

Sliding Tolerance 3-D Point Reduction for Globograms

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ABSTRACT

Vector maps that are plotted using a globogram type of projection may have linework that becomes crowded when approaching the map's horizon line on the hardcopy plot. To reduce this linework clutter on the globogram map, the map can be passed through a point reduction algorithm to thin the points. Point reduction algorithms, whether local or global in nature, typically are two-dimensional geometrical processes, that operate equally on all of the map's digital lines. In the globogram case, this can result in too much point reduction in the central, foreground areas of the map and too little point reduction at the extreme areas near the horizon line, where the oblique 'viewing angle' increases the density of points.

To solve the above problem, this paper will propose and demonstrate an implementation of a three dimensional method of point reduction using a modified local point reduction algorithm. The proposed methodology and algorithm will utilize a variable or sliding tolerance criteria for the point reduction based on a selected point's proximity to the horizon line of the globogram map.

BACKGROUND

In digital mapping, point reduction algorithms are applied to maps to remove excess or unnecessary points while maintaining the basic caricature or shape of the lines. The application of these routines may be required due to the reduction in scale of a map. Added benefits of point reduction include reduced plotting time, reduced storage space of the coordinate pairs, faster vector processing and faster vector to raster conversion (McMaster, 1987).

Over the last twenty years, several algorithms have been developed to perform automated point reduction on digital maps. A review of automated line generalization and point reduction by McMaster (1987) contained discussions of several point reduction algorithms. These algorithms use two dimensional geometrical techniques to remove points from two dimensional maps. The current research sought an improved technique for point reduction for the particular problems associated with the globogram projection.



Figure 1. ORIGINAL MAP - 3445 POINTS



Figure 2. GLOBOGRAM MAP - 3445 POINTS

THE GLOBOGRAM PROBLEM

The globogram is a two-dimensional representation of the three-dimensional map model. Viewed orthogonally, a globogram map gives the appearance of being projected on a sphere, since it is defined in three dimensions and plotted in two dimensions. A map defined in spherical coordinates (R, θ, ϕ) or cartesian coordinates (x, y, z) can be rotated in any of the x , y or z directions, and subsequently plotted using the x and y coordinates on a globogram projection. The problem of line clutter occurs when the z coordinates (in cartesian space, the x, y plane is the map sheet plane) of the map boundaries approach zero, i.e. the map boundaries approach the horizon line. Figure 1 shows the original map of Canada, using the Lambert Conformal projection. Figure 2 shows the globogram of the original map (original map digitally placed on a 400 mm radius sphere, then rotated 58 degrees about the Y axis). Both maps were plotted from the same source basefile containing 3445 coordinates. Due to the nature of the globogram projection, northern portions of the Canadian coastline become crowded and the coordinates become more densely clustered. Less detail is required in these northern areas, and in the general case, less detail is required as map areas approach the horizon of the globogram projection.

TWO-DIMENSIONAL POINT REDUCTION METHODS

Two sequential methods of two-dimensional point reduction may be employed in an attempt to alleviate the point density of the lines. These methods are:

- A. performing a two dimensional point reduction on the original map's x, y coordinates, and then projecting the map onto a globogram,
- B. projecting the original map onto a globogram, and then performing a two dimensional point reduction on the globogram.

Any of the local or global point reduction algorithms are suitable for these two methods; however, these algorithms follow similar methodologies: point reduction is performed equally on all the lines that comprise the map due to a controlling set of fixed tolerance criteria. These criteria may be a tolerance distance, a perpendicular distance or deviation, a field of view angle or deviation, or any other combination of threshold values. The common factor of these algorithms is that this threshold tolerance value is a fixed quantity, usually supplied by the user. Applying these algorithms to two dimensional maps generally produces good results everywhere on the point reduced map, depending on the algorithm used and the selected threshold tolerances. However, the globogram is a special case that requires varying degrees of point reduction that is not provided with traditional two-dimensional methods, as seen in the following examples.



Figure 3. 50% REDUCTION - 1720 POINTS



Figure 4. GLOBOGRAM OF 50% REDUCTION - 1720 POINTS



Figure 5. GLOBOGRAM MAP - 3445 POINTS



Figure 6. 50% REDUCTION OF GLOBOGRAM - 1721 POINTS

TWO-DIMENSIONAL POINT REDUCTION FOR GLOBOGRAMS

A comparison was made between three two-dimensional point reduction algorithms:

1. the proximity method
2. the Jenks modified angular method
3. the perpendicular/proximity method

using two reduction factors, 32.5% and 50%, for the two above sequential methods. After examining the output, it was determined that the perpendicular/proximity method produced the best results in this particular case, and that the 50% reduction demonstrated the most dramatic effects of the algorithm.

Method 'A' Globogram Point Reduction

Figure 3 is the output of the perpendicular/proximity algorithm (50% reduction) as applied to the original map, while Figure 4 shows the globogram of the 50% reduction map. There is a noticeable difference in the point density of the globogram of the point reduced map when compared with Figure 2. However, this reduction in the point density of the lines occurs everywhere on the globogram map, and crowding is still evident in the northern sections. Also, undesirable thinning of the lines has occurred in the southern portions of the map.

Method 'B' Globogram Point Reduction

Figure 5 is the globogram of the original map while Figure 6 shows the results of the perpendicular/proximity algorithm (50% reduction) as applied to the globogram of the original map. Again, there is a noticeable difference in the point density of the reduced globogram when compared with Figure 2. However, as in method A, the reduction in the point density occurs everywhere on the globogram map, and the point crowding in the northern sections has not sufficiently improved, while excessive point reduction has occurred in the southern sections.

THE SLIDING TOLERANCE POINT REDUCTION

The globogram case requires the application of a selective point reduction algorithm that removes more points at the critical areas near the horizon line, and removes relatively fewer points in the less oblique foreground areas.

Since the critical areas of a globogram projection are lines that approach the horizon line, where the z coordinates of the vertices approach zero, point reduction solutions were sought which made effective use of this z coordinate. The z coordinates would be used to evaluate the globogram line's proximity to the globogram horizon line. As a line approaches the horizon, convergence of its vertices increases due to the

oblique viewing angle. To increase the map's legibility, the elimination of a greater number of points in the horizon areas is required.

Global point reduction algorithms, such as the Douglas-Peucker algorithm (Douglas and Peucker, 1973) are not suitable since they operate on a complete line. The group of local processing point reduction algorithms, where a point is tested for redundancy relative to its immediate neighbours, was determined to be appropriate and easily adaptable for this application. Instead of being fixed, the threshold value for each algorithm was allowed to vary from a user-defined minimum value to a user-defined maximum value, following a function that was based on the z coordinate of the test vertex.

Three sliding functions, that varied the range of the tolerance values were tested:

Theoretical Function	Actual Function Used
1. $(R-Z)/R * \text{Tolerance}$	$(R-Z)/R * \text{Tolerance}$
2. $0/90 * \text{Tolerance}$	$[\text{ARCCOS}(Z/R)]/90 * \text{Tolerance}$
3. $\text{SIN}(0) * \text{Tolerance}$	$\text{SQRT}(1-(Z/R)^2) * \text{Tolerance}$

where Tolerance was a user-defined tolerance range. A typical formula for computing the actual value of the tolerance value, using function 1 is:

$$\text{TOLVAL} = \text{MINTOL} + \underbrace{[(R-Z)/R]}_{\text{sliding function}} * \underbrace{(\text{MAXTOL}-\text{MINTOL})}_{\text{tolerance range}}$$

where TOLVAL = computed tolerance value for the test vertex
MINTOL = user-defined minimum tolerance value
MAXTOL = user-defined maximum tolerance value
R = radius of the globogram sphere
Z = z coordinate of the test vertex

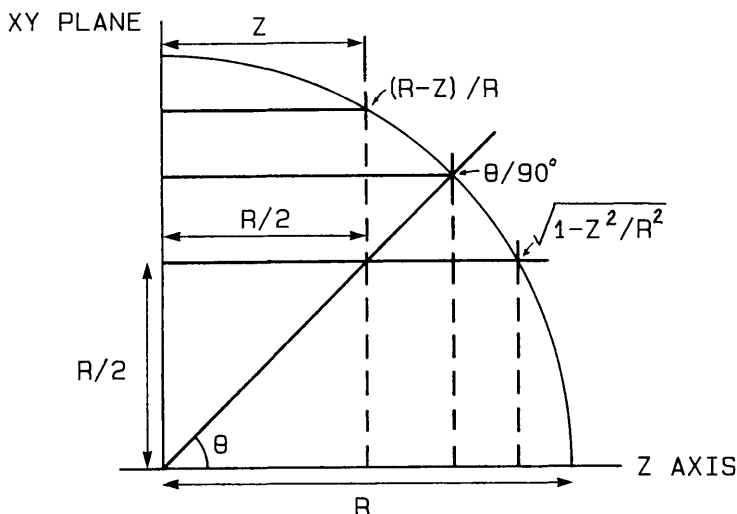


Figure 10. GEOMETRY OF SLIDING FUNCTIONS



Figure 7. GLOBOGRAM OF 50% REDUCTION - 1720 POINTS



Figure 8. 50% REDUCTION OF GLOBOGRAM - 1721 POINTS



Figure 9. 50% GLOBOGRAM 3-D REDUCTION - 1723 POINTS

Each of these functions has its own unique characteristics, being linear, transcendental and quadratic respectively, resulting in different responses or sensitivities on the tolerance values. All the sliding functions range in value from 0 to 1 for Z ranging from R to 0. This results in minimum tolerance values and point reductions when $z=R$ and maximum tolerance values and point reductions when $z=0$. Figure 10 gives a representation of the geometry involved in each of the sliding functions, and shows the position of a point on the sphere where the computed value of TOLVAL is halfway between MINTOL and MAXTOL. The choice of sliding function is not unique, and the final results would clearly depend upon the amount of curvature and rotation in the globogram.

RESULTS

The perpendicular/proximity point reduction algorithm, using the linear function $(R-Z)/R$ to vary the tolerance values, provided the best results, at a 50% reduction in total points, for the Canada example. An illustration of this three-dimensional technique can be seen in Figures 7 through 9. For comparison purposes, Figures 7 and 8 repeat the previous Figures 4 and 6 respectively. Figure 9 shows the results of the adapted algorithm. A significant reduction in the number of points has occurred in the northern sections of the map, near the horizon, whereas much less reduction has occurred in the areas near the foreground. When comparing the three techniques, it is evident that the sliding tolerance method gives significantly better results in areas requiring the most point reduction, leaving the foreground areas relatively intact. Because of the utilization of the sliding tolerance criteria, this approach is superior to the other two methods.

CONCLUSION

The usage of conventional two-dimensional algorithms for enhancing the display of globograms can be significantly improved by incorporating the 3-D sliding tolerance methodology. The utilization of the z dimension as a controlling parameter in determining the tolerance for conventional two-dimensional point reduction algorithms allows for varying degrees of point reduction depending on the relative position of the map on the globogram sphere. This adaptive technique can be successfully applied to any of the 'local' two-dimensional point reduction algorithms, resulting in a more satisfactory presentation of the globogram map.

Future directions and research include evaluations of characteristics of different sliding functions as adapted to various local two-dimensional point reduction algorithms, the display of map projection error as a function of the line detail of the map, and the analysis of the perception of visual density of globograms

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