MULTISCALE DATA MODELS FOR SPATIAL ANALYSIS,
WITH APPLICATIONS TO MULTIFRACTAL PHENOMENA

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ABSTRACT

Most of the discussion in the GIS community is concerned at the highest level with the support of managerial issues and at the lowest level with engineering considerations. Scientific considerations of analytical flexibility and conceptual suitability tend to be slighted. One reason for this situation is the complex, multiscale, and "heterodimensional" world with which the scientist is concerned. The new models of spatial fractals and temporal chaos are making aggressive inroads in our understanding of complex systems, and they deserve to inform considerations of the data models that will fruitfully capture variation in space, time, and scale. I present the outline of a data model that I have found useful, along with examples of its use. The model is well suited to current advances in data organization, hardware, and parallel algorithms.

INTRODUCTION

While definitions of geographical information systems abound, little attention is given to the important differences between the managerial and scientific cultures that use such systems. Consequently, much of the GIS literature is ambiguous about whether a given system—either proposed or actual—is suitable for research as well as management. Because GIS designers and engineers, who are not necessarily the same community as scientific users, are usually concerned about efficiency and ease of use, such matters as data structure adaptability and analytical flexibility tend to be overlooked.

Substantively, the broad class of managers tend to be concerned with the support of decisions and legal issues, rather than the rules of scientific inquiry (Kaplan 1964). Managers (regional planners included) tend to work within a limited range of scales, say one or two orders of magnitude for any given task. (In what follows I shall use "scale" in its everyday, physical interpretation rather than its cartographic sense.) Moreover, at any given scale, managers focus on ideal phenomena of integral dimension: D = 1 points, D = 2 curves, and D = 3 areas (including most especially polygons). GIS engineers seek to serve these needs by pursuing speed, data compactness, and user friendliness in system design.

Scientists, however, and especially workers in a rapidly changing field like spatial analysis, tend to have different concerns. First their style of work tends to be flexible, tentative, often even sloppy by managerial standards: prompting the claim popular in our computer center to the effect that "If we knew what we were doing, it wouldn't be called research!" One manifestation of this flexibility is the willingness—even the enthusiasm—of scientists to patch together disparate pieces of hardware and to use quick and dirty programs to link various software packages to achieve what they want at the moment for a particular research question.

More importantly, spatial analysts are interested in <u>processes</u>, particularly in many different interrelated conceptual realms: physical, biological and human. Consequently, they are trained from professional birth to relate to the world through models of process rather than the more rigid and legalistic structures of management. One consequence of this process orientation is that scientists also often want to <u>simulate</u> data by asking whether a suitably circumscribed set of assumptions can yield results like the complicated phenomena they find in their data. In fact, simulation can be defined as the art of getting complicated results from simple causes, but there is little evidence that GIS designers seek to support such earnest playfulness.

Another consequence of scientific interest in process is that scientists need to look at the world at a very wide range of temporal and spatial <u>scales</u>: often orders-of-magnitude-of-orders-of-magnitude: from millimeters to 10,000 km, from minutes to millennia. For earth scientists in particular this concern with extreme scale ranges is stimulated by a growing anxiety about the need for global monitoring, to link small causes with large effects, to test the limits of scientific ability to capture, store, analyze, and interpret vast amounts of data (IGBP 1988, p.79).

The mind of the scientist is therefore in nearly continual dialogue with his or her model, itself used to extract data from the world and to produce images, maps, tables, plots, and various statements predicting the world's complicatedness by explaining its underlying complexity. The GIS engineer, while concerned about ways of managing the world, is less preoccupied with the extent to which new systems may influence our mental images of the universe, much as did the earlier telescope or microscope (Abler 1987). While I suspect future GIS developments will overcome this limitation, for now the GIS focus is not particularly flexible, process-oriented, or multiscale.

FRACTALS AND MULTIFRACTALS

How then shall we conceive of a GIS that, while not specifically designed for scientific use, nevertheless fosters reasonable analytical ambitions? I cannot presume to fully specify such a system, first, because the above agenda is obviously broad and quite general, and second, because our collective experience with such activities is still in many ways quite limited compared with the more narrowly defined tools and questions of more traditional sciences. But if I were to set a task for a geographical analysis system it would be to address the problem of flexibly handling the input, analysis, and output of data which occur at many scales and which have fractal characteristics.

The first problem with the handling of data from a phenomenon that manifests itself over a range of scales is the requirement of large amounts of storage area and a great deal of computer power. Because

$$\text{DATA VOLUME} = \text{SCALE} \times \text{RESOLUTION} \times \text{VARIABLES} \tag{1}$$

even small increases in any of these terms will bring major jumps in amounts of data if the other is already large. Although I shall be cavalier in what follows by disregarding storage specifications, any concrete attempt to design the system I propose will demand both a lot of hardware and a lot of engineering skill, including heavily parallel and connectionist approaches.

The second problem with scale is that the mind manifests a certain scale inertia: we tend not to make ready mental shifts in magnitude. Although the forest/tree distinction is familiar to everyone, few people recognize that for some analytical purposes it helps to conceive of forests and trees as part of a conceptually unitary phenomenon in which many processes interact at many scales to reveal structures that are, not at all paradoxically, "scaling," i.e. ranging from the nearly infinitely large to the nearly infinitely small (and quite intense). So we talk about trees and forests (read neighborhoods and urban systems, rocks and mountains) as though they were different entities rather than mental images of the same thing. The fractal paradigm is helpful here.

Fractals are phenomena which are self-similar: images and measurements of fractals taken at one scale tend to be similar to images and measurements at another scale. The problems this presents for traditional science can be understood by considering the following history. The goal of classical science has traditionally been to look for regularities (linearities or log-linearities) in data: this often entails measuring variables operationally defined within a narrow range of scales, then modelling relationships, and finally sweeping what is left into an error term. Fractal research challenged this approach by arguing that in many realms virtually everything may be "error". Early geometrical research demonstrated how <u>ideal</u> fractals were scale-invariant, implying that the new regularities were captured by D, a parameter which could be used to "explain" the phenomenon. This conclusion, while naive, is less egregious than the notion that scale is unimportant. Scale (like money) matters, and must be explicitly part of any spatial analysis system.

Later work on <u>stochastic</u> fractal phenomena generated images and data that were more realistic, to be sure, but also spawned a number of different fractal dimensions depending upon the model or the aspect of the phenomenon under study (Stanley 1986). This tends to be upsetting unless one realizes that different facets of a process (say the perimeter of a region versus its "mass") will have different fractal dimensions. The lesson here is for a spatial analysis system to allow clarity about these facets and their measures.

The latest phase of research—and we are now at the cutting edge because an aggressive game is being played with the real world—has begun to focus on <u>real</u>, <u>multi</u>fractal phenomena whose fractal dimensions vary with time, with space, and (although it may seem paradoxical) with scale itself (Feder 1988). Specifically because the universe is made of systems (molecules, people, planets) with characteristic lengths, system behavior changes with scale. Much of this work is still highly theoretical, but some is empirical: perhaps the most relevant to spatial analysts is the research of Lovejoy and others on turbulence in meteorological systems (Gabriel et al 1988) as well as that on earthquakes (Kagan 1980).

The key notion here is that seismological, meteorological, and, to be boldly hypothetical, cultural systems are intermittent processes generating structures in a dissipative cascades from larger scales to smaller. At the largest scale such systems generate space-filling structures of $D \sim 3$, while at the smallest we find "singularities" of $D \sim 0$. The lesson for geographers, in particular, is that such key descriptions as area, intensity, variability, complicatedness, etc. will vary not only with time and space but also with scale. The challenge remains to extract regularity from these data by being able to exert control, in the quasi-experimental sense of collecting lots of data, over scale.

Geographic science is a full participant in this revolution. Geographers have made early and enthusiastic use of these notions and have revealed the multifractality of terrain over space (Mark and Aronson 1984, Roy et al 1986), of coastlines over scale (Goodchild 1980), of sedimentation over time (Plotnick 1986), of variation over scale (Woodcock and Strahler 1987) and of point pattern over density (Harvey 1968, De Cola 1987).

The positive side of all this is that quite parsimonious fractal models are yielding important results in a wide variety of fields and that various dimensions, provided they are operationally defined and displayed for a range of times, places, and scales, are extremely powerful descriptors of real phenomena. Moreover, fractal theory teaches that we often need fewer variables than we thought, which mitigates the Devil's bargain reflected in Equation 1. Still, when it comes to memory and speed, more is quite clearly more, and can sometimes compensate for lack of conceptual rigor.

DATA MODELS

Any geographical analysis system for scientific research must therefore be capable of gathering, managing, and displaying data from fractal phenomena. Rather than narrowly specifying a data structure for such analysis, however, my ambition is more modest. Peuquet (1984, p. 69) defines a data model as "a human conceptualization of reality, without consideration of hardware and other implementation conventions or restrictions." Certainly the conceptual specifications that follow, along with the examples, can be translated into the syntax of any language (such as C or Pascal) permitting variant and dynamic records as well as recursion. Greater specificity would obscure a heuristic approach to structure and an algorithmic approach to process (Smith 1987).

The above outline of fractal research calls for a discussion of four issues. First, we should be clear about the topological realm in which we are working. To begin, I shall not be concerned with phenomena in a third (physically vertical) dimension, although it is clear that the analysis of terrain data is basically a question of analyzing variation in space (Weibel 1988). In terms of integral dimensions, the topologies of the phenomena to be examined range from simple points to space-filling areas, but note how differently these topologies are treated:

DIMENSION	GEOMETRY	SPATIAL ANALYSIS	REMOTE SENSING	GIS
0	Point	Event	Location	Location
1	Curve	Link or Perimeter	Pixel edge	Vector
2	Area	Region	Pixel	Polygon

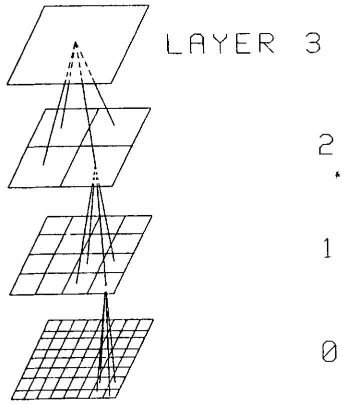
I shall not attempt to do more than acknowledge this incommensurability except to note that what sometimes appears to be a culture conflict may be quite profound: the data structures of GIS are based on atomic units that are not only too small to be detected with remote sensing techniques, they simply do not exist in the two dimensional system (*D* = 2) used by sensors. This is not to make the trivial observation that a benchmark location, a road centerline, or a state boundary is invisible; but rather that they exist in a space of dimension too low even to be incorporated in such a system. Now remote sensing projects 3- and 4-dimensional phenomena into 2-dimensional data, but unfortunately projection only works one way. It is impossible to "project" 1-dimensional GIS entities into a 2-dimensional map. (For a related discussion of this problem see Lovejoy's (1985) discussion of meteorological networks.)

A second area requiring specificity is the distinction between scale and resolution. Although the term <u>scale</u> is used cartographically to represent the size that the representation of an object on a map bears to its "real" size, I choose here to use scale in its more commonplace (and physics) sense of the characteristic size of the real object. In contradistinction, I shall use <u>resolution</u> to mean the smallest areal unit at which a distinction of spatial variation can be made. On the one hand, a data structure must be able to incorporate measurements at the highest resolution made available by sensors (consequently it must be big) while at the same time capturing or permitting the creation of structures of often enormously large scale (Equation 1 above). The data model must also be able to integrate measurements from sources with different resolutions. On the other hand, this need for breadth of scale is mitigated by the fact that scientific data need not be stored with great absolute locational accuracy (Burrough 1988). Third, the data model must reflect clarity of thinking as well as flexibility about such things as features, objects, and entities. I have opted below to call homogeneous clusters of cells "regions," but readers are unlikely to agree that the world can be partitioned into such sets. While the regions are constructed "bottom-up" by aggregating cells, how they are generically defined, functionally specified, or identified by name will be a function of their scale as well as that of the analysis.

Finally, the system also needs to operate "top-down," beginning with a dataset and subdividing it into subsets of greater homogeneity or requiring small-scale analysis. Quadtrees and their variants are a popular approach to this problem, and I would argue that the full pyramid represented by a complete quadtree structure suggests the approach called for. It should also be recognized that the size of a quadtree is a direct reflection of fractal dimension (Samet 1984, p. 227).

EXAMPLES

In the present case, we begin with grid dataset, conceived of as a lattice of $(2^L)^2$ 0-level cells $x_{0,i}$, where $i = 1,\ldots,(2^L)^2$. Let there be a value $f(x_{0,i})$ assumed to be univariate and to be an explicit, monotone function of some underlying measurement: i.e. f could be an observation of events people, photons, trees, votes, etc. Let a λ-level lattice or <u>layer</u> of $(2^{L-\lambda})$ cells be constructed with $f(x_{\lambda,i}) = \Sigma f(x_{\lambda-1,j})$, where the summation is over the $j = 1,\ldots,4$ children cells of $x_{1,i}$. See the following figure:



Three things should be noted. First, for layer $\lambda+1$ to be a true aggregation of layer λ it is important that f be a linear function of the measured phenomenon, otherwise we need a way to recover the measurement from the data (Richards 1986). Second, the possibility of f being multivariate is a complexity I shall not explore. Third, although the present lattice is of dimension 2, the lattice could be of side $(2^L)^3$ or even dimensionally larger, to include time, etc.
A histogram of $\{f_\lambda\}$ would describe each layer λ, and if f were a digital transformation of the underlying count then the abscissa of this histogram would be limited (as in the case of a Landsat band to 2^8 values). In any case various parametric (moments) and nonparametric statistics will also be used to characterize the layers. These statistics would indicate the presence of cells with unusual concentrations (or absences) of events. Higher, more aggregate, levels could be used to scan the image for intense activity.
Next, let $t \in [0, \max\{f_0\}]$ be a threshold. The variable t could also stand, in increasing order of complexity, for an interval, a subset, or even the intersection of multivariate subsets, as well as labelled classes. Obviously this rapidly complicates things but does address the task of image classification and labelling (Campbell 1987). At any level λ consider the regionalization $F_{\lambda,t} = \{x_{\lambda,i}: f(x_{\lambda,i}) \geq 2^\lambda t\}$, i.e. the set of all super-threshold cells (De Cola 1989).

This image segmentation creates a list of disjoint and unconnected regions $\{E_{\lambda,t,k}: k = 1, \ldots, n(\lambda,t)\}$, where $n(\lambda,t)$ is the number of such regions. Each region E can be characterized at least by its location, size (number of cells), and perimeter. We may store this list either in its entirety or, by the use of dynamic variables, in bins containing the above descriptive information. The number, sizes and perimeters of these regions can be used to compute the fractal dimension $D(\lambda,t)$ as well as the Pareto scale parameter $a(\lambda,t)$ for the layer λ, both of which can be expected to be a declining function of t (Lovejoy and Schertzer 1988). From the point of view of memory considerations, it should be noted that in general $\partial^2 n(\lambda,t)/\partial t^2 < 0$, i.e. the number of regions tends to a maximum for some midrange value of t (roughly that value of t for which $p(x_\lambda \in F_{\lambda,t}) = 1/4$ (De Cola 1989)). This bottom-up approach yields regions which are explicitly a function of threshold, of layer, and of such sensing characteristics as resolution, so that we may examine the extent to which regional description and appearance reflects these characteristics. Note therefore that we may explicitly examine resolution effects both as artifact and as explanatory variable.

An example of the multi-scale imaging proposed here is shown in Figure 1, which displays results of a random walk simulation of 1000 steps on a 32^2 torus. Shown in 1d) are all cells visited at least t = 2 times, in 1c) all 2x2 cells visited at least $2^2 \cdot 2 = 8$ times, etc. For this experiment D(t=2) = 1.41. It is helpful to think of each of these figures as an image in itself and not a "defocusing", etc. of some "better" resolution data. Sometimes we are interested in forests, sometimes in trees, still other times in leaves. Each layer tells us something about the process at that scale, characterized by fractal and size distribution parameters, and each threshold generates different statistical characterizations.

Another example of this approach, this time from empirical research, is shown in Figure 2. Figure 2a) presents all of the URBAN-classified $(31.81 \text{ m})^2$ pixels from a from a 2048^2-pixel Landsat image of Northwest Vermont (De Cola in review). Figure 2b) (at the scale of 2a)) locates all level-0 regions of size ≥ 15 pixels. But another way of looking at this information is shown in 2c), all level-4 regions; i.e. all clusters of $2^4 = 16$-sided cells in the study region. (Note that t = URBAN is not a threshold but an imputed land use; nominal values require a form of aggregation different from simple summing (De Cola 1989). This aggregation process reduced the number of regions from the 39,000 of the level-0 image to a manageable 325 in the level-4 image. These regions were then used to estimate the populations of "towns" in Franklin County VT, with results that were more reliable than those derived from the use of level-0 single-pixel regions (De Cola 1988 and see Tobler 1969).

So much for a "bottom-up" approach; next we turn to the recursive subdivision of the highest cell at level L. Each layer L-λ consists of cells representing locations x which can be described in terms of such parameters as $D(\lambda,t)_x$, where the subscript implies locational specificity within the layer. This technique applies as well to Pareto regional size distribution statistics, to the size of the largest region in cell x, etc. Although these parameters can be presented in tables and plots, perhaps the most interesting way of displaying them is as maps. Figure 2d), for example, is a map of the fractal dimension of URBAN pixels from the Vermont study. The scene has been divided into $(2^3)^2 = 64$ cells, bringing the analysis down to the $\lambda = 11 - 3 = 8$ level. The fractal dimension $D(8,\text{URBAN})_x$ is represented by the height of each point. Lack of variation in D_x over space would be a necessary indicator of texture.

The data model presented here is obviously extremely flexible, allowing the scientist 1) to move among scales from layer to layer, 2) to explore the effects of varying threshold, and 3) to roam spatially within layers from cell to cell. As such, it affords ready access to parameters for the scale and location examined, as well as access to images at lower levels and maps at higher. The key is obviously the cell, which is not only the constituent of a region built up from below but also the location of spatial information for a window of subdivided space. For example, in the example above, the URBAN spectral classification was determined at level 0, while the identification of actual "urban" features was made at level 4, and the exploration of spatial variation in URBAN fractal dimension was made at level 8. The difference between the urban names here is intended explicitly to recognize that URBAN is a pixel group operationally defined by the specification of a spectral classification process, while "urban" is a word I have chosen to denote level 4 regions with a specific spatial morphology (connectedness, size, disjointedness, and form).

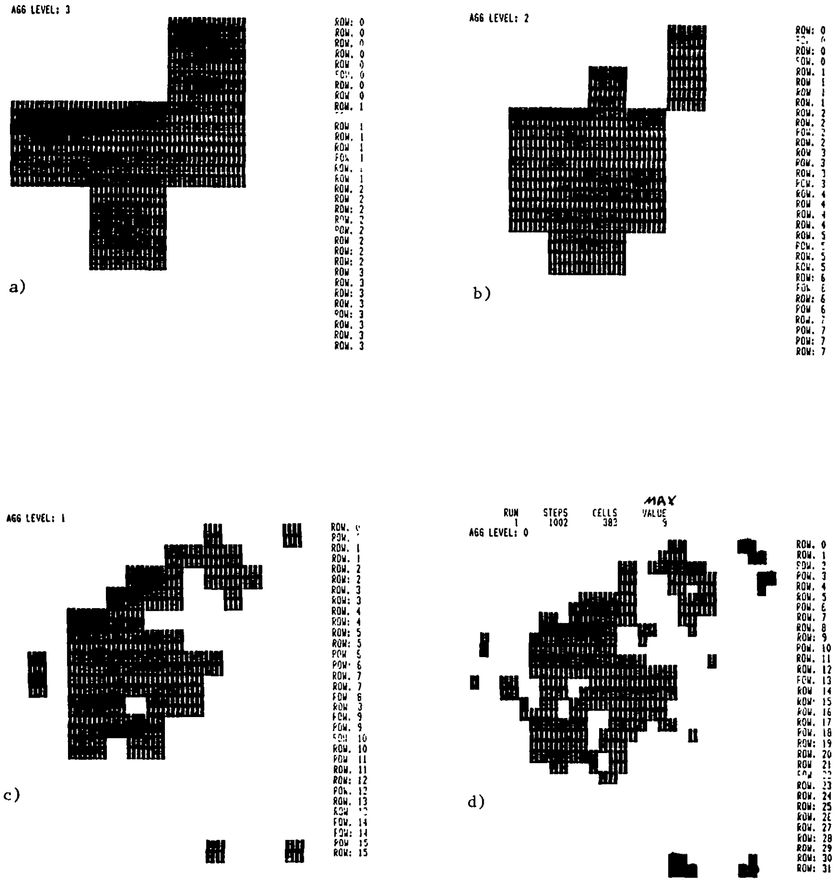


Figure 1. Random walk simulation with first 4 aggregation levels.

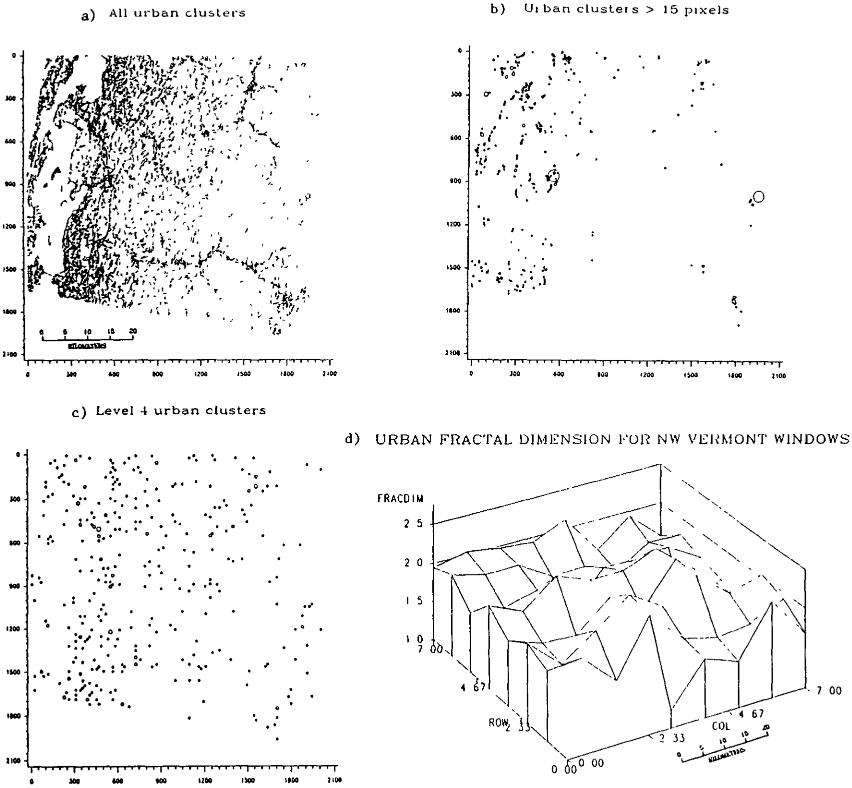


Figure 2. Images and a map of URBAN pixels from a Landsat scene for Northwest Vermont.

PROSPECT

Admittedly, it is no easy feat to translate the above requirements into a full-blown geographical analysis system for the study of events, interactions, and regions in space and time, but we are moving in this direction. At least three kinds of developments are cause for optimism. First, data structures, such as quadtrees and iterated function systems (Barnesley 1988), as well as data hardware, such as faster and more capacious chips as well CD-ROM (Lambert and Ropiequet 1986) give us powerful command over and broad access to large numbers of measurements. Second, sheer improvements in computational speed are always welcome, but probably the greatest advances will come from parallel and connectionist architectures and algorithms (Toffoli 1987 and Mower 1988). There seems no reason why a parallelist approach to spatial data (Bhaskar, Rosenfeld and Wu 1988) cannot be adopted to the multidimensional analysis of digitized spectral data as well. Finally, multifractal approaches to real world phenomena offer new ways of integrating multisource data in ways that make analysts less burdened by hitherto supposedly incompatible resolutions. While I am not optimistic about the near-term integration of vector ($D = 1$) and raster ($D = 2$) data, I am excited about the fact that scientific research can only flourish in these tumultuous times.

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