A SPATIAL LOW-PASS FILTER
WORKING FOR TRIANGULAR IRREGULAR NETWORK (TIN)
AND RESTRICTED BY BREAK LINES

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ABSTRACT

Spatial low-pass filters are widely used for smoothing 3-D surfaces. However they only work on raster data structure. A new approach here presents a spatial low-pass filter which works directly on triangular irregular network (TIN) structure. This approach has two goals. The first goal is reducing data structure conversion. It avoids a conversion between TIN and raster for smoothing. The second goal is to restrict filtering by break lines. Comparing with conventional direction-homogenous smooth procedures, this filter provides a capability to limite filtering region.

INTRODUCTION

Spatial filtering

A spatial object can be converted from spatial domain to frequency domain, and vice versa. For an one-dimensional object f(x), we have

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \text{COS } nx + b_n \text{SIN } nx)$$

where a and b are Fourier factors.

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \text{ COS } nx \, dx \qquad (n = 0,1,2,...)$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \text{ SIN } nx \, dx \qquad (n = 1,2,...)$$

This formula shows us that a spatial object can be represented as a sum of a series items in frequecy domain. Each item corresponds a component with one spatial wavelength. These items are ordered from long wavelength to short wavelength in the series. Also, this formula shows that the whole series of components in frequency domain can be reversed to the original object in spatial domain.

Theoretically, information has not been lost during a conversion in either direction, if both conversions carry complete components, i.e. all items. However, if we only convert part of items from frequecy domain to spatial domain, the restoring object is also an incompleted object. Two properties should emphasis on the incomplete conversion. First, the more items are counted to convert, the more precise object can be restored. Second, ignoring different part of components generated various effects. All kinds of filters use the phenomena.

In frequency domain, details of a spatial object are represented by short wavelength components, while large features are represented by long wavelength components. If we only convert long wavelength components from frequecy domain to spatial domain, then only those features with the long wavelengths are restored in spatial domain, while all details of the spatial object are eliminated. The procedure is called low-pass filter. The effect is smoothing.

There are two purposes for user to use low-pass spatial filtering (smoothing) in applications. The first is generalization. It means that we only use a limited number of items from the beginning of the series in frequency domain to instead of all items of the whole series. The restored spatial object from the part of items is similar to the original one in overall, except those very details. The second is noise (e.g. random errors) removing. It is useful when we are aware that some noises (e.g. random errors) exist in original data. Usually noises have shorter wavelengths. They only damage the accuracy of last items in frequency domain. If we select a wavelength threshold longer than the wavelength of all noises, those noises can be deleted from the original data after a low-pass filtering.

In many applications, user needs different degree of smoothing. This option can be obtained for user by selecting a wavelength as a threshold. It corresponds that user determines how many items are counted in frequency domain. Any detail with wavelength shorter than the threshold in frequency domain is erased. Only those features with longer wavelength in frequency domain are kept and restored in spatial domain.

In this way, user can control how strongly they want to smooth the surface. If a very short wavelength is selected, little smoothing is done, result is very similar to the original surface, except very details disappeared. In an extreme case, when the wavelength is zero, the filtered result is identical to the original data. If select a wavelength threshold longer than the dimension of the research area, result is just a flat surface with average elevation.

All concepts from previous discussion are also good for two and three dimension situations, although their formulas are not so simple.

Algorithm of filters depends on data structure

There are many well-known algorithms developed for implementing spatial filtering, such as Fast Fourier Transform (FFT), Fast Walsh Transform (FWT), etc. However they only work for raster data structure. This limitation causes conversion between different data structures, because once a smooth operation is necessary, the data must be converted to raster data structure.

A spatial filter on polygons was proposed in 1986 by author. It directly works on vector data structure. More particularly, a spatial filter presenting in this paper directly works on TIN structure.

Break lines and filtering

The feature of using break lines is a capability of local control. A spatial filter working with break lines has two properties. It can recognize existence of break lines, and it can be effected by break lines. Thus, for example, a lake surface can be kept flat while the new filter is executed around it because the boundary of the lake, as a break line, can prohibit the smoothing effect into the lake region.

A LOW-PASS SPATIAL FILTER ON TIN

Algorithm

The algorithm of the filter proposed here is similar in concept to an algorithm for raster structure. For a point (X0,Y0) on the surface of a TIN, its filtered elevation ZF (X0,Y0) is an average of elevations of its neighbor points.

$$ZF\ (X0,Y0) = \sum_{i=1}^{N} Z_i \,/\, N \qquad (i = 1, \ldots, N)$$

Where Z_i is the elevation of the ith neighbor point, N is the amount of points counting in as neighbor. These counted neighbor points should be in a nearby range, because if two points are separated far away, they have little effect on each other. This range can be a horizontal circle whose center is at the point and its radius is given by user's option. The radius is actually the threshold wavelength of the filter. The larger the radius is, the more details are erased.

$$ZF\ (X0,Y0) = \iint_C Z(x,y)\ dx\ dy \,/\, \pi R$$

$$\text{where } (x - X0)^2 + (y - Y0)^2 <= R^2$$

here R is the radius of the circle C.

Differently from raster data struture, the sum of elevations of neighbor points is essentially the volume of the range, because TIN represents a continuous surface.

$$d\ V(x, y) = Z(x, y)\ dx\ dy$$

Thus the formula can be represented as:

$$ZF\ (X0,Y0) = \iint_C d\ V(x,y) \,/\, \pi R$$

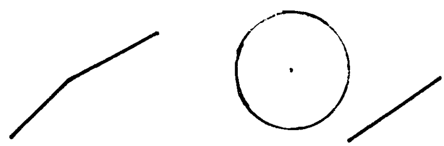
$$\text{where } (x - X0)^2 + (y - Y0)^2 <= R^2$$

This formula provides a basic calculation of filtered value for any point on the TIN surface. Then a filtered surface can be produced from the input surface.

RESTRICT FILTER OPERATION BY BREAK LINES

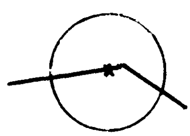
Break lines are linear feature with more precise data. A terrain surface should exactly follow break lines wherever they exist. Filtering should not effect the break lines. The break lines should have a power to restrict filtering. More disscussion about break lines is in reference (Chen, 1988).

When we use low-pass filter, for those points that none of break lines passes in its neighbor circle range, the filter result should not been influenced by any break lines. The result point value is exactly generated by the previous calculation without any consideration of break lines.

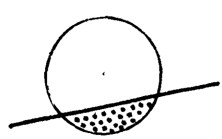


When a break line passes through its neighbor circle range, two major cases are in consideration.

First case, if the break line exactly passes the center point, the point should have an identical elevation value of the point on the break line, according to the fact that break lines are more precise. Thus, wherever the break line exists, we use break line elevations replace old values along each break line.



If a break line does not pass through the center point, but cuts a part of the neighbor circle range, as shown on the following picture, the effects from the shaded area should not be counted to influence the filtered value of the center point. In this way, the filter effects do not pass over any break line. A break line prohibits filtering from one side to another side.



EXPERIMENTS

The test area is in Misato, Japan. Original data are digitized from a contour map. Only the reservoir and the dam have more precise engineering data. They are collected as break lines.

Figure 1 shows a 3-D view of the terrain surface after a smoothing procedure. This surface is directly filtered from TIN structure. The smoothing threshold wavelength is 50 meters. We can see that the smoothing procedure influences the whole area including the reservoir. For example, the dam is too fuzzy to see.

Figure 2 shows a map of break lines at the area. These break lines are reservoir boundary and dam edges, etc.

Figure 3 shows a 3-D view at the same area, but is a result of smoothing procedure restricted by break lines. The difference of existence of the dam between two views is obvious.

CONCLUSIONS

Two goals of the proposed spatial filter are reached. A TIN surface can be filtered in TIN structure. Also, break lines can restrict the filter procedure. Further observation will be on its performance.

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Figure I. A 3-D view of filtered terrain surface in Misato, Japan
(Using the Low-pass filter with a threshold wavelength 50 meter)

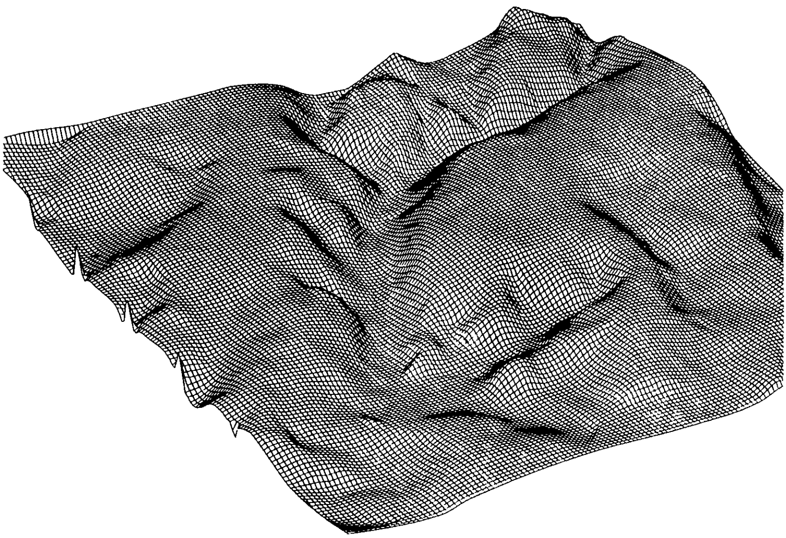

Figure II. A map of break lines in Misato reservoir area.

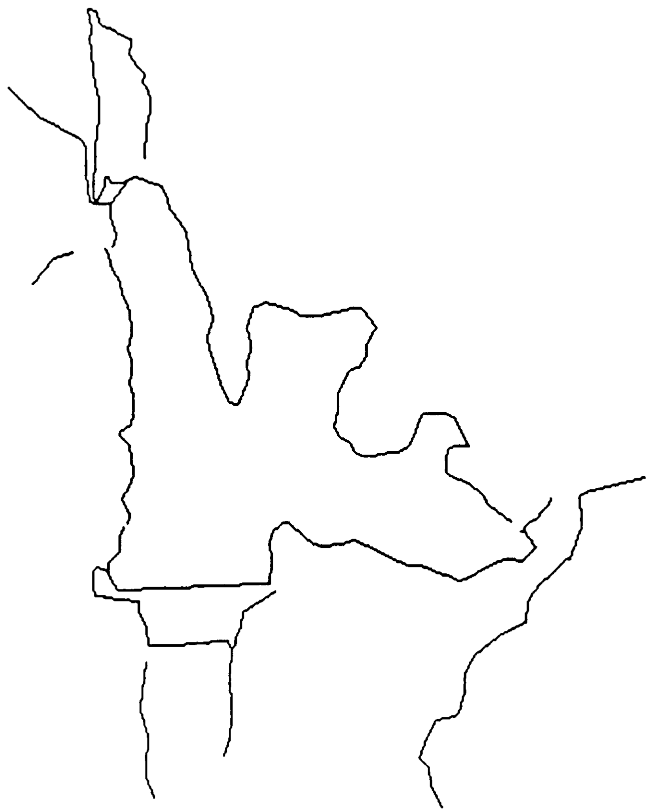

Figure III. A 3-D view of filtered terrain surface in Misato, Japan
but effected by break line on Figure II.
(Using the Low-pass filter with a threshold wavelength 50 meter)