TOPOGRAPHIC GRAIN AUTOMATED FROM DIGITAL ELEVATION MODELS

Richard J. Pike[1], and William Acevedo[2]
U.S. Geological Survey
Don H. Card[2]
National Aeronautics and Space Administration
[1]Menlo Park, CA 94025; [2]Ames Research Center, Moffett Field, CA 94040

ABSTRACT

Relief at topographic grain is an estimate of local relief optimized by varying unit-cell size. In homogeneous terrain, local relief (Y) within nested circles increases with circle size (X) and then levels off at a diameter termed "grain," a measure of characteristic local ridgeline-to-channel spacing. To map relief and grain as continuous variates, we automated their estimates from digital elevation models (DEMs). The computer calculates values of elevation dispersion within nested sample areas in a DEM, plots them against sample size, and analyzes this function to identify the Knick, or break-point. The resulting quantities grain and relief at grain appear to correspond to "range" and "sill", two parameters of spatial autocovariance.

INTRODUCTION

Once-intractable problems in regional geomorphology and physiography are beginning to yield to analysis of digital elevation models (DEMs) manipulated on fast computers by spatial-analysis software. A long-standing goal in landform interpretation is to abstract the character of continuous topography (Pike, 1988). Numerical methods for such representation of terrain require measures of land form that minimize chances of misinterpretation and can be readily communicated and mapped. Many parameters have been devised to describe topographic geometry, at different scales, in both horizontal (XY) and vertical (Z) domains (Evans, 1972). This is our first report on experiments with the automation of two related measures, grain and local relief.

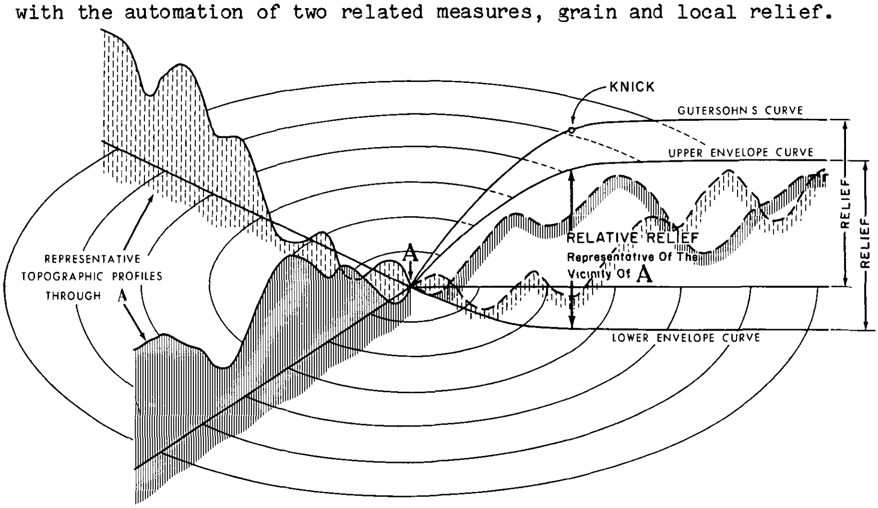


Figure 1. Gutersohn's (1932) concept of local relief measured on varying areas, here circles, optimized at a Knick resulting from the addition of two height envelopes (from Thompson, 1959, 1964).

RELIEF AND GRAIN

Topographic grain is the characteristic horizontal spacing of major ridges and valleys. Grain is inherent in Johnson's (1933) restricted definition of texture as "the average size of the units comprising a given topography." The grain concept arose from the need for a variable and nonarbitrary unit-cell size within which to calculate another parameter, local relief, rather than from any perceived need to measure texture per se (Johnson, 1933). Defined as elevation range ($Z_{max}-Z_{min}$) within a limited area, local relief has a serious operational drawback: estimates of it from unit cells of one size do not represent a wide spectrum of terrain types with equal fidelity (Trewartha & Smith, 1941; Wood & Snell, 1960). This problem reflects the varied dominance of topography by local features that differ widely in relief and spacing (Johnson, 1933; Thompson, 1959, 1964).

Gutersohn (1932) devised a calculation for local relief such that size of the unit cell would be neither arbitrary nor uniform. According to his concept, envelope curves of maximum and minimum elevation vary with distance in a way that defines the optimal areas for measuring local relief (Figure 1). Unused until its adoption by Wood and Snell (1959) to optimize the sample design for their oft-cited quantitative taxonomy of terrain (Wood & Snell, 1960), the method entails measuring relief in nested squares centered at a sample point on a topographic map and plotting relief (Y) against length of the side of the square (X). In homogeneous topography, relief generally increases rapidly with size of the square until the full range of local elevation has been encountered, after which it increases much more slowly. The cell size corresponding to the relief value at this breakover, or inflection, is large enough, but no larger than required, to include the most important features typifying that topography (Figure 2).

The varying area of a topographic sample, adjusted "for the degree of coarseness or fineness of the relief pattern" (Trewartha & Smith, 1941), seems to have been termed "grain" by someone at the University of Wisconsin, likely before its use by Young (1954). We think that W.F. Wood, Young's contemporary at the Department of Geography at Madison, adopted "grain" for his implementation of Gutersohn's (1932) approach to calculating local relief (Wood & Snell, 1959, 1960). Neuenschwander's (1944) review of morphometric analysis included Gutersohn's breakthrough, which evidently was first described outside the German literature by Hook (1955), a student of W.F. Wood at Iowa State University. Hook did not mention "grain." The choice of terms was unfortunate, for "grain" conflicts with accepted usage describing map patterns and trends. However, it is so well entrenched in the literature that we decline to propose any alternative here.

PAST WORK

In practice, grain varies widely with topographic texture. Values obtained by Wood & Snell (1960) from 1:100,000-scale contour maps (n=413 samples) in central Europe range from 2 to 14 miles. Thompson's (1959, 1964) study of the Alps from 1:250,000-scale maps (n= nearly 300), by a method differing from Gutersohn's in practice but not in concept, yielded grain measurements between 2 and 28 miles. A regional analysis of southern New England by the Wood-Snell method (Pike, 1963) from 1:24,000-scale USGS quadrangles (n=142) resulted in grain values of 1 to 11 miles. Grain varied from 1 to 6 miles in Georgia at 1:62,500 (n=76) and southern New York at 1:24,000 (n=94) (Autometric, 1964). All these studies used circular samples.

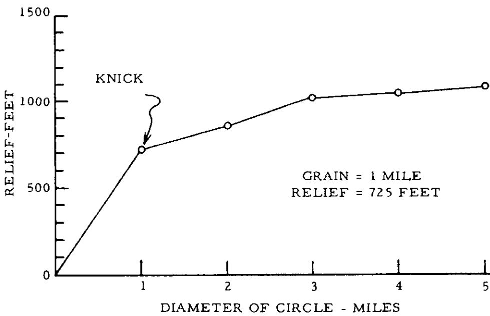
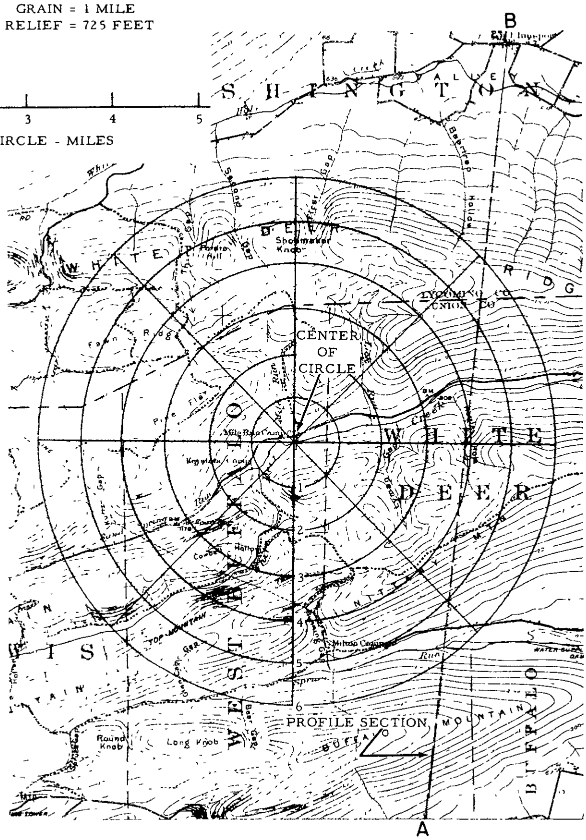
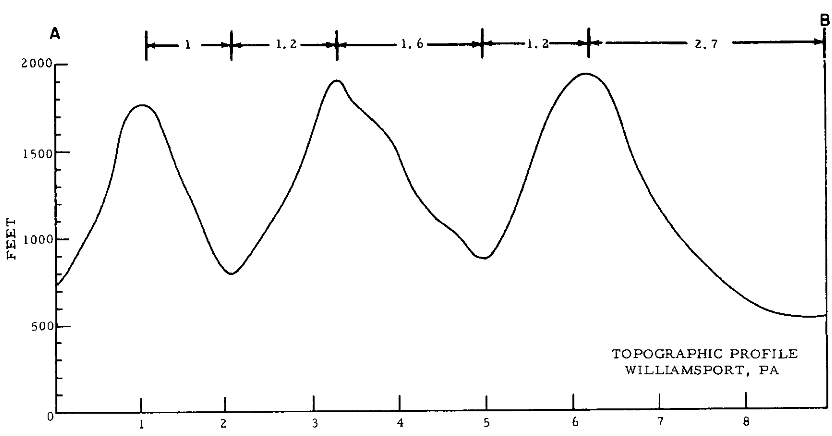


Figure 2. Relation of topographic grain, relief at grain, and Knick to the source contour map and to a nearby topographic profile (AB). Grain approximates observed ca. 1-km ridge-to-valley spacings along the two-mile-distant profile, but relief at grain underestimates the ca. 2000-foot local height differences along that profile. Sample from valley-&-ridge topography of central Pennsylvania, 100 km northeast of Aughwick (compare with Figure 7). (From Carr and Van Lopik, 1962)





Although grain is an important descriptor of meso-scale topographic texture, its relation to other attributes of land form differs so much, and so unsystematically, by locale and map scale that its geomorphic significance has never been properly ascertained. The only grain results that have been mapped and contoured are those from the Alps (Thompson, 1959, 1964) and southern New England (Pike, 1963). In the latter region grain correlates strongly with local relief and mean elevation (r= 0.79 and 0.67, respectively, from unpublished results).

Grain measurement has problems, aside from the obvious tedium of the technique. Its results are unrepresentative in heterogeneous (nonstationary) topography (where a sample includes contrasting physiographic units), in very low-relief terrain, or where the sample center lies near the intersection of major valleys or ridges (Wood & Snell, 1960). Even under favorable conditions, the relief/distance curve may not inflect crisply, and its visual appraisal can be subjective. Pike (1963) found that using circle area rather than diameter often sharpened the inflection, or *Knick* (plural *Knicke*), literally a break or bend (Gutersohn, 1932), of the relief/distance function. However, this modification does not remove all ambiguity.

Grain was first automated for topographic profiles, rather than areas, by Pike 20 years ago (Schaber et al., 1980). The algorithm was part of a terrain-analysis package inspired directly by the early DEM work of Tobler (1968). The method computes relief in nested segments of a sampling traverse (beginning in its center), plots relief against segment length, draws the relief/distance curve, and selects as grain the segment length where convex change in slope along the curve is sharpest (the *Knick*). Automated values of grain for 12 profiles on 1:62,500-scale maps (Pike, 1988) range from 6 to 26 km.

THIS STUDY

We are developing algorithms to automate estimates of grain and relief over areas within large DEMs, thus complementing the automation of relief and other measures for invariant sample cells (Pike & Acevedo, 1988). The procedure follows that devised for profiles. The computer searches successively larger nested circles or squares around a point in a DEM, computes relief or another measure of elevation dispersion, and graphs the results (Y) against the corresponding sample sizes (X). To reduce subjectivity in selecting the *Knick*, we fit the relief/sample-size curve with many pairs of complementary, linear, intersecting equations. Topographic grain is defined, on the X-axis, at the intersection of the pair of equations that minimizes least-squares. Relief at grain is defined, on the Y-axis, as the corresponding value of local relief. Maps of grain and relief at grain result from moving the sampling procedure through a DEM.

We automated the analysis of grain on two datasets. Developmental work was done on 15 new minimal-error USGS 1:24,000-scale DEMs (derived from digitized contours, rather than from stereo profiling) for San Mateo County, California (resolution 30m). Further tests of automated against manual grain values were run on 1:250,000-scale data from the Defense Mapping Agency Topographic Center (DMATC) digital terrain tapes of southern New England (63-m-resolution). We used Sun 3/260* and 4/260* workstations and our own software.

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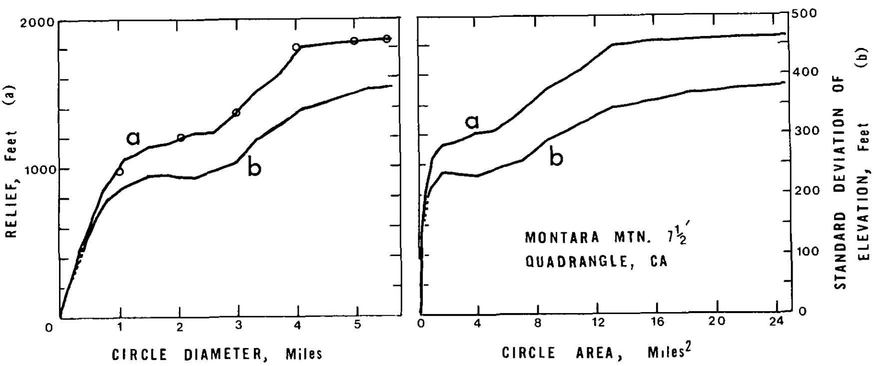


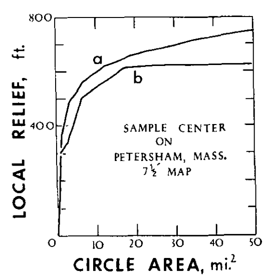
Figure 3. Automated grain curves from 30-m DEM: Relief (a) and standard deviation of elevation (b), each as a function of sample diameter (left) and area (right). Manual measurements on map (dots) yield a similar curve. Grain here is about 1 - 1.5 miles.

The experiments addressed three main issues: (1) Obtaining the curve of relief/sample size from DEMs automatically, (2) Determining how to best locate the optimum inflection, or Knick, on this curve, and (3) locating the Knick automatically, without human judgment.

RESULTS: STANDARD GRAIN ANALYSIS

The first tests showed that manual and automated techniques yield virtually identical relief/sample-size curves from 1:24,000-scale maps and DEMs (Figure 3), when sample locations and cells are similar (squares or circles). Curves differed more in tests of 1:250,000-scale DMATC data (automated) against 1:24,000-scale maps (Pike, 1963, manual) (Figure 4), which we ascribe to the contrasting information content of the data. Quite different curves resulted where sample locales differed or if one method used squares and the other circles.

Figure 4. Relief/area curves for same locale but different methods and data. (a) Manual method: 1:24,000-scale maps (Pike, 1963). (b) Automated method: DMATC 1:250,000-scale digital data. In both cases grain occurs at a circle diameter of about 3 to 4 miles and a relief at grain of about 600 feet.



The next tests suggested that such robust statistics of elevation dispersion as variance and standard deviation (Figure 3) yield at least as sharp Knicke as local relief (elevation range), for both circle diameter and area. Elevation range can be unrepresentative (because chances of including an unrepresentative height value are so high; Wood & Snell, 1960; Evans, 1972), even though it may reflect the land surface more faithfully than other parameters (just as modal elevation always indicates such observed features as terraces, flood plains, and accordant summits, whereas the mean may not).

Plotting local relief or standard deviation as a function of window area, as opposed to diameter, also sharpens the Knick in automated determinations of grain (Figures 3, 5). We tried to enhance the Knick even more, by the first derivative of the curve taken at 5-pixel windows along it (Figure 5). Lastly, we attempted an optimal solution for the Knick by least-squares fits to the relief/sample-size curve. The example in Figure 5, centered in the La Honda CA 30-m DEM, used square windows increasing in edge length by 60-m increments. Results suggest a roughly 1.0-km-diameter grain and a 220-m relief at grain.

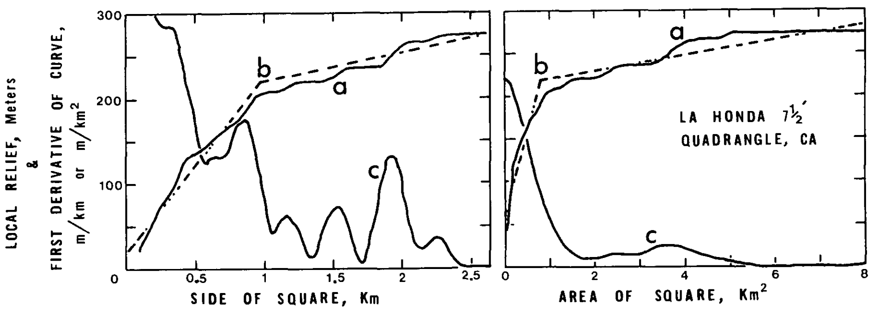


Figure 5. Topographic grain, for relief/distance (left) and relief/area (right), from three different automated calculations on 30-m DEM data: (a) relief/sample-size curves, (b) least-squares fit to curves, and (c) first derivative of curves. Grain is about 1 km and relief at grain is about 220 meters (see text).

The most robust values of topographic grain seem to result from plots of local relief (Figure 5) or standard deviation of elevation (Figure 3) against window area, plus choice of the Knick by least-squares. The area-versus-distance comparison in Figure 5 shows that least-squares analysis (b) yields the most similar grain values (0.1 km apart) among the three pairs of curves: For distance, X = 1.0 km; for area, X (distance equivalent of area) = 0.9 km. The curves from first derivatives (c) are the least satisfactory. That for circle diameter is too irregular to yield an unambiguous Knick and the (much smoother) curve for area yields a high grain of 1.5 km; the two grain values are at least 0.4 km apart. Knicke in just the raw relief/sample-size curves (a), at roughly 0.8 km distance and 1.1 km area, are between the two in definition (that derived from area is the sharper); the two grain values are a high 0.3 km apart. These tests suggest that the first derivative of the curve may not supply the desired enhancement.

We are not wholly satisfied that least-squares fitting yields optimal grain values. Further tests, from eight samples of DMATC data in southern New England, show that automated grain values from the least-squares technique do not always coincide with those selected by eye from the same relief/sample-size curves. Ambiguity in the computer arises from weak or multiple Knicke and from the absence of uniform criteria for graph coordinates; choices of vertical and horizontal scales critically affect the shape of the curve, in both visual and least-squares analyses. These problems have been evident since the work of Wood and Snell (1960), Thompson (1959, 1964), and Pike (1963).

Automating the procedure on DEMs enables grain and relief at grain to be mapped regionally at various resolutions. Figures 6b and c are the first maps of grain ever made by machine. We calculated 63,504 grain values for the Montara Mountain 7.5' quadrangle in San Mateo County CA, using least-squares to identify the Knick, by moving 13 nested circular windows (diameter increment 0.65 miles) through the DEM one pixel (30 m) at a time. Grain values are low, ranging from 1.1 km (dark tones) to 2.9 km (light tones). Overall pattern reflects dominant ridgelines and stream channels as well as other contrasts in the local landforms, notably that between lineated topography to the northeast and more randomly oriented terrain to the southwest.

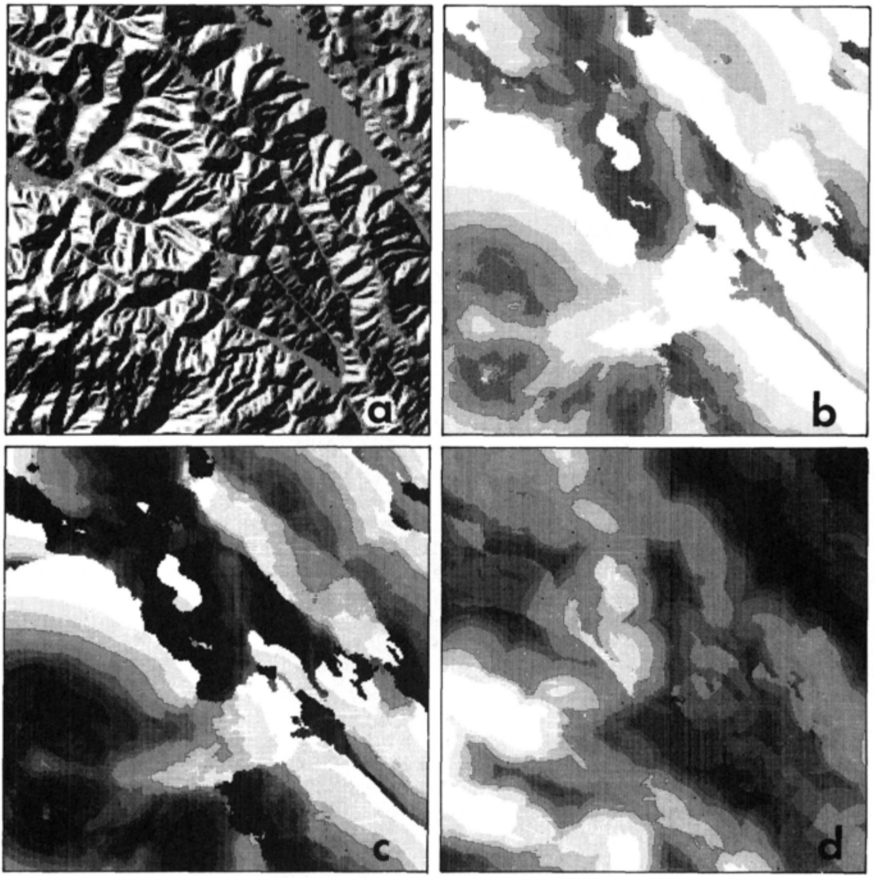


Figure 6. Maps of topographic grain (b, expressed as circle diameter; c, as circle area; see text) and relief at grain (d). Dark tones, low values; light tones, high values. Shaded relief image (a) of Montara Mountain, CA, quadrangle. The images, made from a 30-m-resolution DEM by automated methods on a Sun workstation and a Calcomp plotter, are 7.56 km across.

Figure 6d is the map of the accompanying values of relief at grain, on circles that vary from about 1 km to 3 km across. Like a slope map, Figure 6d numerically expresses the roughness of the Santa Cruz Mountains in this area. Relief at grain varies from 50 m (darkest tone) to 475 m (lightest tone). Because both this map and those of grain were made at maximum resolution (30 m), to produce fine-grained images, the analysis is highly CPU intensive. The three maps together required 50 hours on the Sun 3/260 or 8 hours on the 4/260.

RESULTS: GRAIN AND GEOSTATISTICS

The fact that topographic elevation is a regionalized variable (Olea, 1977) leads us to believe that the methods of geostatistics (e.g., Oliver & Webster, 1986) apply directly to the problem of topographic grain. Accordingly, we are experimenting with the measurement of grain from autocorrelograms and variograms. In the first phase of this work published variograms are used to test our main hypothesis: that the relief/sample-size function yielding topographic grain is similar to the variogram of elevation for the same area. A variogram is a plot of squared differences between paired observations (Y), averaged by distance bins, against distance between those observations (X). We think the geostatistical parameters termed "range" and "sill" on elevation variograms (Olea, 1977) are equivalent to grain and relief at grain (Figure 7) and that both methods describe the same attribute of topography, spatial autocovariance of elevation.

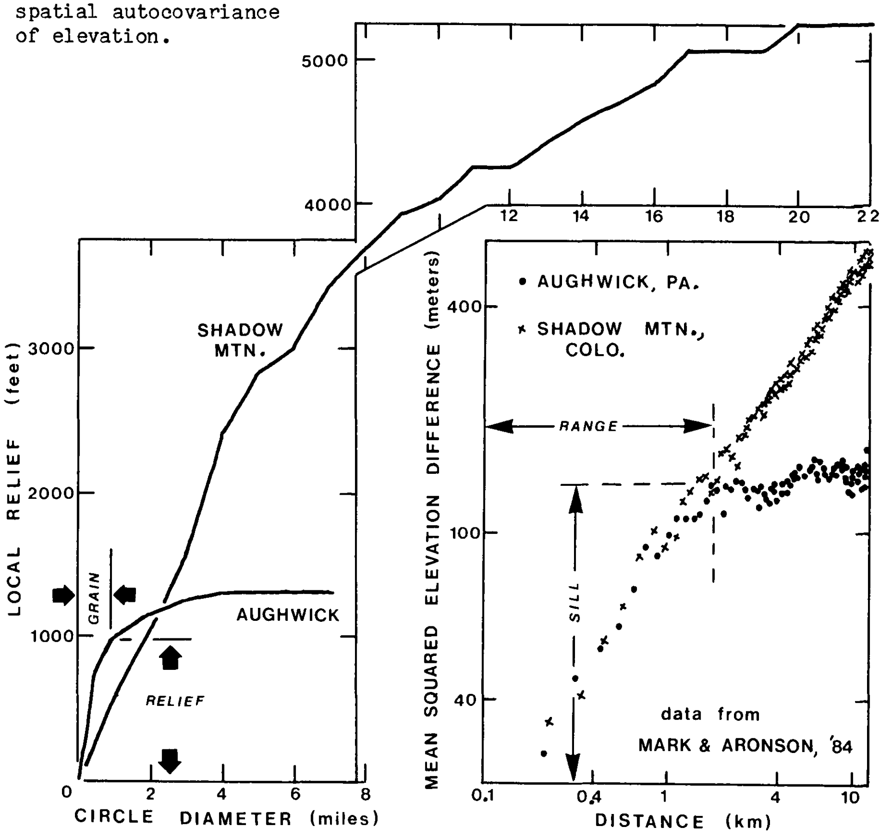


Figure 7. Correspondence of grain and relief at grain to their geostatistical equivalents "range" and "sill" near Aughwick, Pennsylvania (see Figure 2). Manual relief/diameter curves from 1:250,000-scale maps (left); automated variograms (right) from 1:24,000-scale DEMs (Mark & Aronson, 1984) for same areas. Grain of Colorado sample is not reached until about 10 miles (16 km).

Variograms computed from 1:24,000-scale DEMs (Figure 1 of Mark & Aronson, 1984) are consistent with relief/distance functions of the same areas on 1:250,000-scale contour maps (Figure 7). We manually measured grain (1-mile-circle increment) for Mark & Aronson's Aughwick Pennsylvania and Shadow Mountain Colorado samples, the only variograms showing data points. Both the Aughwick grain curve and its variogram inflect at a distance of one mile ± 0.5 mi. The variogram for Shadow Mountain does not inflect, but that is only because its computation stopped at a distance of 12.5 km (7.8 mi.) and the Knick, which is not crisply defined in this area, does not occur until at least a 9- to 11-mile-circle diameter (Figure 7). These comparisons suggest that grain values might be obtained from topographic variograms, and perhaps more objectively than is possible by the traditional procedures.

W.F. Wood long ago contended that autocorrelation ultimately would be the best way to estimate topographic grain (personal communication, 1964). We believe that our results, however preliminary, confirm his view. Perhaps more important, formal geostatistics may supply a much-needed theoretical basis for the relief/grain concept.

CONCLUSIONS

Topographic grain, a threshold phenomenon of spatial autocorrelation, measures the areal dominance of terrain by its characteristic local relief. Grain and relief at grain can be computed automatically from DEMs and mapped regionally. Automated grain values agree with those derived manually under similar conditions. Subjectivity in selecting the Knick is reduced by plotting relief against sample area instead of diameter and by least-squares partitioning of the relief/sample-size function. Additionally, standard deviation of elevation and the raw relief/sample-size function yield crisper Knicke, respectively, than relief (elevation range) and the first derivative of the curve. Lastly, although variograms may replace relief/distance functions for estimating grain and relief at grain, much further work remains before these and other issues attending topographic grain, to say nothing of its significance for landscape evolution, are understood and solved.

ACKNOWLEDGMENTS

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