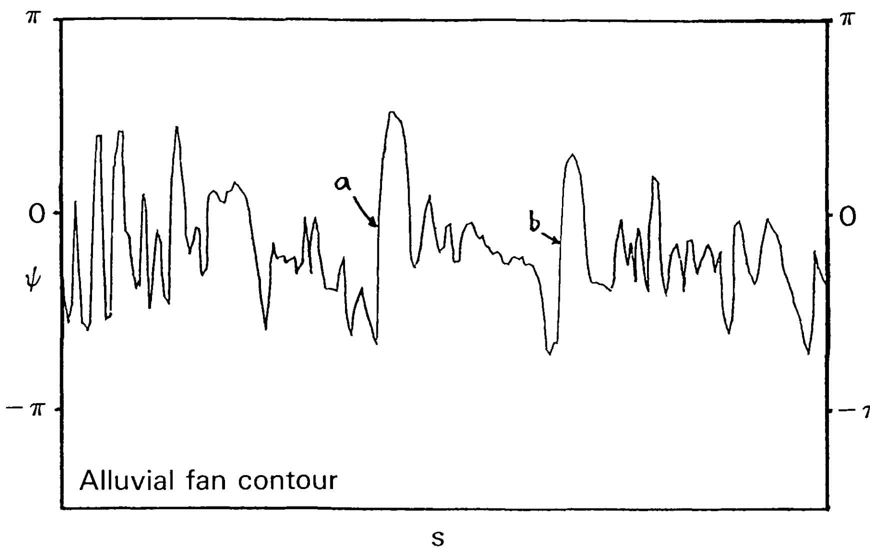


Figure 3: Psi-s plot of a portion of the 4800 foot contour on an alluvial fan in Arizona. Two contour crenulations (see text below) are marked at "a" and "b".

A Contour Line. Contours and shorelines are geographical lines which are not usually composed of circular arcs. (Exceptions might be found in cirques and other glacial landforms, or along spits.) Thus, it probably will take as many straight line segments to represent the contour on the Psi-s plot as it would on the x-y map (for an equivalent level of generalization). This impression is support by the irregular appearance of the Psi-s plot, which is presented at about the same scale as the other lines.

Whereas the Psi-s transformation may not reduce storage requirements in this case, it can be very valuable in the identification of features on contour lines (Mark, 1986). This will be discussed in more detail below.

THE PSI-S PLOT AND RIVER MEANDER RESEARCH

In their classic theoretical paper on river meander form, Langbein and Leopold (1966) developed a solution to the path of a river which, while fitting a total channel length L into a straight line distance $D < L$, minimized the variance of angular change along the path. Their probability analysis was most easily developed when stream azimuth was expressed as a function of distance along the channel (in other words, on a Psi-s plot). They found that the angle-change variance was minimized when the plot was a sine function (in Psi-s space), corresponding with what they called a "sine-generated curve" in cartesian coordinates (Langbein and Leopold, 1966, p. H3).

Langbein and Leopold went on to note that sine-generated curves have "a relatively uniform radius of curvature in the bend portion" (Langbein and Leopold, p. H3), which corresponds with "the fact that a sine curve has quite a straight segment as it crosses the x-axis" (Langbein and Leopold, p. H3). This straight segment represents about 1/3 of the meander length. Also, they noted that the slope of the Psi-s curve "is the reciprocal of the local radius of curvature of the meander" (Langbein and Leopold, p. H3). They constructed Psi-s plots of several natural rivers and one meander trace from a flume study, and then fitted sine-curves to portions of these, apparently 'by eye'. These curves fit rather well, but it should be noted that fits were over just one or two bends, and the reaches selected were mostly 'well-known' meanders. Langbein and Leopold did not report fitting straight lines to the Psi-s plots.

Brice (1974a, p. 582) credited Langbein and Leopold (1966) with having devised the technique of plotting azimuth (Psi) against distance as measured along the stream (s). However, Brice himself clarified the relation between planform geometry and straight segments of the Psi-s plot, stating that "segments of the resulting plot, which have a uniform slope, represent arcs of uniform curvature" (Brice, 1974a, p. 582). Brice claimed that arcs of constant curvature are more common in natural channels than the 1/3 proportion suggested by Leopold and Langbein (1966, p. H4) for equilibrium channels. Brice fitted line segments to portions of the Psi-s plot "by eye" (p. 586) to identify segments of constant curvature. In another paper published the same year, Brice (1974b) restated the description of his method, and emphasized the fact that horizontal segments of the Psi-s curve represent straight lines in cartesian space (Brice, 1974b, p. 185).

PSI-S PLOTS AND THE ANALYSIS OF DIGITAL CARTOGRAPHIC LINES

One major advantage of using a Psi-s representation is that when digitized points are equally-spaced along the cartographic line (spacing = ds), the line is completely characterized by a vector of direction (Psi) values. Clearly, the storage of a cartographic line is greatly reduced if direction (Psi) measurements are stored as a vector with known spacing of s rather than a paired set of x-y coordinates. Additionally, techniques of line generalization can be applied to the Psi-s representation to further reduce storage requirements of the line.

Line Generalization Using the Psi-s Curve.

O'Neill (1987) used the Psi-s representation in a new approach to cartographic line generalization. Just as a complicated cartographic line can be approximated (to some specified accuracy) by a series of straight line segments in the plane (Douglas and Peucker, 1973), so can such a line be approximated by straight line segments on the Psi-s plot. Note that this is equivalent to approximating the line by a sequence of straight lines and circular arcs in cartesian space. A slightly modified version of the Douglas-Poiker line generalization algorithm (Douglas and

Peucker, 1973) can be used here; the modification is needed because the axes of the Psi-s plot are not dimensionally homogeneous.

One solution is to measure deviations from a straight line in a direction parallel to the Psi-axis. If all points over some range of s lie within the specified tolerance, then the segment can be represented as a circular arc; otherwise, the point of maximum deviation is found, and the test is applied recursively to the two halves of the segment.

Psi-s Curves and Feature Identification for Cartographic Lines.
Recently, Mark (1986) discussed how the Psi-s representation can be used to identify contour crenulations in digitized versions of contour lines on pediments and alluvial fans. In the Psi-s representation, a contour crenulation appears as an abrupt change of almost 90

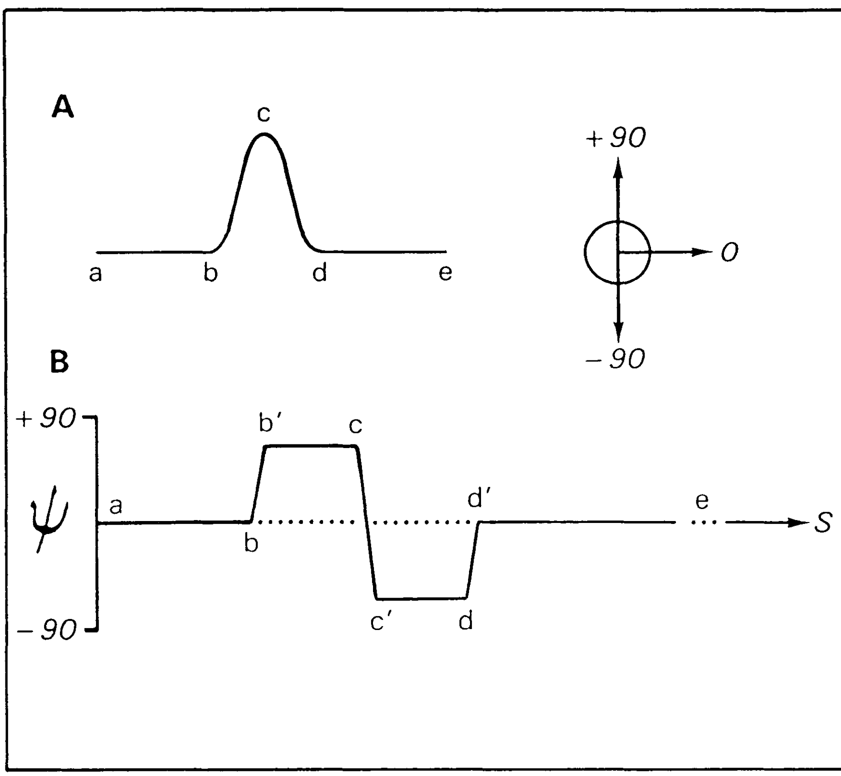


Figure 4: A portion of a contour crenulation (A) and the associated Psi-s plot (B). The place where the contour crosses the channel ("c") has the largest angular change on the Psi-s plot. [after Mark, 1986, p. 231].

degrees (pi) as the contour enters the small valley in which the channel lies ("b" in Figure 4), a change of almost 180 degrees (2 pi) in the opposite direction as the contour crosses the channel ("c" in Figure 4), and finally another 90-degree (pi) turn as the contour leaves the valley ("d" in Figure 4). The recognition of these features is facilitated through the calculation of the first differences of this series (departures from a straight line at each polyline vertex). Note that the near 180-degree change uniquely identifies the down-slope direction in this environment. Line segments "a" and "b" in Figure 3 represent two clearly-marked 180-degree direction changes at contour crenulations.

SUMMARY

The Psi-s curve has already proven to be a powerful representation for the analysis of river meander planform. It also has considerable potential in automated classification and feature recognition for digital cartographic lines. The use of the Psi-s curve should provide more compact yet effective generalizations of certain cartographic lines than can geometric generalizations in x-y space. Any geographic lines which include substantial proportions of circular arcs should be more effectively handled in the Psi-s representation, whereas the representation should be of little or no advantage for irregular lines (those which resemble fractals). Circular arcs are common components of rivers, roads, and railroads. Transformation to a Psi-s plot may not be worthwhile for coastlines and contours, unless feature recognition or shape characterization is the objective.

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